Math 240 Name: Key

Date: 09/30/16
Quiz 3

Directions: You have 15 minutes to complete this quiz. Read each problem carefully. There are two problems on the back of this page.

1. (2 points) Let $a, b \in \mathbb{Z}$, $a \neq 0$. State the definition of the phrase:

"a divides b".

What is the notation we use for this phrase?

"a divides b" means $b = ak$ for some integer k.

We use the notation $a \mid b$.

2. (3 points) Let $a, b, m \in \mathbb{Z}$, $m \geq 2$.

(a) State the definition of the phrase:

"a is congruent to b modulo m".

What is the notation we use for this phrase?

"a is congruent to b modulo m" means $m \mid a-b$.

We write $a \equiv b \pmod{m}$.

(b) Give an example of two integers that are congruent modulo 3.

$4 \equiv 7 \pmod{3}$ since $3 \mid (4-7)$

(c) Give an example of two integers that are <u>not</u> congruent modulo 4.

$1 \not\equiv -6 \pmod{4}$ since $4 \nmid (1-(-6))$.

3. (3 points) Let $x, y \in \mathbb{Z}$.

   Prove: If $5|x$ and $5|x+y$, then $5|y$.

Proof: Assume that $5 | x$ and $5 | y+x$. Then $x = 5k$ and $x+y = 5\ell$ for some $k, \ell \in \mathbb{Z}$. Therefore

$$5\ell = x+y = 5k + y,$$

So $y = 5\ell - 5k = 5(\ell - k)$. Since $\ell - k \in \mathbb{Z}$, we see that $5|y$, as desired. $\blacksquare$

4. (2 points) Let A, B and C be sets.

   Prove: $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$

Proof Let $z \in A \times (B \cap C)$. Then z is an ordered pair and we write $z = (a, d)$, where $a \in A$ and $d \in B \cap C$. Then $d \in B$ and $d \in C$, so we know that

$$(a,d) \in A \times B \quad \text{and} \quad (a,d) \in A \times C.$$

Hence $(a,d) \in (A \times B) \cap (A \times C)$.

We conclude that $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$. $\blacksquare$