Section 2.1: Cartesian Coordinates and Distance

Lecture 1
Announcements

• Calculator rental opportunity
  – Supplies are very limited
  – You’ll need to visit the Mathematics Department office (BSS 320) with $20 cash if you want a chance!

• The syllabus and timeline are on our course website: http://users.humboldt.edu/pgoetz/115.htm. Please go there, print copies of the syllabus and timeline, and put them in a convenient place for reference.
The Rectangular (Cartesian) Coordinate System
Points in the Cartesian coordinate system
The Distance Formula

The distance between two points $(x_1, y_1)$ and $(x_2, y_2)$ in a plane is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
The Distance Formula

\[ (x_1, y_1) \quad \text{and} \quad (x_2, y_2) \]

\[ d^2 = a^2 + b^2 \]
The Distance Formula

\[ d^2 = (x_2 - x_1)^2 + (y_1 - y_2)^2 \]
Using the distance formula

From previous page:

\[ d^2 = (x_2 - x_1)^2 + (y_1 - y_2)^2 \]
\[ = (x_1 - x_2)^2 + (y_1 - y_2)^2 \]

The distance \( d \) between the points \((x_1, y_1)\) and \((x_2, y_2)\) is given by

\[ d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \]

Ex: Find the distance between the points \((2, 3)\) and \((-5, 3)\).
Examples

• Write an equation in $x$ and $y$ that a point $(x, y)$ will satisfy if and only if its distance to the origin $(0, 0)$ is 10.

• Find a point in the first quadrant with coordinates of the form $(2a, a)$ such that the distance from $(2a, a)$ to $(1, 3)$ is 5.
The Midpoint Formula

\[
(x, y) = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]
The Midpoint Formula

\[
(x_1, y_1) \quad \text{and} \quad (x_2, y_2)
\]

Midpoint:

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]

Distance between points:

\[
s = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

Slope of the line:

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]
The **midpoint** \((x, y)\) of the segment connecting \((x_1, y_1)\) and \((x_2, y_2)\) is given by the \[ (x, y) = \left( \frac{x_2 + x_1}{2}, \frac{y_1 + y_2}{2} \right) \]

Ex: Find the midpoint of the line segment between \((1, 3)\) and \((-2, -3)\).
Examples

• Suppose that \( A = (2, 1) \) and \( B = (x, y) \) and the segment \( AB \) has midpoint \( M = (3, -2) \). Find \( x \) and \( y \).

• Prove that the diagonals in a parallelogram bisect each other.