Math 240 Name: Key

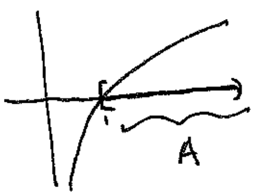
Date: 12/02/16
Quiz 5

Directions: You have 20 minutes to complete this quiz. Read each problem carefully. There are two problems on the back of this page.

1. (4 points)

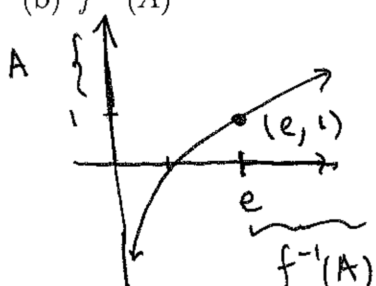
   Let $f : (0, \infty) \to \mathbb{R}$ be the function defined by $f(x) = \ln(x)$. Let $A = \{x \in \mathbb{R} \mid x \geq 1\}$. Write the following sets using interval notation.

   (a) $f(A)$



$f(A) = \boxed{[0, \infty)}$

   (b) $f^{-1}(A)$

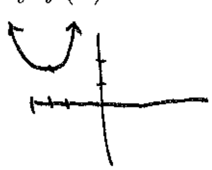


$f^{-1}(A) = \boxed{[e, \infty)}$

2. (4 points)

   Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = x^2 + 3x + 4$.

   (a) Show that f is not injective.



graph does not pass HLT so not injective.

$\cancel{f(x) = (x+1)(x+4)}$

vertex: $\left(-\frac{3}{2}, \left(-\frac{3}{2}\right)^2 + 3\left(\frac{-3}{2}\right) + 4\right) = \left(\frac{-3}{2}, \frac{9}{4} - \frac{9}{2} + 4\right) = \left(\frac{-3}{2}, \frac{7}{4}\right)$

   (b) Show that f is not surjective.

range$(f) = [\frac{7}{4}, \infty)$, so f is not surjective.

3. (1 point) $\mathbb{R}-\{0,1\}$

   Let $A = \cancel{\mathbb{R}-\{0}\}$, and let $f : A \to A$ be the function defined by $f(x) = 1 - \frac{1}{x}$. Show that $f \circ f \circ f = \mathrm{id}_A$.

$$(f \circ f)(x) = f(f(x)) = 1 - \frac{1}{1-\frac{1}{x}} = 1 - \frac{x}{x-1} = \frac{x-1-x}{x-1} = \frac{-1}{x-1}$$

$$(f \circ f \circ f)(x) = f((f \circ f)(x)) = f\left(\frac{-1}{x-1}\right) = 1 - \frac{1}{\frac{-1}{x-1}} = 1 + x - 1 = x$$

Therefore $f \circ f \circ f = \mathrm{id}_A$. //

4. (1 point)

   Let A and B be nonempty sets and let $f : A \to B$ and $g : B \to A$ be functions such that $g \circ f = \mathrm{id}_A$. Prove that f is one-to-one and g is onto.

<u>Proof that f is one-to-one</u>

Let $a_1, a_2 \in A$ and assume that $f(a_1) = f(a_2)$. Then $g(f(a_1)) = g(f(a_2))$, so since $g \circ f = \mathrm{id}_A$, $a_1 = a_2$. Therefore f is one-to-one.

<u>Proof that g is onto</u>

Let $a \in A$ be arbitrary. Then we have $(g \circ f)(a) = \mathrm{id}_A(a) = a$, so $g(f(a)) = a$. This shows that $f(a) \in B$ is a preimage of a, so g is onto.