Test #2 EXTRA (Keys)

#1. Length $Y \sim$ normal with $\mu = 8$.

(a) (1) $P(\bar{Y} > 10) = P\left(Z > \dfrac{10-8}{\sigma/\sqrt{15}}\right) = P\left(Z > \dfrac{2\sqrt{15}}{\sigma}\right)$

On the other hand (2) $P(\bar{Y} > 10) = P\left(Z > \dfrac{10-8}{\sigma/\sqrt{50}}\right) = P\left(Z > \dfrac{2\sqrt{50}}{\sigma}\right)$.

So, the probability of (1) will be greater!

(b) $P(\bar{Y} > 10) = P\left(Z > \dfrac{10-8}{\sigma/\sqrt{50}}\right) = P\left(Z > \dfrac{2}{0.3}\right) = P(Z > 6.67) \approx 0$.

(c) $P(\bar{Y} < 7.5) = P\left(Z < \dfrac{7.5-8}{\sigma/\sqrt{50}}\right) = P\left(Z < \dfrac{-0.5}{0.3}\right) = P(Z < -1.67) = 0.0475$.

#2. (a) 95% C.I. on $\mu_1 - \mu_2$ (pooled method)

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = (30.94 - 30.65) \pm (2.101)(8.18)\sqrt{0.1+0.1} = (-7.4,\ \ 7.98)$$

NO, not significantly different

♣ $s_p = \sqrt{\dfrac{9(8.02^2) + 9(8.34^2)}{10+10-2}} = 8.18$, and $df = 18$

(b) Test on $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$ (*2-tailed test*)

Test Statistic $t = \dfrac{(\bar{x}_1 - \bar{x}_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \dfrac{30.94 - 30.65}{(8.18)\sqrt{0.1+0.1}} = 0.08$, *p*-value = greater than α

We do not reject H_0. There is no strong evidence that the two methods have significantly different average bacteria amounts.

#3. (a) The *z*-value that corresponds to the 70th percentile is 0.52.

So, solve for *X* from $0.52 = \dfrac{X - 125}{6.5}$. *X*=125+(0.52)(6.5)=128.38

(b) $P(\bar{X} \geq 130) = P\left(Z \geq \dfrac{130-125}{6.5/\sqrt{5}}\right) = P(Z \geq 1.72) = 0.0427$

(c) $P(X \geq 130) = P\left(Z \geq \dfrac{130-125}{6.5}\right) = P(Z \geq 0.77) = 0.2206$

(d) In part (c), we got probability is 0.2206. So the answer = $(0.2206)^3 = 0.0107$

#4. (a) $\bar{x} \pm t_{0.025}^{df=49} \dfrac{s}{\sqrt{n}} = 4.3 \pm \left(2.009 \times \dfrac{3.2}{\sqrt{50}}\right) = 4.3 \pm 0.91 = (3.39,\ 5.21)$ (b) No

#5. (a) 1-tailed
(b) We do not reject H_0. There is no significant difference between the two procedures.
(c) Type II error is possible.

#6. $\bar{X} = 82,\ s = 26$

(a) 95% c.i. on the mean = $\bar{X} \pm \left(t_{0.025}^{df=9}\right)\left(\dfrac{s}{\sqrt{n}}\right) = 82 \pm (2.262)(8.22) = 82 \pm 18.59 = (63.41, 100.59)$

(b) SE of the mean = $\dfrac{s}{\sqrt{n}} = \dfrac{26}{\sqrt{10}} = 8.22$, (c) ME = 18.59