Math 240 Name: Key

Date: 09/16/16
Quiz 2

Directions: You have 15 minutes to complete this quiz. Read each problem carefully. There is one problem on the back of this page.

1. (3 points) Give a direct proof of the following statement.

   Let $n \in \mathbb{Z}$. If n is even, then $(n+1)^2 - 1$ is even.

Proof: Assume that $n \in \mathbb{Z}$ is even. Then $n = 2k$ for some $k \in \mathbb{Z}$.

Therefore $(n+1)^2 - 1 = (2k+1)^2 - 1 = 4k^2 + 4k = 2(2k^2 + 2k)$.

Since $2k^2 + 2k \in \mathbb{Z}$ we know that $(n+1)^2 - 1$ is even. $\boxtimes$

2. (3 points) Give a proof by contrapositive of the following statement.

   Let $n \in \mathbb{Z}$. If $(n+1)^2 - 1$ is even, then n is even.

Proof: We start by assuming n is odd. Then $n = 2k+1$, for some $k \in \mathbb{Z}$. Therefore

$$(n+1)^2 - 1 = (2k+1+1)^2 - 1 = 4k^2 + 8k + 4 - 1 = 2(2k^2 + 4k + 1) + 1.$$

Since $2k^2 + 4k + 1 \in \mathbb{Z}$ we know that $(n+1)^2 - 1$ is odd, as desired. $\boxtimes$

3. (4 points) Prove that if $n \in \mathbb{Z}$, then $n^2 + 5n + 20$ is even.

Proof let $n \in \mathbb{Z}$. We use a case analysis.

Case 1 n is even

Then $n = 2k$ for some $k \in \mathbb{Z}$. So

$$(n^2 + 5n + 20) = (2k)^2 + 5(2k) + 20 = 4k^2 + 10k + 20$$

$$= 2(2k^2 + 5k + 10).$$

Since $2k^2 + 5k + 10 \in \mathbb{Z}$ we know that $n^2 + 5n + 20$ is even.

Case 2 n is odd

Then $n = 2l + 1$ for some integer l. So

$$n^2 + 5n + 20 = (2l+1)^2 + 5(2l+1) + 20$$

$$= 4l^2 + 4l + 1 + 10l + 5 + 20$$

$$= 2(2l^2 + 7l + 13).$$

Since $2l^2 + 7l + 13 \in \mathbb{Z}$ we know that $n^2 + 5n + 20$ is even. ∎