

Name: Key

Signature : \_\_\_\_\_

### EXAM I

- Please **DO NOT OPEN THIS EXAM** until told to do so.
- You have 50 minutes to complete this exam.
- The exam is worth 50 points total, there are ten problems, and each problem is worth 5 points.
- Please read the directions to each problem carefully and write your solutions (computations) neatly in the space provided **showing all work**. Use the back side of a page in the event you need more space.
- Partial credit will be awarded for partial solutions, but you should work carefully and check all calculations for accuracy.
- All problems should be **answered explicitly** using appropriate **units** and **notation**.
- Please **put a box around your final answer** in each problem.
- All numerical answers should be **exact** (e.g.,  $7/9$ ,  $\sqrt{2}$ ,  $\pi$ , ...). No decimal approximations unless directed otherwise.
- You may use one 3" by 5" notecard, written, not typed, with your name printed on the card. Please turn in your notecard with your exam when you are finished.
- You may **NOT** use any other notes, books or other people during this exam. No calculators or any other electronic devices are allowed.
- By signing your name above, you are stating that the work given on this exam is your own, that you agree to the above directions, and you agree to adhere to the highest standards of academic honesty.
- Good luck!

Problem 1 (5 points) Solve the following equation.

$$|2x - 1| + 4 = 11$$

$$|2x - 1| = 7 \Rightarrow 2x - 1 = 7 \quad \text{or} \quad 2x - 1 = -7$$

$$\Rightarrow 2x = 8 \quad \text{or} \quad 2x = -6$$

$$\Rightarrow \boxed{x = 4 \quad \text{or} \quad x = -3}$$

Problem 2 (5 points) Solve the following quadratic equation by **factoring**.

$$2x^2 - x - 6 = 0$$

$$(2x + 3)(x - 2) = 0$$

$$\Rightarrow 2x + 3 = 0 \quad \text{or} \quad x - 2 = 0$$

$$\Rightarrow 2x = -3 \quad \text{or} \quad x = 2$$

$$\Rightarrow \boxed{x = -\frac{3}{2} \quad \text{or} \quad x = 2}$$

**Problem 3** (5 points) Solve the following inequality. Write your answer in interval notation.

$$3|x+1| > 6$$

$$3|x+1| > 6 \Rightarrow |x+1| > 2 \Rightarrow x+1 > 2 \text{ or } x+1 < -2$$

$$\Rightarrow x > 1 \text{ or } x < -3$$

interval:  $\boxed{(-\infty, -3) \cup (1, \infty)}$

**Problem 4** (5 points) Find the slope-intercept form of the line passing through the points  $(-1, 2)$  and  $(3, -6)$ .

$$m = \frac{-6-2}{3-(-1)} = \frac{-8}{4} = -2$$

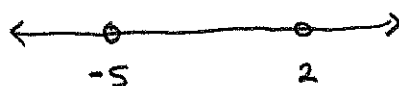
$$y = -2x + b \quad \text{plug in } (-1, 2) : 2 = -2(-1) + b \Rightarrow b = 0$$

$$\boxed{y = -2x}$$

**Problem 5** (5 points) Find the **domain** of the following function. Write your answer in **interval notation**.

$$f(x) = \frac{x+1}{(x+5)(x-2)}$$

$$x \neq -5, x \neq 2$$



$$\text{domain}(f) = (-\infty, -5) \cup (-5, 2) \cup (2, \infty)$$

**Problem 6** (5 points) The following equation is the equation of a circle. Use the method of completing the square to determine the **center** and **radius** of this circle.

$$x^2 + y^2 - 2x + 6y + 1 = 0$$

$$x^2 - 2x + y^2 + 6y = -1 \Rightarrow x^2 - 2x + 1 + y^2 + 6y + 9 = -1 + 1 + 9$$

$$\Rightarrow (x-1)^2 + (y+3)^2 = 9$$

$$\text{center} = (1, -3)$$

$$\text{radius} = 3$$

**Problem 7** (5 points) Parts (a) and (b) of this problem refer to the following two functions.

$$f(x) = \frac{1}{2x}, \quad g(x) = \frac{1}{x^2 + 1}$$

(a) Determine an expression for the function  $(f - g)(x)$ . Simplify your answer.

$$\begin{aligned} (f - g)(x) &= f(x) - g(x) = \frac{1}{2x} - \frac{1}{x^2 + 1} = \frac{x^2 + 1 - 2x}{2x(x^2 + 1)} = \frac{x^2 - 2x + 1}{2x(x^2 + 1)} \\ &= \boxed{\frac{(x-1)^2}{2x(x^2 + 1)}} \end{aligned}$$

(b) Determine the **domain** of the function  $(f - g)(x)$ . Write your final answer in **interval notation**.

$$x \neq 0,$$

$$\boxed{\text{domain } (f - g) = (-\infty, 0) \cup (0, \infty)}$$

**Problem 8** (5 points) Use the following description to find an **algebraic expression** for the function  $g(x)$ .

"The graph of  $g(x)$  is formed by reflecting the graph of  $f(x) = \sqrt{x}$  in the  $y$ -axis, then moving to the left by 2 units, then translating down by 4 units."

~~$g(x) = \sqrt{x}$~~

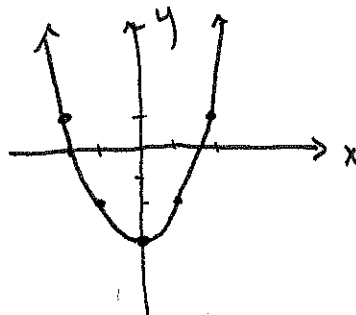
$$\sqrt{x} \longrightarrow \sqrt{-x} \longrightarrow \sqrt{-(x+2)} \longrightarrow \sqrt{-(x+2)} - 4$$

$$\boxed{g(x) = \sqrt{-(x+2)} - 4}$$

**Problem 9** (5 points) Parts (a) and (b) refer to the function  $f(x) = x^2 - 3$ .

(a) Sketch the graph of  $f(x)$ .

$x$	$x^2 - 3$
-2	1
-1	-2
0	-3
1	-2
2	1



(b) Use the graph in part(a) to determine the **domain** and **range** of this function. Write your answers in **interval notation**.

$$\text{domain}(f) = (-\infty, \infty)$$

$$\text{range}(f) = [-3, \infty)$$

**Problem 10** (5 points) The functions  $f$  and  $g$  are determined by the following two tables.

$x$	0	1	2	3
$f(x)$	-1	1	3	-3

$x$	0	1	2	3
$g(x)$	2	1	0	0

(a) Evaluate  $(fg)(0)$ .

$$(fg)(0) = f(0)g(0) = (-1)(2) = \boxed{-2}$$

(b) Evaluate  $(f \circ g)(0)$  if possible; otherwise state "undefined".

$$(f \circ g)(0) = f(g(0)) = f(2) = \boxed{3}$$

(c) Evaluate  $(g \circ f)(3)$  if possible; otherwise state "undefined".

$$(g \circ f)(3) = g(f(3)) = g(-3) \quad \boxed{\text{undefined}}$$