Test #2 (Keys)

#1. Burn time $X \sim$ normal with $\mu = 18$ and $\sigma = 1.2$

(a) $P(X < 15.6) = P\left(Z < \frac{15.6 - 18}{1.2}\right) = P(Z < -2) = 0.0228$

(b) $200 \times 0.0228 = 4.56$, i.e., about 5

(c) From part (a), we got probability 0.9772. So the answer = $(0.9772)^4 = 0.9119$

#2. Height $Y \sim$ normal with $\mu = 68$ and $\sigma = 2.5$

The z-value that corresponds to the 95th percentile is 1.645.

So, solve for X from $1.645 = \frac{X - 68}{2.5}$. Y=68+(1.645)(2.5)=72.11

#3. Weight $Y \sim$ normal with $\mu = 11.6$ and $\sigma = 0.2$

(a) $P(Y < 11.5) = P\left(Z < \frac{11.5 - 11.6}{0.2}\right) = P(Z < -0.5) = 0.3085$

(b) $P(\overline{Y} < 11.5) = P\left(Z < \frac{11.5 - 11.6}{0.2/\sqrt{50}}\right) = P(Z < -3.54)$ =almost 0

#4. (a) SE of the mean = $\frac{s}{\sqrt{n}} = \frac{21.88}{\sqrt{10}} = 6.92$

(b) 95% c.i. on the mean = $\overline{X} \pm \left(t_{0.025}^{df=9}\right)\left(\frac{s}{\sqrt{n}}\right) = 167.5 \pm (2.262)(6.92) = 167.5 \pm 15.65 = (151.85, 183.15)$

(c) No (because the c.i. includes 160)

#5. (a) Data are from a normal distribution.

(b) From (b) of #4, ME = 15.65

(c) 99% ME = $\left(t_{0.005}^{df=9}\right)\left(\frac{s}{\sqrt{n}}\right) = (3.25)(6.92) = 22.49$

(d) $H_0 : \mu = 160$ vs $H_1 : \mu \neq 160$ (*2-tailed test*)

Test Statistic $t = \frac{\bar{x} - 160}{s/\sqrt{n}} = \frac{167.5 - 160}{21.88/\sqrt{10}} = \frac{7.5}{6.92} = 1.08$ (*df*=9)

p-value = between 0.15 and 0.2 (1-tailed) ; between 0.3 and 0.4 (2-tailed)

Conclusion: We do not reject H_0. There is strong evidence that the mean number of growing days is not significantly different from 160. (This conclusion agrees with (c) of #4.)

#6. 95% C.I. on $\mu_1 - \mu_2$ (pooled method)

$$(\bar{x}_1 - \bar{x}_2) \pm t_{0.025}^{df=18} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = (0.4732 - 0.4692) \pm (2.101)(0.128)\sqrt{0.1 + 0.1} = (-0.116, \quad 0.124)$$

NO, **not** significantly different

♣ $s_p = \sqrt{\frac{9(0.1414^2) + 9(0.1129^2)}{10 + 10 - 2}} = 0.128$, and $df = 18$

#7. Note first "between 13.4 and 18.3" = 15.85 ± 2.45

(a) ME = 2.45

(b) Because ME = $\left(t_{0.025}^{df=41}\right)SE = 2.021 \times SE = 2.45$, Therefore SE = $\frac{2.45}{2.021} = 1.21$

(c) Because SE = $\frac{SD}{\sqrt{42}}$ = 1.21, Therefore SD = $1.21 \times \sqrt{42}$ = 7.84

(d) Using 68-95 rule, one can lose ***up to*** 15.85+(2)(7.84)=15.85+15.68=31.53 pounds