Math 240

Name: Key

Date: 09/30/16
Quiz 3

Directions: You have 15 minutes to complete this quiz. Read each problem carefully. There are two problems on the back of this page.

1. (2 points) Let \(a, b \in \mathbb{Z}, a \neq 0\). State the definition of the phrase:

\[\text{"a divides } b\text{".}\]

What is the notation we use for this phrase?

\[\text{"a divides } b\text{" means } b = ak \text{ for some integer } k.\]

We use the notation \(a \mid b\).

2. (3 points) Let \(a, b, m \in \mathbb{Z}, m \geq 2\).

(a) State the definition of the phrase:

\[\text{"a is congruent to } b\text{ modulo } m\".}\]

What is the notation we use for this phrase?

\[\text{"a is congruent to } b\text{ modulo } m\" \text{ means } m \mid (a-b).\]

We write \(a \equiv b \pmod{m}\).

(b) Give an example of two integers that are congruent modulo 3.

\[4 \equiv 7 \pmod{3} \text{ since } 3 \mid (4-7)\]

(c) Give an example of two integers that are not congruent modulo 4.

\[1 \not\equiv -6 \pmod{4} \text{ since } 4 \nmid (1-(-6)).\]
3. (3 points) Let \( x, y \in \mathbb{Z} \).

Prove: If \( 5 \mid x \) and \( 5 \mid x + y \), then \( 5 \mid y \).

**Proof:** Assume that \( 5 \mid x \) and \( 5 \mid x + y \). Then \( x = 5k \) and \( x + y = 5l \) for some \( k, l \in \mathbb{Z} \). Therefore

\[
x + y = 5k + y,
\]

so \( y = 5l - 5k = 5(l-k) \). Since \( l-k \in \mathbb{Z} \), we see that \( 5 \mid y \), as desired. \( \Box \)

4. (2 points) Let \( A, B \) and \( C \) be sets.

Prove: \( A \times (B \cap C) \subseteq (A \times B) \cap (A \times C) \)

**Proof:** Let \( z \in A \times (B \cap C) \). Then \( z \) is an ordered pair and we write \( z = (a, d) \), where \( a \in A \) and \( d \in B \cap C \). Then \( d \in B \) and \( d \in C \), so we know that

\[
(a, d) \in A \times B \quad \text{and} \quad (a, d) \in A \times C.
\]

Hence \( (a, d) \in (A \times B) \cap (A \times C) \).

We conclude that \( A \times (B \cap C) \subseteq (A \times B) \cap (A \times C) \). \( \Box \)