**HW #1** (You do NOT turn in these problems! Complete the “online” problems)

<< Chapter 1 >>

- These are good sample problems of the class material that has been covered. These will also help you prepare for the online homework and upcoming test problems.

#1 The following table shows the preference schedule for an election with four candidates (A, B, C, and D). Use the plurality method to
  (a) find the winner of the election.
  (b) find the complete ranking of the candidates.

<table>
<thead>
<tr>
<th>Number of voters</th>
<th>29</th>
<th>21</th>
<th>18</th>
<th>10</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>D</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>2nd</td>
<td>C</td>
<td>C</td>
<td>A</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>3rd</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>A</td>
<td>D</td>
</tr>
<tr>
<td>4th</td>
<td>B</td>
<td>D</td>
<td>D</td>
<td>D</td>
<td>A</td>
</tr>
</tbody>
</table>

#2 The following table shows the preference schedule for an election with four candidates (A, B, C, and D). Use the plurality method to
  (a) find the winner of the election.
  (b) find the complete ranking of the candidates.

<table>
<thead>
<tr>
<th>Number of voters</th>
<th>6</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>A</td>
<td>B</td>
<td>B</td>
<td>D</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>2nd</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>A</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>3rd</td>
<td>D</td>
<td>A</td>
<td>D</td>
<td>C</td>
<td>D</td>
<td>C</td>
</tr>
<tr>
<td>4th</td>
<td>B</td>
<td>D</td>
<td>A</td>
<td>B</td>
<td>B</td>
<td>D</td>
</tr>
</tbody>
</table>

#3 (Use the table of #2). The table shows the preference schedule for an election with four candidates (A, B, C, and D). Use the Borda count method to
  (a) find the winner of the election.
  (b) find the complete ranking of the candidates.

#4 (Use the table of #2). The table shows the preference schedule for an election with four candidates (A, B, C, and D). Use the plurality-with-elimination method to
  (a) find the winner of the election.
  (b) find the complete ranking of the candidates.

#5 Use the following table to illustrate why the Borda count method violates the Condorcet criterion.

<table>
<thead>
<tr>
<th>Number of voters</th>
<th>6</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>2nd</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>3rd</td>
<td>C</td>
<td>D</td>
<td>B</td>
</tr>
<tr>
<td>4th</td>
<td>D</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

*
HW #2
<< Chapter 1 >>

#1 (Use the table of #2 of HW#1). The table shows the preference schedule for an election with four candidates (A, B, C, and D). Use the method of pairwise comparisons to
(a) find the winner of the election.
(b) find the complete ranking of the candidates.

#2 An election with five candidates (A, B, C, D, and E) is decided using the method of pairwise comparisons. If B loses two pairwise comparisons, C loses one, D loses one and ties one, and E loses two and ties one,
(a) find how many pairwise comparisons A loses.
(b) find the winner of the election.

#3 An election with six candidates (A, B, C, D, E, and F) is decided using the method of pairwise comparisons. If A loses four pairwise comparisons, B and C lose three, D loses one and ties one, and E loses two and ties one,
(a) find how many pairwise comparisons F loses. [Hint: First compute the total number of pairwise comparisons for six candidates.]
(b) find the winner of the election.

#4 Use the following table.
(a) find the total number of pairwise comparisons.
(b) find the winner by the method of pairwise comparisons.
(c) find the complete ranking of the candidates.

<table>
<thead>
<tr>
<th>Number of voters</th>
<th>6</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>2nd</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>3rd</td>
<td>C</td>
<td>D</td>
<td>B</td>
</tr>
<tr>
<td>4th</td>
<td>D</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

HW #3
<< Chapter 2 >>

#1 Consider the weighted voting system [q: 10, 6, 5, 4, 2].
(a) What is the smallest value that the quota q can take?
(b) What is the largest value that the quota q can take?
(c) What is the value of the quota q if at least two-thirds of the votes are required to pass a motion?
(d) What is the value of the quota q if more than two-thirds of the votes are required to pass a motion?

#2 Consider the weighted voting system [q: 7, 5, 3]. Find the smallest value of q for which
(a) all three players have veto power.
(b) P_2 has veto power but P_3 does not.

#3 Consider the weighted voting system [q: 10, 8, 6, 4, 2]. Find the smallest value of q for which
(a) all five players have veto power.
(b) P_3 has veto power but P_4 does not.

#4 Consider the weighted voting system [10: 6, 5, 4, 2].
(a) Which players are critical in the winning coalition \( \{P_1, P_2, P_4\} \)?
(b) Write down all winning coalitions.
(c) Find the Banzhaf power distribution of this weighted voting system.

#5 In a weighted voting system with four players the winning coalitions are: \( \{P_1, P_2, P_3\} \), \( \{P_1, P_2, P_4\} \), \( \{P_1, P_3, P_4\} \) and \( \{P_1, P_2, P_3, P_4\} \).
(a) Which players are critical in the winning coalition \( \{P_1, P_2, P_4\} \)?
(b) Mark all the critical players in all of the winning coalitions.
(c) Find the Banzhaf power distribution of this weighted voting system.

HW #4

<< Chapter 2 >>

#1 Consider the weighted voting system [16: 9, 8, 7].
(a) Write down all the sequential coalitions, and in each sequential coalition identify the pivotal player.
(b) Find the Shapley-Shubik power distribution of this weighted voting system.

#2 In a weighted voting system with three players the winning coalitions are: \( \{P_1, P_2\} \) and \( \{P_1, P_2, P_3\} \).
(a) List the sequential coalitions and identify the pivotal player in each sequential coalition.
(b) Find the Shapley-Shubik power distribution of the weighted voting system.

#3 In a weighted voting system with four players the winning coalitions are: \( \{P_1, P_2, P_3\} \), \( \{P_1, P_2, P_4\} \), \( \{P_1, P_3, P_4\} \) and \( \{P_1, P_2, P_3, P_4\} \).
(a) Which player is pivotal in the winning coalition \( \{P_1, P_2, P_3, P_4\} \)?
(b) List all of the sequential coalitions and identify the pivotal players.
(c) Find the Shapley-Shubik power distribution of this weighted voting system.

#4 Let A be a set with 10 elements.
(a) Find the number of subsets of A.
(b) Find the number of subsets of A having one or more elements.
(c) Find the number of subsets of A having exactly one element.
(d) Find the number of subsets of A having two or more elements. [Hint: Use the answers to parts (b) and (c).]

#5 (a) Given that 20! = 2,432,902,008,176,640,000, find 19!
(b) Find \( \frac{20!}{19!} \).
(c) Find \( \frac{20!}{199!} \).
(d) Find \( \frac{1!}{8!} \).

HW #5

<< Chapter 3 >>

#1 The Placerville General Hospital has a nursing staff of 225 nurses working in four shifts: A (7:00 a.m. to 1:00 p.m.), B (1:00 p.m. to 7:00 p.m.), C (7:00 p.m. to 1:00 a.m.), and D (1:00 a.m. to 7:00 a.m.). The number of nurses apportioned to each shift is based on the average number of patients treated in that shift, shown in the table below.

<table>
<thead>
<tr>
<th>Shift</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Total</th>
</tr>
</thead>
</table>

\[ \text{Note: } A + B + C + D = 225. \]
Patients

<table>
<thead>
<tr>
<th></th>
<th>871</th>
<th>1,029</th>
<th>610</th>
<th>190</th>
<th>2,700</th>
</tr>
</thead>
</table>

(a) Find the standard divisor.
(b) Explain what the standard divisor represents in this problem.
(c) Find the standard quotas.

#2 Tasmania State University is made up of five different schools: Agriculture, Business, Education, Humanities, and Science (A, B, E, H, and S for short). The total number of students at TSU is 12,500. The faculty positions at TSU are apportioned to the various schools based on the schools' respective enrollments. The following table shows each school's standard quota.

<table>
<thead>
<tr>
<th>School</th>
<th>A</th>
<th>B</th>
<th>E</th>
<th>H</th>
<th>S</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard quota</td>
<td>32.92</td>
<td>15.24</td>
<td>41.62</td>
<td>21.32</td>
<td>138.90</td>
<td>250</td>
</tr>
</tbody>
</table>

(a) Find the number of faculty positions at TSU.
(b) Find the standard divisor.
(c) Find the number of students enrolled in each school.

#3 Plainville Hospital has three wings (A, B, and C). The nurses in the hospital are assigned to the three wings based on the number of beds in each wing, shown in the following table.

<table>
<thead>
<tr>
<th>Wing</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of beds</td>
<td>154</td>
<td>66</td>
<td>30</td>
<td>250</td>
</tr>
</tbody>
</table>

(a) Suppose the total number of nurses working at the hospital is 20. Use Hamilton's method to apportion the nurses to the wings based on the table.
(b) Suppose an additional nurse is hired at the hospital, bringing the total number of nurses to 21. Use Hamilton's method to apportion the nurses to the wings based on the table.
(c) Compare your answers in (a) and (b). What is strange about the two apportionments?

#4 A small country consists of five states: A, B, C, D, and E. The standard quotas for each state are given in the following table.

<table>
<thead>
<tr>
<th>State</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard quota</td>
<td>25.496</td>
<td>14.491</td>
<td>8.486</td>
<td>30.449</td>
<td>21.078</td>
<td>100</td>
</tr>
</tbody>
</table>

(a) Find the number of seats being apportioned.
(b) Find the apportionment under the Huntington-Hill method.

#5. Round each number using the Huntington-Hill rounding rules. (Hint: $\sqrt{8 \times 9} = \sqrt{72} \approx 8.48528$)

(a) 8.5  (b) 8.4  (c) 8.49  (d) 8.483  (e) 8.486

* HW #6

<< Chapter 4 >>

#1 Four players (Carlos, Sonya, Tanner, and Wen) are sharing a cake. Suppose that the cake had previously been divided into four slices ($s_1$, $s_2$, $s_3$, and $s_4$). The following table shows the values of the slices in the eyes of each player.

<table>
<thead>
<tr>
<th></th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$s_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carlos</td>
<td>$3.00$</td>
<td>$5.00$</td>
<td>$5.00$</td>
<td>$3.00$</td>
</tr>
<tr>
<td>Sonya</td>
<td>$4.50$</td>
<td>$3.50$</td>
<td>$4.50$</td>
<td>$5.50$</td>
</tr>
<tr>
<td>Tanner</td>
<td>$4.25$</td>
<td>$4.50$</td>
<td>$3.50$</td>
<td>$3.75$</td>
</tr>
<tr>
<td>Wen</td>
<td>$5.50$</td>
<td>$4.00$</td>
<td>$4.50$</td>
<td>$6.00$</td>
</tr>
</tbody>
</table>

(a) Which of the four slices are fair shares to Carlos?
(b) Which of the four slices are fair shares to Sonya?
(c) Which of the four slices are fair shares to Tanner?
(d) Which of the four slices are fair shares to Wen?
(e) Find all possible fair divisions of the cake using $s_1$, $s_2$, $s_3$, and $s_4$ as shares.
#2 Four players (Allen, Brady, Cody, and Diane) are sharing a cake valued at $20 using the lone-divider method. The divider divides the cake into four slices \( s_1, s_2, s_3, \) and \( s_4 \). The following table shows the values of the slices in the eyes of each player.

<table>
<thead>
<tr>
<th></th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( s_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allen</td>
<td>$4.00</td>
<td>$5.00</td>
<td>$4.00</td>
<td>$7.00</td>
</tr>
<tr>
<td>Brady</td>
<td>$6.00</td>
<td>$6.50</td>
<td>$4.00</td>
<td>$3.50</td>
</tr>
<tr>
<td>Cody</td>
<td>$5.00</td>
<td>$5.00</td>
<td>$5.00</td>
<td>$5.00</td>
</tr>
<tr>
<td>Diane</td>
<td>$7.00</td>
<td>$4.50</td>
<td>$4.00</td>
<td>$4.50</td>
</tr>
</tbody>
</table>

(a) Who was the divider?  
(b) Find a fair division of the cake.

#3 Three players (Ana, Belle, and Chloe) are dividing four pieces of furniture using the method of sealed bids. The following table shows the players' bids on each of the items.

<table>
<thead>
<tr>
<th></th>
<th>Ana</th>
<th>Belle</th>
<th>Chloe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dresser</td>
<td>$150</td>
<td>$300</td>
<td>$275</td>
</tr>
<tr>
<td>Desk</td>
<td>$180</td>
<td>$150</td>
<td>$165</td>
</tr>
<tr>
<td>Vanity</td>
<td>$170</td>
<td>$200</td>
<td>$260</td>
</tr>
<tr>
<td>Tapestry</td>
<td>$400</td>
<td>$250</td>
<td>$500</td>
</tr>
</tbody>
</table>

(a) Find the value of each player's fair share.  
(b) Describe the first settlement (who gets which item and how much do they pay or get in cash).  
(c) Find the surplus after the first settlement is over.  
(d) Describe the final settlement (who gets which item and how much do they pay or get in cash).

#4 Three players (A, B, and C) are dividing the array of 13 items as shown below using the method of markers. The players' bids are indicated in the figure.

(a) Which items go to A?  
(b) Which items go to B?  
(c) Which items go to C?  
(d) Which items are left over?

HW #7

<<< Chapter 5 >>>

#1 For the graph shown below,

(a) give the vertex set (i.e., write all the vertices).  
(b) give the edge list (i.e., write all the edges).  
(c) give the degree of each vertex.  
(d) draw a version of the graph without crossing points.
#2 Consider the graph with vertex set \{A, B, C, X, Y, Z\} and edge list \{AX, AY, AZ, BB, CX, CY, CZ, and YY\}.
(a) List all the vertices adjacent to \(Y\).
(b) List all the edges adjacent to \(AY\).
(c) Find the degree of \(Y\).
(d) Find the sum of the degrees of the vertices.

#3 Consider the graph shown below.

(a) Find all circuits of length 1.
(b) Find all circuits of length 2.
(c) Find all circuits of length 3.
(d) Find all circuits of length 4.
(e) Find all circuits of length 5.
(f) What is the total number of circuits in the graph?

#4 Use the graph shown below

(a) Does it have an Euler circuit? Why or why not?
(b) Does it have an Euler path? Why or why not?
(c) Eulerize the graph.

#5 Consider the following graph.

(a) Does it have an Euler circuit? Why or why not? If yes, find it by labeling the edges 1, 2, 3, and so on in the order in which they are traveled.
(b) Does it have an Euler path? Why or why not?

HW #8
<< Chapter 5 >>

#1
(a) Give an example of a connected graph with eight vertices such that each vertex has degree 3.
(b) Give an example of a disconnected graph with eight vertices such that each vertex has degree 3.
(c) Give an example of a graph with eight vertices such that each vertex has degree 1.
#2 For (a), (b), and (c), choose from one of the following answers and provide a short explanation for your answer.

(A) The graph has an Euler circuit.
(B) The graph has an Euler path.
(C) The graph has neither an Euler circuit nor an Euler path.
(D) The graph may or may not have an Euler circuit.
(E) The graph may or may not have an Euler path.

(a) Figure (a)
(b) Figure (b)
(c) A graph with six vertices, all of degree 2.

---

#3 This exercise comes to you courtesy of Euler himself. Here is the question in Euler's own words, accompanied by the diagram shown below.

Let us take an example of two islands with four rivers forming the surrounding water. There are fifteen bridges marked a, b, c, d, etc., across the water around the islands and the adjoining rivers. The question is whether a journey can be arranged that will pass over all the bridges but not over any of them more than once.

What is the answer to Euler's question? If the “journey” is possible, find it. If it isn’t, explain why not.

*Hint:* Here is a better diagram of the figure.

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#4 For the graph shown below,
(a) find three different Hamilton circuits.
(b) find a Hamilton path that starts at A and ends at B.
(c) find a Hamilton path that starts at D and ends at F.

#5 For the weighted graph shown below,

(a) find a Hamilton path that starts at A and ends at C, and give its weight.
(b) find a second Hamilton path that starts at A and ends at C, and give its weight.
(c) find the optimal (i.e., least weight) Hamilton path that starts at A and ends at C, and give its weight.

#6 For the weighted graph shown below,

(a) find a Hamilton path that starts at B and ends at D, and give its weight.
(b) find a second Hamilton path that starts at B and ends at D, and give its weight.
(c) find the optimal (i.e., least weight) Hamilton path that starts at B and ends at D, and give its weight.

*HW #9*

<< Chapter 5 >>

#1
(a) How many edges are there in $K_{20}$?
(b) How many edges are there in $K_{21}$?
(c) If the number of edges in $K_{50}$ is $x$ and the number of edges in $K_{51}$ is $y$, what is the value of $y - x$?

#2 In each case, find the value of $N$.
(a) $K_n$ has 120 distinct Hamilton circuits.
(b) $K_n$ has 45 edges.
(c) $K_m$ has 20,100 edges.

#3 Find an optimal tour for the TSP shown in figure, and give its cost. [Hint: Do exhaustive search.]
#4 A space mission is scheduled to visit the moons Callisto (C), Ganymede (G), Io (I), Mimas (M), and Titan (T) to collect rock samples at each and then return to Earth €. The travel times (in years) are shown in the weighted graph below. *(Note: This is the interplanetary TSP.)*

(a) Find the nearest-neighbor tour with starting vertex E. Give the total travel time of this tour.

(b) Find the nearest-neighbor tour with starting vertex T. Write the tour as it would be traveled by an expedition starting and ending at E. Give the total travel time of this tour.

HW #10

<< Chapter 5 >>

#1 Consider the network shown in figure.

(a) How many degrees of separation are there between C and E?
(b) How many degrees of separation are there between A and E?
(c) How many degrees of separation are there between A and H?

#2 Consider the network shown in figure.

(a) Find a spanning tree of the network.
(b) Calculate the redundancy of the network.
(c) What is the largest degree of separation between a pair of vertices in the network?

#3 Consider the network shown in figure.
(a) Find a spanning tree of the network.
(b) Calculate the redundancy of the network.
(c) What is the largest degree of separation between a pair of vertices in the network?

#4 Find an MST of the network shown below using Kruskal’s algorithm, and give its weight.

#5 Find an MST of the network shown below using Kruskal’s algorithm, and give its weight.

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**HW #11**

1. Evaluate each of the following.
   (a) \( \log(100) \)  (b) \( \log(10000) \)  (c) \( \log(0.01) \)  (d) \( \log(1) \)  (e) \( \log(300) \)

2. Solve for \( x \) in each of the following.
   (a) \( 10^x = 100 \)  (b) \( 10^x = 10000 \)  (c) \( 10^x = 0.01 \)  (d) \( 10^x = 1 \)  (e) \( 10^x = 300 \)

3. Solve for \( x \) in each of the following.
   (a) \( 2(10^x) = 8 \)  (b) \( 3(10^x) = 10000 \)

4. A population of beetles is growing according to a linear growth model. The initial population (week 0) was \( P_0 = 3 \), and the population after 8 weeks is \( P_8 = 67 \).
   (a) Write a recursive formula for the number of beetles in week \( n \).
   (b) Write an explicit formula for the number of beetles in week \( n \).
   (c) Calculate \( P_{20} \).
   (d) How many weeks will it take for the beetle population to reach 187?
#5. A bacteria culture is started with 300 bacteria. After 4 hours, the population has grown to 500 bacteria. If the population grows exponentially,

(a) write a recursive formula for the number of bacteria.

(b) write an explicit formula for the number of bacteria.

(c) if this trend continues, how many bacteria will there be in 1 day?

(d) how long does it take for the culture to triple in size?

HW #12

#1 Suppose you borrow $1,250 for a term of three years at simple interest and 5.1% APR. How much is the total (principal plus interest) you must pay back on the loan?

#2 Suppose you purchase a 15-year U.S. savings bond with an APR of 4%. The face value of the bond is $8,000. Find the purchase price of the bond.

#3 Find the future value of an investment of $P=1,237.50 compounded annually with a 8.25% APR for a term of

(a) three years.

(b) four and a half years.

#4 Consider a CD paying a 3% APR compounded daily.

(a) Find the periodic interest rate. Leave your answer in fractional form.

(b) Find the future value of the CD if you invest $1,580 for a term of three years.

#5 Suppose you purchase a car and you are going to finance $18,700 for 60 months at an APR of 4.8% compounded monthly. Find the monthly payments on the loan.

HW #13

#1 Suppose you purchase a car and you are going to finance $14,500 for 48 months at an APR of 6% compounded monthly. Find the monthly payments on the loan.

#2 Lynn bought $300,000 house, paying 10% down, and financing the rest at 6% interest for 30 years.

(a) Find her monthly payments.

(b) How much interest will she pay over the life of the loan?

#3 You can afford a $700 per month mortgage payment. You've found a 30 year loan at 5% interest.

(a) How big of a loan can you afford?

(b) How much total money will you pay the loan company?

(c) How much of the money is interest?

#4 Marie can afford a $250 per month car payment. She has found a 5 year loan at 7% interest.

(a) How expensive of a car can she afford?

(b) How much total money will she pay the loan company?

(c) How much of that money is interest?

#5 Suppose you want to buy a $200,000 house. You plan to pay 10% as a down payment, and take out a 30 year loan for the rest.

(a) How much is the loan amount going to be?

(b) What will your monthly payment be if the interest rate is 5%?

(c) What will your monthly payment be if the interest rate is 6%?
Math103

* Use your own blank paper and write in a similar format in case you have to miss a class.

#1. Your attendance points will be taken off unless this form is given to your group leader in advance (or after the fact with a doctor’s or someone’s signed note).

#2. You will get 4 points for turning in this form ahead of absence, or 3 points for turning in belatedly (but within 2 weeks after absence).

Section: ________________  Name: ________________________________

I have (had) to miss Math103 class on ______________________________ because __________

______________________________________________________________

______________________________________________________________

Date: ___________________________  Signature: ______________________