Math 240 Name: Key

Date: 11/04/16
Quiz 4

Directions: You have 20 minutes to complete this quiz. Read each problem carefully. There are two problems on the back of this page.

1. (2 points)

   Let $A = \{a, b, c, d\}$ and let

   $$R = \{(a,a), (a,b), (a,c), (a,d), (b,b), (b,c), (b,d), (c,c), (c,d)\}$$

   be a relation on A.

   Which of the properties reflexive, symmetric and transitive does R possess? Explain your answers.

   R is not reflexive since $d \not R d$. R is not symmetric since $(a,b) \in R$ but $(b,a) \notin R$. R is transitive since whenever $(x,y) \in R$, $(y,z) \in R$, then $(x,z) \in R$ as is clear by inspection.

2. (2 points)

   Let $A = \{1, 2, 3, 4, 5, 6\}$. The relation

   $$R = \{(1,1), (1,5), (2,2), (2,3), (2,6), (3,2), (3,3), (3,6), (4,4), (5,1), (5,5), (6,2), (6,3), (6,6)\}$$

   is an equivalence relation on A. Determine the distinct equivalence classes.

   $[1] = \{1, 5\}$, $[2] = \{2, 3, 6\}$, $[4] = \{4\}$.

3. (2 points)

   Let $A = \{1, 2, 3, 4\}$. Give an example of a nonempty relation on A that is transitive, but neither reflexive nor symmetric.

   $R = \{(1,2), (2,3), (1,3)\}$, for example.

4. (2 points)

   Construct the multiplication table of $\mathbb{Z}_4$.

$\odot$	[0]	[1]	[2]	[3]
[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]
[2]	[0]	[2]	[0]	[2]
[3]	[0]	[3]	[2]	[1]

5. (2 points)

   Define a relation S on the set of all real numbers $\mathbb{R}$ as follows: for $x, y \in \mathbb{R}$,

$$xSy \text{ if } x - y \in \mathbb{Z}.$$

Prove that S is an equivalence relation on $\mathbb{R}$. Describe the equivalence class of $\sqrt{2}$ by listing the elements of this set.

Proof: Let $x, y, z \in \mathbb{R}$.

First of all xSx since $x - x = 0$ and $0 \in \mathbb{Z}$.

Second, assume that xSy. Then $x-y \in \mathbb{Z}$. So $y-x \in \mathbb{Z}$. So ySx.

Finally, suppose xSy and ySz. Then $x-y \in \mathbb{Z}$ and $y-z \in \mathbb{Z}$. Hence $x-z = x-y+y-z \in \mathbb{Z}$, so xSz. ∎

$[\sqrt{2}] = \{\sqrt{2}+n \mid n \in \mathbb{Z}\}$

$= \{\ldots, \sqrt{2}-1, \sqrt{2}, \sqrt{2}+1, \sqrt{2}+2, \ldots\}$.