Summary of Convergence Tests for Infinite Series

Test	Series	Converges if	Diverges if	Comment
Divergence	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	cannot be used to show convergence
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\|r\| < 1$	$\|r\| \geq 1$	Sum: $S = \frac{a}{1-r}$
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Integral	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	f is continuous, positive, and decreasing
Comparison	$\sum_{n=1}^{\infty} a_n$	$0 \leq a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 \leq b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	a_n, b_n positive
Limit Comparison	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	a_n, b_n positive Test fails if $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ or $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} b_n$	$b_{n+1} \leq b_n$ $\lim_{n\to\infty} b_n = 0$	$\lim_{n\to\infty} b_n \neq 0$	b_n positive
Absolute Convergence	$\sum_{n=1}^{\infty} a_n$	$\sum_{n=1}^{\infty} \|a_n\|$ converges		cannot be used to show divergence
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\| < 1$	$\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\| > 1$	Test fails if $\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\| = 1$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \|a_n\|^{1/n} < 1$	$\lim_{n\to\infty} \|a_n\|^{1/n} > 1$	Test fails if $\lim_{n\to\infty} \|a_n\|^{1/n} = 1$

Comments: The following general guidelines are useful:

1. Does the nth term approach zero as n approaches infinity? If not, the Divergence Test implies the series diverges.
2. Is the series one of the special types - geometric, telescoping, p-series, alternating series?
3. Can the integral test, ratio test, or root test be applied?
4. Can the series be compared favorably to one of the special types?