Math 340, Number Theory, Fall 2016

Homework 1 Solutions

In the next 3 problems, assume a, b, and c are arbitrary nonzero integers.

1.1.20 True or False: If $c|a$ and $c|b$, then $[a,b] \leq ab/c$.

This is **false**. Let $a = -2$, $b = 4$, $c = 1$. Then $[a,b] = 4$ but $ab/c = -8$.

1.1.22 True or False: $(a,b)|[a,b]$.

This is **true**. We know that $[a,b]$ is a multiple of a so $[a,b] = ma$ for some integer m. Also (a,b) divides a. Therefore $[a,b]/(a,b) = ma/(a,b)$ is an integer. So $(a,b)|[a,b]$.

1.1.24 True or False: If $b|c$, then $[a,b] \leq [a,c]$.

This is **true**. Any multiple of c is also a multiple of b. So the set of positive common multiples of a and c is a subset of the positive common multiples of a and b. So $[a,b] \leq [a,c]$.

1.2.29 Prove that $(a, a+2)$ is 2 if a is even and 1 if a is odd.

Proof. Let $a \in \mathbb{Z}$. Suppose that $d \in \mathbb{Z}_{>0}$ divides a and $a+2$. Then d divides $a+2-a = 2$. This shows that $(a, a+2)$ is either 1 or 2 in any case. Now if a is odd, then $2 \nmid a$ and so $(a, a+2) = 1$. On the other hand, if a is even, then 2 divides a and $a+2$, so $(a, a+2) = 2$.

This completes the proof. $\square$

1.2.32 Prove that if $d|a$, $d|b$, and $d|c$, and if x, y, and z are any integers, then d divides $ax + by + cz$.

Proof. Let a, b, c, d, x, y, z be integers. We assume that $d|a$, $d|b$, and $d|c$. One application of Theorem 1.2 yields $d|ax + by$. A second application of Theorem 1.2 implies $d|ax + by + cz$.

$\square$