Puzzles Make Math Less Puzzling
How'd the Bunny Vanish?

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Dedicated to

“In fact, I believe one reason I am in Mathematics today is that I began reading Gardner’s books and articles in Junior High and High School. Browse and Enjoy!”
H.E. Dudeney* (1857-1930) and Sam Loyd (1841-1911)

• These 2 greatest puzzle inventors may have worked together, or they may have stolen from each other.
• Both wrote newspaper columns.
• Dudeney was English. Loyd was American.
• Dudeney was the stronger mathematician.
• Loyd exploited and popularized his puzzles, primarily for advertising purposes.

* Not to be confused with AK Dewdney
From Dudeney’s 536 Puzzles and Curious Problems:

FINDING A SQUARE

• Here are six 7-digit numbers:

4,784,887     2,494,651     8,595,087
1,385,287     9,042,415     9,406,087

• Three of these numbers added together will form a perfect square. Without resorting to a “...no other course but laborious trial...., the answer may be found directly by very simple arithmetic and without any experimental extraction of a square root.”

• In other words, solve it without a calculator and without calculating any square roots. Note: there is only one solution.
Loyd was the master at creating geometric vanishing area (and disappearing rabbits) puzzles, one of many types of mechanical puzzle.
The most famous puzzle of them all is Sam Loyd’s “Get Off the Earth”, from 1896.

Millions were made.

Used for all kinds of advertising and campaigning (people tend to keep flyers longer if they contain something worth keeping)

Loyd offered all kinds of prizes for the best explanation, including a new bicycle. He received literally 1000s of letters.

Here’s a modern version of GOTE:
http://www.samuelloyd.com/gote/index.html
Sam Loyd Originals
In 1896, Republican candidate William McKinley was in trouble. As a means to get people to listen to his message, his campaign contacted Sam Loyd and licensed GOTE. While the back of the puzzle stated McKinley’s platform, the bigger message was the disappearing stereotyped Chinese man. American anti-Chinese prejudice was widespread; Denis Kearney’s Workingman’s Party platform played off fears that Asian immigrants take jobs from Whites. Its slogan bluntly stated “The Chinese Must Go”.

With this not-so-subtle message, the Republican Party was subliminally using Kearney’s platform. Over 10 million GOTE puzzles are said to have been distributed.

To emphasize this disgraceful bit of history, Chinese were officially banned from Humboldt County from 1885-1959.

http://users.humboldt.edu/ogayle/hist383/CentralPacific.html
One more example of how advertisers took advantage of 1890s America and its prejudices:

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For the **BEST** explanation of the principle of the puzzle on the opposite side received within one year from September 1st, 1897, from any policy-holder of the Company whose policy is in force September 1st, 1898, the Company will present a gratuity of $100 in Gold. For the **next Best**, $95. For the **third**, $90. For the **fourth**, $85; and so on to $5, making twenty gratuities in all, the **Highest $100**, the **Lowest $5**.

There must appear at the top of the sheet on which the Explanation is made, the number and date of the policy held, together with the address (street, number, city and state) of the competitor, and the name of the agent who last collected on the policy. The explanation is to be sent by mail addressed to the “Metropolitan Life Insurance Company, New York City, N.Y., Puzzle Department.”

The announcement of the successful competitors will be made in the Company’s Paper “The Metropolitan,” first issued after the competition closes.

This curious puzzle illustrates the uncertainty of life. We see a little family circle of Japs suddenly broken up, and yet cannot tell beforehand which one is to go. We can only hope that the right one was insured when the miniature earthquake occurred.

The moral is plain. The only way to make sure that the first member of a family who dies is insured, is for every member of the family to have a policy.

Every one of those policies will be realised on if they are kept in force, for, though we may go through life without sickness or accident, none can evade death. From that there is no escape and no postponement.

Life policies, payable at the death of the insured, and Endowment policies payable to the insured during his lifetime, are issued by the Metropolitan Life Insurance Co.

on both sexes and at all ages, between two and seventy. Premiums run from five cents per week, upward, and are collected from the home of the insured by the Company’s agents. Claims are paid immediately upon receipt and approval of proofs of death. The Company is now paying one every seven minutes of each business day, and is disbursing $16 a minute in doing so! It has already distributed among its policy-holders a sum which, including that now invested for their security, exceeds Eighty-five millions of dollars. Its assets amount to Thirty-four millions, and its surplus to considerably more than five millions. It has close to four millions of policies in force, and yet there is room for you, reader, if you are not insured, or if you want to increase your present insurance.

The Company has also an Intermediate Branch, in which policies for $3000 are issued, on males or females from 21 to 65; premiums payable quarterly, half-yearly or annually.

It has also an Ordinary Department, in which policies from one to twenty-five thousand dollars are issued on adult lives.

For full particulars send to the home office, or to any of the branch offices.

**BRANCH OFFICE,**
Not all advertisers exploited racism and politics.
Some versions didn’t sell anything (I think)
GOTE continues to be used for political messages.

From Esquire Magazine, 1955?

Look at the pictures at right; cut out the large square (dotted line) above. Now cut out the inner circle along the dotted line. This will leave you a square piece of paper with a hole in it. Glue this on a piece of shirt cardboard. Take the larger circle and fasten it back where it belongs with a pin, so it will swivel. Swivel this arrow to N.E. Count the Red Chinese. Now swivel to N.W. and count again. What happened to the thirteenth man? Why can't the American Government solve its recognition problems this neatly? The answer to the first question (thirteen entities may be redistilled to make twelve, but each of the twelve will be a trifle larger) plays a fundamental role in chemistry; it applies to conservation of mass in ordinary chemical reactions—no matter how two or more chemicals may combine to form other compounds, their total mass does not change noticeably. (In nuclear physics, the conversion of mass into energy accords with Einstein's equation, $e=mc^2$.) We suggest you try this one on your friends, though not if they're Red Chinese.
To Protect his rights, Loyd Patented His Invention
But it didn’t stop the countless bootleggers. Many unauthorized versions were produced.

Here’s a German one:
Here’s one from Canada:

WHERE DOES HE GO?

"SOUVENIR" STOVES & RANGES ARE THE WORLDS BEST

Buy one and all your trouble in cooking or heating will disappear as quickly as does the JOLLY TAR.

When the button is down there are thirteen Jack Tars. Count them. Study their postures. Then move the button up and count again. You will find there are only twelve. Can you tell WHICH ONE HAS VANISHED? Where does he go?

"SOUVENIR" STOVES AND RANGES
Are the World’s Best

We are the Largest Manufacturers of STOVES and RANGES in Canada.

SOLD BY LEADING DEALERS IN EVERY TOWN IN THE DOMINION OF CANADA.

THE GURNEY, TILDEN CO., LIMITED.
HAMILTON, ONT.

WESTERN AGENCY:
The GURNEY STOVE & RANGE CO., LTD.
Winnipeg, Man.

EASTERN AGENCY:
The GURNEY MASSEY CO., LTD., Montreal, Que.

The Greatest Variety of Styles & Sizes in Canada.
Political Themes are still popular:

“ONE INTERNATIONAL WAR CRIMINAL ALWAYS HAS TO LOOK OVER HIS SHOULDER. IF YOU THINK HE CAN’T BE FOUND....

O Sumwon bin Lyin

THAT JOKER GEORGE HAS CONVENIENTLY FORGOTTEN HIM. BUT IT DOESN’T TAKE A CRYSTAL BALL TO SEE THE FUTURE.
IF YOU WANT TO SEE HOW I WOULD SERVE HIM UP TO YOU, TURN THE INNER CIRCLE TO THE RIGHT AND LET THE JOKER ENTERTAIN THE DOG.” — Hillary

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PRINTED AND MANUFACTURED IN THE U.S.A.
So what’s going on? If you want to solve this yourself, I’ll understand if you leave class now. But if you want more hints, solving this one will help you solve the others!

You choose where to cut the card
Let’s take away all the fancy stuff and take another look at it in its most basic form.
Here’s another way to look at it:

Gain a pile: move 4/5, then 3/5, then 2/5, then 1/5.
Then lose a pile: move 1/4 move 2/4, then 3/4, then 4/4.
This was one puzzle I knew Jerry didn’t have. It was sent to me by a professor who attended an earlier version of this presentation (10 years ago)

Stuart,
I enjoyed your talk yesterday.
Last night I had an amusing thought about which of the six men in tophats disappears. If you print their names between the heads, then we can talk about exactly which guy goes.

J
O R D R A
E O O O L
-----------------------------------------
Y N N B L
A N E E
L I R N
D E T

See what happens?
After the shift we have Joey, Ronald, Donnie, Robert, and Allen

Just a thought. Best wishes, Allen Schwenk
Just when you might be thinking it’s starting to make sense.....
From Martin Gardner’s
MATHEMATICS, MAGIC AND MYSTERY, 1956:

©Mel Stover, 1956
If these puzzles provide insights into the important issues of the time, then what does this one by Robin DeBreuil and titled “Who Turned to Doggie Doo?” say about our current era?

http://debreuil.com/ddw/puzjava/picmove.htm
The puzzles so far have all been essentially one-dimensional, that is, objects get shorter or longer. Now, let’s move to two dimensions and explore puzzles where area appears either to vanish or appear from nowhere.
Sebastiano Serlio’s *Architettura*, 1545

“a man should finde a Table of ten foote long, and three foote broade: with this Table a man would make a doore of seven foote high, and foure foote wide...and you shall yet have (two) three cornerd pieces” (with a combined area of 3 square feet)
William Hooper, *Rational Recreations*  
1st ed. 1774, 4th ed. 1794  

Did Hooper understand the paradox or was he just plagiarizing Edme Guyot’s *Nouvelles Recreations Physiques et Mathematiques*, which had a major error in the 1st edition in 1770 and was corrected in the 2nd ed. 1775
Hooper’s Geometric Money

1st ed: \[ 3 \times 10 = 5 \times 4 + 3 \times 6 \]
\[ 30 = 38 \]

4th ed: \[ 3 \times 10 = 5 \times 4 + 2 \times 6 \]
\[ 30 = 32 \]
A French version from the 1800s
Let’s explore other cut-up squares that have been changed to rectangles:

Area = 64

Area = 65

Area = 168

Area = 169

Area = 25

Area = 24

2 observations:
• Fibonacci numbers
• area changes by ± 1
What happens when we move the pieces very carefully?

You explore before we continue with the slides.........
Add coordinates to the diagram:

(0, 5) (5, 3) (8, 2) (13, 0)

Calculate the slopes of the 4 segments:

\[ m_1 = \frac{5 - 3}{0 - 5} = -\frac{2}{5} \quad m_2 = \frac{5 - 2}{0 - 8} = -\frac{3}{8} \]
\[ m_3 = \frac{2 - 0}{8 - 13} = -\frac{2}{5} \quad m_4 = \frac{3 - 0}{5 - 13} = -\frac{3}{8} \]

What first appeared to be a diagonal of the rectangle is actually a parallelogram shaped hole!!

What’s the area of this parallelogram?
To better see what’s happening here, let’s redraw the rectangle (not to scale) and divide the parallelogram into 2 congruent triangles with sides $a$, $b$, and $c$:

![Diagram of a parallelogram divided into two congruent triangles](image)

Using the distance formula:

- $m(\overline{AB}) = \sqrt{(0 - 8)^2 + (5 - 2)^2} = \sqrt{73}$ = side $a$
- $m(\overline{AC}) = \sqrt{(0 - 13)^2 + (5 - 0)^2} = \sqrt{194}$ = side $b$
- $m(\overline{BC}) = \sqrt{(8 - 13)^2 + (2 - 0)^2} = \sqrt{29}$ = side $c$

Next use Heron’s Formula:  

$$\text{Area } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

with  

$$s = \frac{a + b + c}{2}$$
Using TI-Nspire:

\[
\text{Area} \triangle ABC = \sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)}
\]

with \( s = \frac{a+b+c}{2} \)

Define \( s = \frac{\sqrt{73} + \sqrt{194} + \sqrt{29}}{2} \)

\[
\frac{\sqrt{s \cdot (s-\sqrt{73}) \cdot (s-\sqrt{194}) \cdot (s-\sqrt{29})}}{2} = 0.5
\]

Since \( \triangle ABC \cong \triangle CDA \)

Area parallelogram ABCD = .5 + .5 = 1

Therefore we have found the extra unit of area!!
This leads to an interesting generalization that helps us to better understand both this paradox puzzle and Fibonacci numbers, too.

\[ \begin{array}{ccccccccccc}
1 & 1 & 2 & 3 & 5 & 8 & 13 & 21 & 34 & 55 & 89 & 144 & \ldots \end{array} \]

From the previous slides, we see that:

- \( 5 \times 5 = 3 \times 8 + 1 \)
- \( 8 \times 8 = 5 \times 13 - 1 \)
- \( 13 \times 13 = 8 \times 21 + 1 \)

\[ F_n^2 = F_{n-1} \times F_{n+1} \pm 1 \]

The square of any Fibonacci number is one more than or one less than the product of the two Fibonacci numbers on either side.
These vanishing area puzzles so captivated people in the late 1800’s that they, like Sam Loyd’s circular puzzles, were used in many advertising campaigns.

And a New Puzzle

Good puzzles are always in demand. Nothing gets more attention or is more certain to be passed about from hand to hand and talked about. Our “Elusive Square” Puzzle has them all guessing. Made of four pieces of cardboard printed with squares. Arranged in one way, there are sixty-four squares; arranged another way, there are sixty-five squares. The puzzle—where does the extra square come from? A puzzle that appeals to the children and to the mathematically and scientifically inclined as well. Your ad printed right on the face of the puzzle.

A sample free to any business concern.

CHARLES SCHINDLER
Manufacturer of Ad Novelties that appeal to children

Toledo, Ohio

July 1916
Randi’s Remarkable Rugs

From Martin Gardner’s GOTCHA!
Paul Curry first used this version of the paradox in the 1950’s.
6 by 13 = 78 rabbits     6 by 13 = 77 rabbits and
1 rabbit hole
Fit the Square inside the Puzzle
Each of these squares contains 8 right triangles; 4 with an area of 12 and 4 with an area of 5 sq cm. with a total area of 68 sq cm. Note also that the triangles with area of 5 are all congruent and the triangles with an area of 12 are all congruent.

The remaining polygon in the upper square has an area of 124 while the polygon in the lower square has an area of 128. Explain the discrepancy.
DISAPPEARANCES

I wonder how magicians make their rabbits disappear;
Enchanted words like “hocus pocus” can not interfere
With laws of science and facts of mathematics that are clear.
The prestidigitators, making use of devious schemes,
(although they never tell you how) transport things as in dreams:
At times suspended, banished, null and void-or so it seems.
There must be something secret, yes, a trick that will involve
when done with sleight of hand- a force that’s able to dissolve.
The Cheshire Cat family all grinned when they saw Alice.
And, as usual, one of the cats vanished quite slowly,
beginning with the tail, and ending with the grin, which
remained some time after the rest of it had gone.

Gathering for Gardner II • January 17–21, 1996 • Atlanta, Georgia
Binary Arts Corporation • Alexandria, Virginia
From MoMath (on the -1st floor)

*Simple geometrical vanishes are not hard to make.*

On a strip of paper, draw a horizontal dotted line. Then pick an object that looks like itself even if you cut some of the top or bottom off – like a skyscraper or a tree. Draw some of those objects evenly spaced across the horizontal line, but make sure that the one on the far left is completely above the line and the one on the far right is completely below the line. Now cut along the horizontal line to make two strips, and watch the objects disappear as you slide the bottom strip back and forth.
Don’t Do This At Home!

According to Martin Gardner, in 1968 a man in London was sentenced to 8 years in jail for doing this with 5 pound notes.
Teachable moment: The more times you explain a concept, the better your explanations will get.

Slocum, 1996

Look at the hint again. If I had 12 full glasses of water, and put a little bit of each one into the empty 13th glass, at the end of it all, it would LOOK as if I magically filled 13 glasses with the water from only 12 full glasses. Of course I haven’t. It’s just that the difference is too small to pick up visually. With the disappearing pirates, the principle is the same. A little bit of each of the 12 pirates is all added up to make the 13th. Of course the art is carefully drawn to hide what’s going on.

Slocum, 2011

Cover: The Disappearing Bicyclist! Notice when the arrow on the wheel points to A there are 13 boys and the first boy on the lower left is almost completely outside of the wheel. Each subsequent clockwise boy has been drawn slightly further toward the center of the wheel, until the last boy is almost all inside the wheel/tire. When the wheel section is rotated from A to B, 12 boys gain some part of their body and are \( \frac{1}{2} \) larger. So in reality no boy has actually disappeared.
The New Home for Jerry’s Puzzles
The Lilly Rare Book Library at Indiana University, Bloomington
REFERENCE LIST

Personal visit with Jerry Slocum at his home and Private Museum, Beverly Hills, CA
February 18, 2011

Personal visit to Slocum Puzzle Collection, Lilly Rare Book Library, Indiana University,
Bloomington, IN April 14, 2011.

Hooper’s Paradox with a Java Applet
http://www.cut-the-knot.org/Curriculum/Fallacies/HooperParadox.shtml
http://www.samuelloyd.com/gote/index.html
http://www.slocumpuzzles.com/index2.htm

Who Turned to Doggie Doo? animated
http://debreuil.com/ddw/puzjava/picmove.htm

Explanations for several Java animated puzzles
http://library.thinkquest.org/28049/geometrical_vanishes.htm

David Singmaster’s Annotated Bibliography on Recreational Mathematics
http://www.g4g4.com/MyCD5/SOURCES/singmaterial.htm

New York Times Obituary for Martin Gardner
http://www.nytimes.com/2010/05/24/us/24gardner.html?_r=2

Dr. Gayle Olson-Raymer’s lecture notes on Anti-Chinese policies in California
http://users.humboldt.edu/ogayle/hist383/CentralPacific.html
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