Dawn of the Differential Equations

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Abstract

You find yourself in the middle of a zombie apocalypse. People are scared, there's widespread panic, and human-devouring zombies are roaming the streets. However, you are in luck because this paper will discuss the mathematical modeling of a system of differential equations that could potentially save your life. More specifically, we will start by establishing a basic model with specific characteristics for each class in the zombie apocalypse (Zombies, Humans, and Dead). We will then expand on our basic model by adding equations to characterize the events under other circumstances. In these additional models, we will discuss the basic model with a latency equation added, a model added to incorporate a quarantine class, a model where humans are able to discover and implement a treatment/cure against the zombies, and a model with intelligent resistance from the humans, where they will learn how to fight zombies more effectively over time.

1. Introduction

The modern zombie is typically thought to be a reanimated corpse of a previously live person who was infected by a disease and subsequently died from another foreign causation, or was bitten and therefore infected by a zombie who had attacked them. Although much of the history pertaining to zombies comes from the Afro-Caribbean cultural beliefs often involving their spiritual system, these beliefs are typically known as the religion of Voodoo. In Voodoo, a zombie is a dead person who was revived by a sorcerer. When revived, the zombie would be controlled by the sorcerer, and the zombie would have no will of its' own. These beliefs and folklore helped form the idea of the modern zombie, although similar ideas and beliefs originated from China, Japan, India, Persia, the Pacific Oceania, the Arabian Peninsula, and the Native Americans.

Modern zombies, like the ones featured in the T.V. show, "The Walking Dead," are actually very different from the zombies found in voodoo and other seasoned cultures. The modernized zombies obey a stricter set of rules; one of which is that the modern zombie is always portrayed as a mindless monster of sorts who has an insatiable appetite for human flesh. Zombies have one goal in mind, and that is to kill, eat, and infect the living. At times this task can be difficult (with decomposing arms and disintegrating legs), but overall, most people are no match for the hordes of zombies that would be present in a zombie outbreak.

2. Basic Model with Modern Zombie Characteristics Found in Pop Culture

For the basic model, there will be three categories of beings: Humans (H), Zombies (Z), and the Dead (D). Each of the populations will be changing with respect to time. The Humans can die through natural causes, any death not having to do with a zombie, which will be represented with the parameter ζ. The dead category will contain beings who have died through both zombie attacks and natural causes. The Humans in the Dead class can re-animate as Zombies, and will thus be counted as the zombie population (parameter δ). Assuming zombies do no attack other zombies and only attack humans, another factor that will affect the population will

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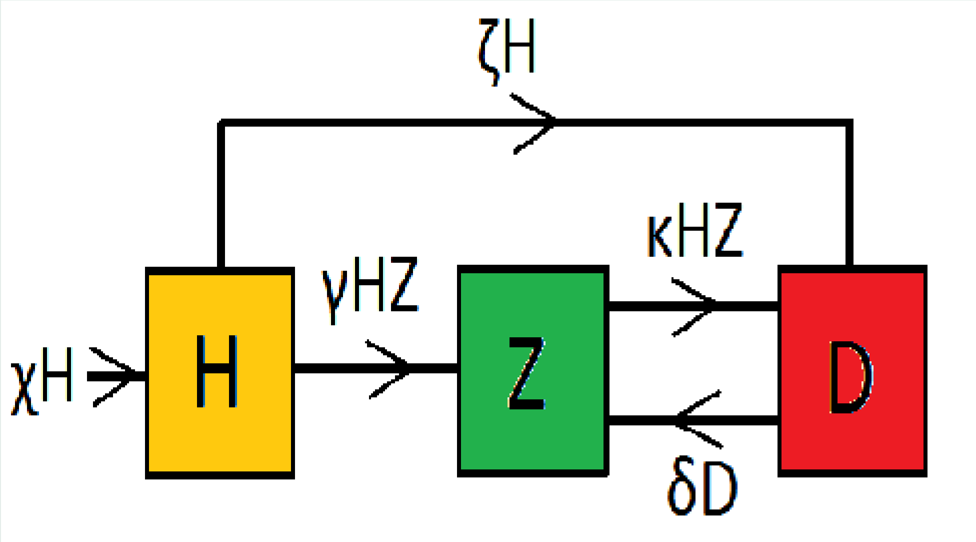

Figure 1: Basic model flowchart

be the parameter γ, which will represent the human conversion to zombies through accidental encounters on the human part. Only the humans in this system will become infected by contact with zombies, while the zombies will primarily thirst for human flesh. Therefore, we do not need to talk about any other animals or species that could satisfy the zombies hunger. In this model, there will be only two possible origins of a zombie; the first being that they are re-animated from the very recently deceased, and the second is that a human will become a zombie after losing an encounter with a zombie. In this basic model we will assume the birth rate is (χ). Humans can defeat a zombie only by removing the head of the zombie or by damaging the zombies brain significantly, this will be represented in parameter κ. After considering all of these factors we have found the basic model to be given by the system of differential equations in Figure 1

$$H' = \chi H - \gamma HZ - \zeta H$$
$$Z' = \gamma HZ + \delta D - \kappa HZ$$
$$D' = \zeta H + \kappa HZ - \delta D$$

Again, recall that

1. χ is the growth rate of humans, we used .01.
2. ζ is the natural death of humans, we used .00596.
3. δ is the rate at which humans re-animate from the dead class into zombies, we used .02.
4. γ is the rate at which zombies kill humans through random encounters, we used .0002 for Figure 2.
5. κ is the rate at which the humans will kill the zombies through any random encounter, we used .00015 for Figure 2.

In Figure **??**, we see the result of the Mathematica plot of the following system with these given parameters. One can see that the Human civilization is quickly overwhelmed by the zombies, as the amount of zombies rises with little limitations. We used initial conditions such that the Humans had a population of 500, the Zombies at 1, and the Dead at 0.

When considering this system of differential equations, one can assume that a significant population without infection (N) will eventually make contact with a Zombie. These interactions go one of two ways; either the zombie defeats the human or the human is able to kill the zombie. This implies that the infection will transmit at a rate γN per some measurement of time. Obviously, in this model the 'Infection' is actually just zombiefication caused only by another zombie, where the chance that any human will have an encounter with a zombie is represented by H/N. Because of this, the amount of new zombies being formed from this random encounter process per unit of time can be shown through:

$$(H/N)(\gamma N)Z = \gamma HZ$$

Although, as touched on previously, Humans can also prevent themselves from becoming a zombie by defeating one in a random encounter. Every human (H) is able to defeat, or resist

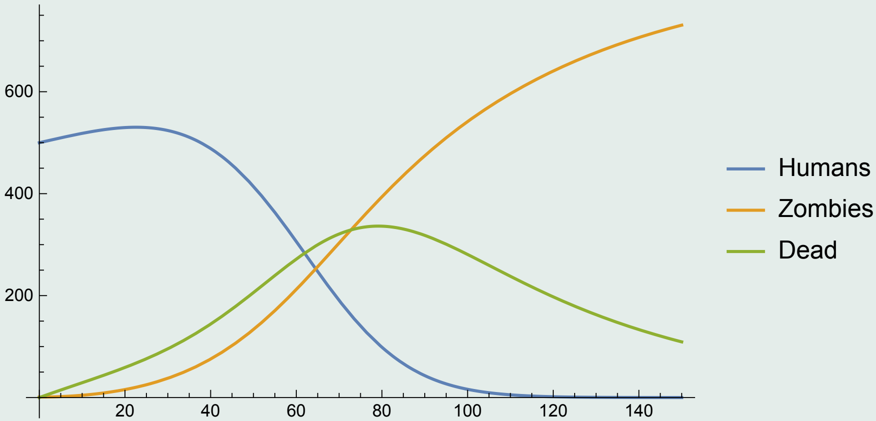

Figure 2: Zombies more effective

zombiefication at a rate of (κ). Hence the chance a zombie running into a human would be Z/N, this is very similar to the opposite probability idea mentioned last paragraph. From this idea, we can surmise the amount of zombies destroyed from humans in the random interaction process.

$$(Z/N)(\kappa N) = \kappa H Z$$

$$\begin{aligned}
&H' + Z' + D' = \chi \\
&\chi \to \infty \\
&H + Z + D \to \infty
\end{aligned}$$

In Figure 3 we see the result of the Mathematica plot of the same system of differential equations except that we switched the values for parameter 4 and 5, so that $\gamma < \kappa$, where γ is now .00015 and κ is now .0002, this makes the Humans actually more effective at killing zombies than zombies at killing humans. One would find that this only delays the inevitable

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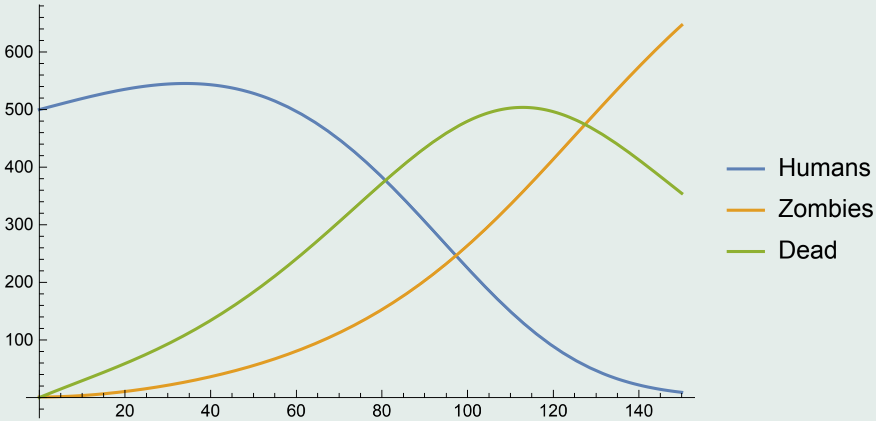

Figure 3: Humans more effective

as the Human population is still utterly destroyed, only in a longer period of time. The initial conditions are the same as figure 2, Where the potential for the population growth (χ) will go to infinity, and all of the beings added together will equal everyone on earth and its potential for any further growth (birth rate). While time goes to infinity, the birth rate (χ) cannot be equal to 0. If all of the human population has infinite potential for growth, and the Zombie is the predator, implying its population will grow the fastest, this means that the human population cannot reach infinity. Therefore, in the basic model, humans will go extinct as $t \to \infty$. If we speculate that the outbreak happens over a very short amount of time, or if we just look at a small period of time, then we are able to ignore the birth rate as well as the death rate. This will set $\chi = \zeta = 0$. And we will set the system of differential equations equal to 0.

$$
\begin{aligned}
-\gamma HZ &= 0 \\
\gamma HZ + \delta D - \kappa HZ &= 0 \\
\kappa HZ - \delta D &= 0
\end{aligned}
$$

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We know that γ is a parameter and therefore cannot be equal to 0. This implies from the first differential equation that either $H = 0$ or that $Z = 0$ when the whole system is equal to 0. When considering the scenario that $H = 0$ we will see that the Human population goes extinct. However, when $Z = 0$, we see an infection free world, or a zombie ridden scenario. Under a "doomsday" scenario, we are left with the equilibrium:

$$(\overrightarrow{H}, \overrightarrow{Z}, \overrightarrow{D}) = (0, \overrightarrow{Z}, 0)$$

The disease-free equilibrium when Z=0 becomes

$$(\overrightarrow{H}, \overrightarrow{Z}, \overrightarrow{D}) = (\overrightarrow{N}, 0, 0)$$

Now we can perform a Jacobian

$$J = \begin{vmatrix} \gamma Z & -\gamma S & 0 \\ \gamma Z - \alpha Z & \gamma S - \kappa S & \delta \\ \kappa Z & \kappa & -\delta \end{vmatrix}$$

The Jacobian at the disease free equilibrium is

$$J(N,0,0) = \begin{vmatrix} 0 & -\gamma N & 0 \\ 0 & \gamma N - \kappa N & \delta \\ 0 & \kappa N & -\delta \end{vmatrix}$$

Now we have

$$det(J - \lambda I) = -\lambda[\lambda^2 + (\delta - (\gamma - \kappa)N)\lambda - \gamma\delta N]$$

This implies that the characteristic equation must always have positive real eigenvalues. Since this is the case, the disease-free equilibrium will always be unstable on the right hand side of the trace-determinant plane.

$$J(0,\vec{Z},0) = \begin{vmatrix} -\gamma\vec{Z} & 0 & 0 \\ \gamma\vec{Z} - \kappa\vec{Z} & 0 & \delta \\ \kappa\vec{Z} & 0 & -\delta \end{vmatrix}$$

Which gives us

$$det(J - \lambda I) = -\lambda(-\gamma\vec{Z} - \lambda)(-\delta - \lambda)$$

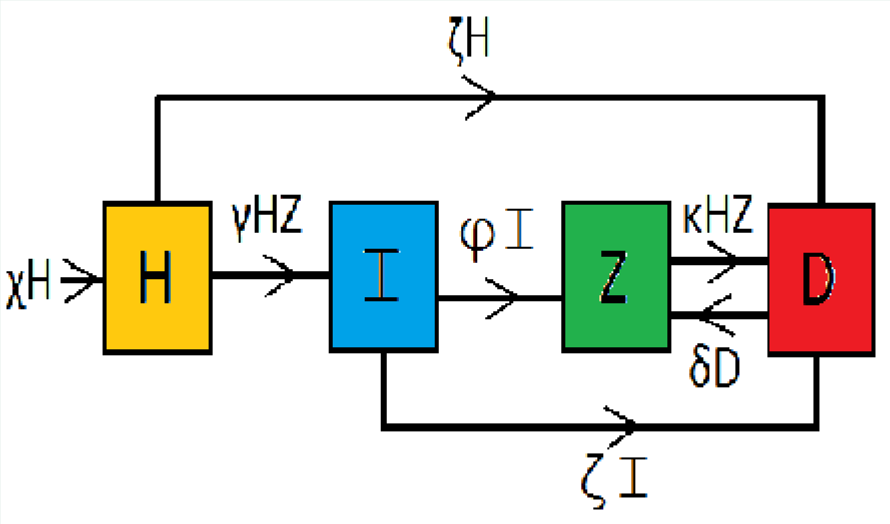

Figure 4: Latency flowchart

3. Latency of Zombie Infection Leads to New Infected Class

The official definition of latency in a virus, is a particular virus's ability to lie dormant within the infected individual before overcoming that person with disease. Applying this theory to a zombie apocalypse would imply that this new class of individuals (the Infected) we are creating would be in a sort of purgatory or limbo between zombie and human form because they have the potential to be re-animated as zombie very soon. To account for Latency, we can modify the previous system to incorporate the chance that a human would become infected, but not turn into a zombie immediately. The most important changes to the basic model will be that the Humans will now move to the Infected category for a certain length of time before re-animating.

The second is that the infected Humans can still die a death other than a zombie attack, and will therefore not turn into one (parameter ϕ).This new change in the model will be shown in Figure 4, and in the following system:

$$
\begin{aligned}
H' &= \chi H - \gamma HZ - \zeta H \\
I' &= \gamma HZ - \phi I - \zeta I \\
Z' &= \phi I + \delta D - \kappa HZ \\
D' &= \zeta H + \zeta I + \kappa HZ - \delta D
\end{aligned}
$$

When looking at this system we must remember that

1. χ is the growth rate of humans, we used .01.
2. ζ is the natural death of humans, we used .00596.
3. δ is the rate at which humans re-animate from the dead class into zombies, we used .02.
4. γ is the rate at which zombies kill humans through random encounters, we used .0002.
5. κ is the rate at which the humans will kill the zombies through any random encounter, we used .00015.
6. ϕ is the new parameter for Latency, it represents the death rate of the infected population, we used .006.

In Figure 5, we see the result of the Mathematica plot of the Latency system with an infected class added. One can see that the majority of the humans simply become infected before turning into a zombie, which simply delays the downfall of the human civilization by about 50 days. The only new initial conditions were for the infected class, which had a population of zero at time equals zero.

Our equilibria is

$$
\begin{aligned}
Z &= 0 \rightarrow (\vec{H}, \vec{I}, \vec{Z}, \vec{D}) = (N, 0, 0, 0) \\
H &= 0 \rightarrow (\vec{H}, \vec{I}, \vec{Z}, \vec{D}) = (0, 0, \vec{Z}, 0)
\end{aligned}
$$

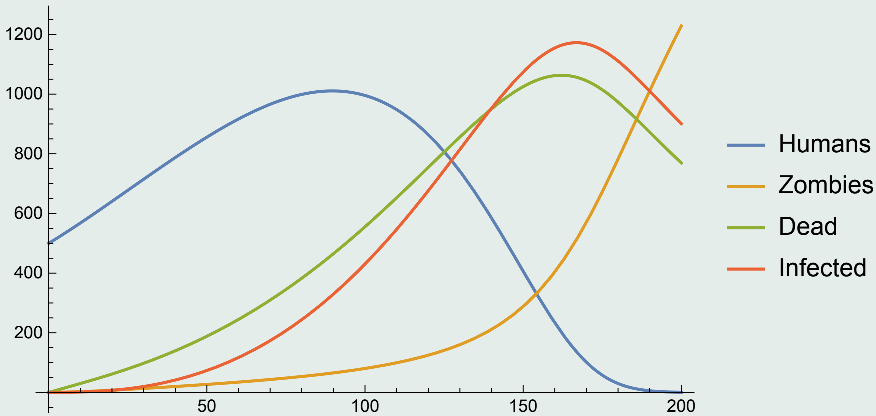

Figure 5: Model with latency

Which means that existence between humans and zombies and infected individuals is not possible.

With this scenario, our Jacobian is

$$J = \begin{vmatrix} -\gamma Z & 0 & -\gamma H & 0 \\ \gamma Z & -\phi & \gamma H & 0 \\ -\kappa Z & \phi & -\kappa H & \delta \\ \kappa Z & 0 & 0 & -\delta \end{vmatrix}$$

First, we get

$$det(J(N,0,0,0) - \lambda I) \;=\; det \begin{vmatrix} -\lambda & 0 & -\gamma N & 0 \\ 0 & -\phi - \lambda & \gamma N & 0 \\ 0 & \phi & -\kappa N - \lambda & \delta \\ 0 & 0 & \kappa & -\delta - \lambda \end{vmatrix}$$

$$= -\lambda det \begin{vmatrix} -\phi-\lambda & \gamma N & 0 \\ \phi & -\kappa N-\lambda & \gamma \\ 0 & \kappa N & -\delta-\lambda \end{vmatrix}$$

$$= \; -\lambda[-\lambda^3 - (2\phi+\kappa N)\lambda^2 - (\phi\kappa N + \phi^2 - \phi\gamma N)\lambda + \phi^2\gamma N]$$

Because $\phi^2\gamma N > 0$, $det(J(N,0,0,0) - \lambda I)$ has an eigenvalue with a positive, real part. This means that the disease-free equilibrium is unstable.

Next, we have

$$det(J(0,0,\vec{Z},0) - \lambda I) \;=\; det \begin{vmatrix} -\gamma\vec{Z}-\lambda & 0 & 0 & 0 \\ \gamma\vec{Z} & -\phi-\lambda & 0 & 0 \\ -\kappa\vec{Z} & \phi & -\lambda & \delta \\ \kappa\vec{Z} & 0 & 0 & -\delta-\lambda \end{vmatrix}$$

The eigenvalues are thus $\lambda = 0, -\gamma Z, -\phi, -\delta$. Since all eigenvalues are non- positive, the doomsday equilibrium is stable. Even with a latent period of infection, zombies will again take over the population. We plotted numerical results from the data again using Euler?s method for solving the ODEs in the model.

4. Quarantine of the Zombies and Infected Classes

In this next model we imagine a scenario in which the zombies begin to get themselves trapped in a field, barn,etc.. at a certain rate. The zombies then begin to die at a separate rate, which represents the chance of them falling off cliffs, getting impaled by tools on accident, etc... This will only slow down the infection rate, perhaps buying the humans more time to devise a successful escape from inevitable doom. For this model, we create a new parameter ω, which accounts for the portion of the infected population that are entering the quarantine zones. Once these infected individuals turn into zombies, they will not be able to infect new humans. Another significant addition to the model will be that the quarantine regions will also contain a portion of the zombie population that also is no longer able to infect new healthy individuals

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(ϱ). Some of the zombies or infected individuals will attempt to escape from the quarantine zone, resulting in death, and thus they will be moved to the dead class. This will be shown through parameter θ. However, there is a reverse affect where the infected individuals moved to the dead class could potentially come back to life as a zombie outside of the quarantine. These changes are shown in Figure **??** and the following system:

$$\begin{aligned}
H' &= \chi H - \gamma H Z - \zeta H \\
I' &= \gamma H Z - \phi I - \zeta I - \omega I \\
Z' &= \phi I + \delta D - \kappa H Z - \varrho Z \\
D' &= \zeta H + \zeta I + \kappa H Z - \delta D + \theta Q \\
Q' &= \omega I + \varrho Z - \theta Q
\end{aligned}$$

In order to understand why this system makes sense, it helps to recall the following:

1. χ is the growth rate of humans, we used .01.
2. ζ is the natural death of humans, we used .00596.
3. δ is the rate at which humans re-animate from the dead class into zombies, we used .02.
4. γ is the rate at which zombies kill humans through random encounters, we used .0002.
5. κ is the rate at which the humans will kill the zombies through any random encounter, we used .00015.
6. ϕ is the new parameter for Latency, it represents the death rate of the infected population, we used .006.
7. ω the infected entering quarantined zones, we used .004.
8. ϱ zombies entering quarantined zones, .003 was used in figure 4.
9. θ is equal to the rate at which the quarantined zombies and infected class die through failed escape attempts, we used .006.

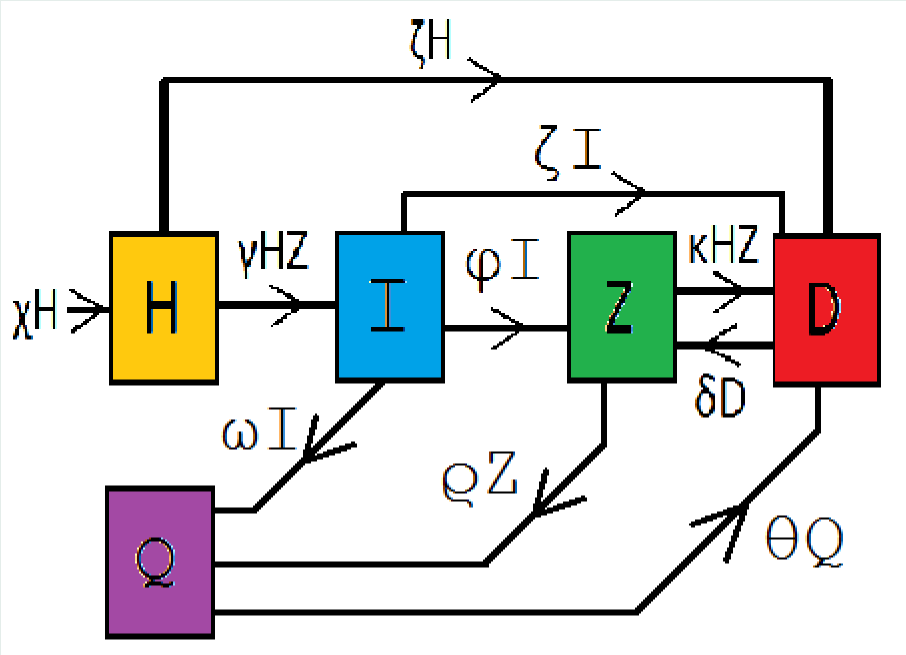

Figure 6: Quarantine flowchart

With a short outbreak ($\chi = \zeta = 0$), we have the equilibria

$$(\vec{H}, \vec{I}, \vec{Z}, \vec{D}, \vec{Q}) \; = \; (N, 0, 0, 0, 0), (0, 0, \vec{Z}, \vec{D}, \vec{Q})$$

The F matrix represents new infections, and the V matrix represents transfers between compartments.

$$F = \begin{vmatrix} 0 & \gamma N & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix},$$

$$V = \begin{vmatrix} \phi + \omega & 0 & 0 \\ -\phi & \kappa N + \varrho & 0 \\ -\omega & -\varrho & \theta \end{vmatrix}$$

$$V^{-1} = \frac{1}{\theta(\phi + \omega)(\kappa N + \varrho)} \begin{vmatrix} \theta(\kappa N + \varrho) & 0 & 0 \\ \phi\theta & \theta(\phi + \omega) & 0 \\ \phi\varrho + \omega(\kappa N + \varrho) & \varrho(\phi + \omega) & (\phi + \omega)(\kappa N + \varrho) \end{vmatrix}$$

In Figure 7, we see the result of the Mathematica plot with a quarantine differential equation added to the system. The graph shows that the zombies quickly infect nearly all of the humans, moving them into the infected class. One might also notice that the infected and zombie classes have much faster deterioration rates than any other previous models. This current model will eventually result in a complete doomsday, as all of the populations in each class will reach zero as time goes to infinity.

$$FV^{-1} = \frac{1}{\theta(\phi + \omega)(\kappa N + \varrho)} \begin{vmatrix} \gamma N \phi\theta & \gamma N \theta(\phi + \omega) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

Which gives us

$$D_0 \; = \; \frac{\gamma N \phi}{(\phi + \omega)(\kappa N + \varrho)}$$

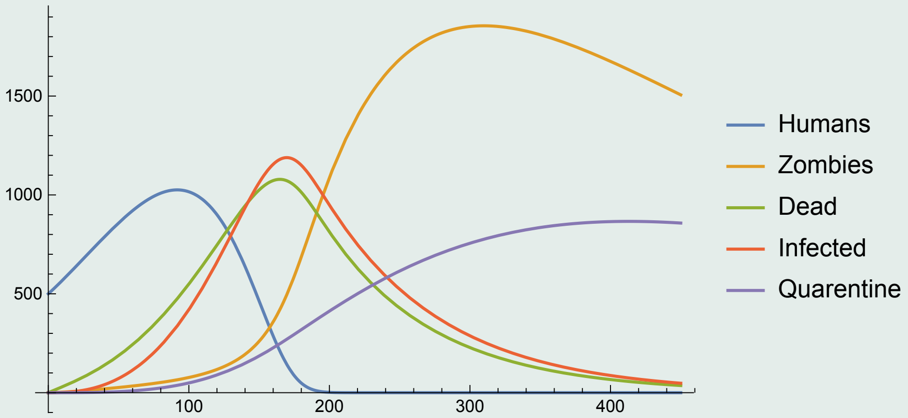

Figure 7: Model with quarantine

This shows that the equilibrium is only stable if $D_0 < 1$. To stay stable, we have to either increase ω or ϱ. If the population is very large, then

$$D_0 \approx \frac{\gamma\phi}{(\phi+\omega)\kappa}$$

If $\gamma > \kappa$, then complete eradication of the zombies requires quarantining people in the primary stages of infection.

5. Treatment of Zombies

In this newest model, we will conceptualize the discovery of a treatment for the spread of the zombies that will be implemented 150 days after the start of the zombie apocalypse. This new treatment will be able cure a zombie, turning them back to their previous human form. Although this new treatment will be able to cure zombies, it is not able to prevent humans

from becoming zombies. This will allow cured zombies (now humans) to once again join the zombie class if attacked or infected by a zombie. To put it simply, the treatment does not in any way administer immunity to humans. Zombies who were created from previously dead humans coming back to live in there other form, now, with the help of the treatment, have a second shot at surviving the zombie apocalypse.We will no longer need to address the quarantine section in this model because these quarantined zombies can be cured. As one can see in the flowchart in Figure 8, the cured zombies move to the human class. In the following system, the only new parameter will be c", which will represents cured zombies.

$$
\begin{aligned}
H' &= \chi H - \gamma HZ - \zeta H + cZ \\
I' &= \gamma HZ - \phi I - \zeta I \\
Z' &= \phi I + \delta D - \kappa HZ - cZ \\
D' &= \zeta H + \zeta I + \kappa HZ - \delta D
\end{aligned}
$$

The following parameters apply to the above system of differential equations.

1. χ is the growth rate of humans, we used .01.
2. ζ is the natural death of humans, we used .00596.
3. δ is the rate at which humans re-animate from the dead class into zombies, we used .02.
4. γ is the rate at which zombies kill humans through random encounters, we used .0002.
5. κ is the rate at which the humans will kill the zombies through any random encounter, we used .00015.
6. ϕ is the new parameter for Latency, it represents the death rate of the infected population, we used .006.
7. c the rate at which humans are able to cure the zombies with their new treatment.

In Figure 9, we see the result of the Mathematica plot with a treatment discovered and implemented on the zombie population at a rate c, 150 days after the beginning of the zombie

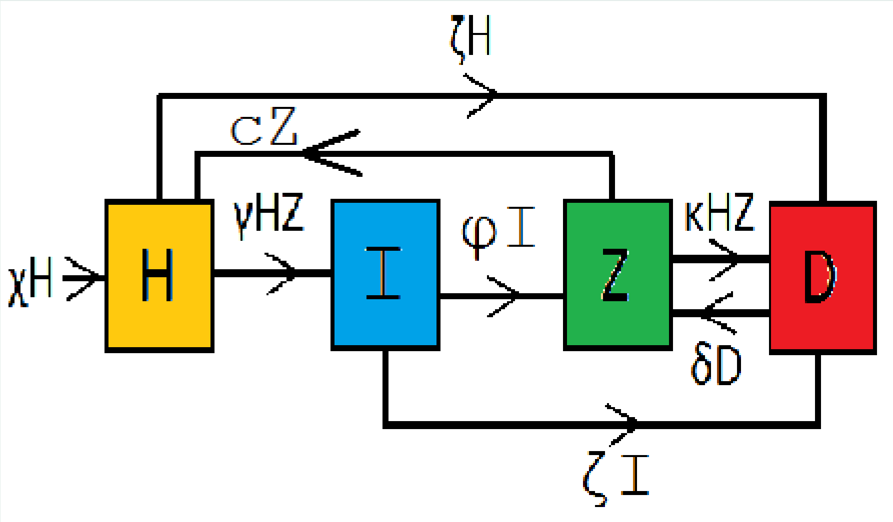

Figure 8: Treatment flowchart

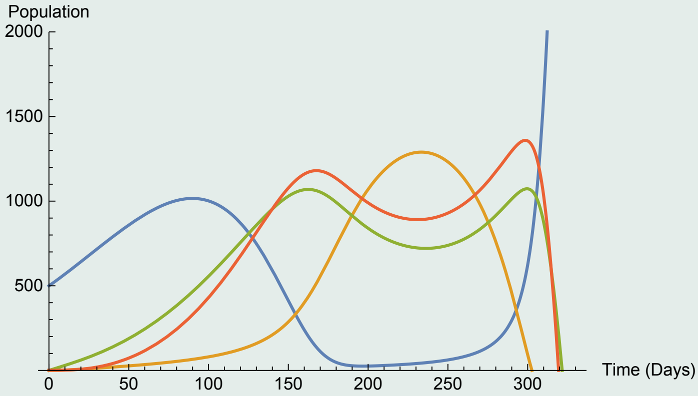

Figure 9: Model with treatment implemented after 150 days

apocalypse. One can see that the humans narrowly escaped annihilation from the zombies by fighting back with the treatment. For this treatment model, we used the initial conditions where the human population was 500, the zombies were at 0, the dead were at 0, and the infected class were at 1 individual. One might think that if no zombies existed at the start, then the rise of the zombies would never happen. However, one infected individual would soon turn into that one zombie, which in turn would initiate the apocalypse.

Just like our other models, if $\chi = 0$, then $H + I + Z + D \to \infty$, so we set $\chi = \zeta = 0$. When $Z = 0$, we get our disease-free equilibrium,

$$(\vec{H}, \vec{I}, \vec{Z}, \vec{D}) \ = \ (N, 0, 0, 0, 0)$$

But, because of the cZ term in the first equation, we now have the possibility of an endemic equilibrium $(\vec{H}, \vec{I}, \vec{Z}, \vec{D})$ satisfying

$$\begin{aligned}
-\gamma\vec{H}\vec{Z} + cZ &= 0\\
\gamma\vec{H}\vec{Z} - \phi\vec{I} &= 0\\
\phi\vec{I} + \delta\vec{D} - \kappa\vec{H}\vec{Z} &= 0\\
\kappa\vec{H}\vec{Z} - \delta\vec{D} &= 0
\end{aligned}$$

So, our equilibrium is

$$(\vec{H},\vec{I},\vec{Z},\vec{D}) = \left(\frac{c}{\gamma}, \frac{c}{\phi}\vec{Z}, \frac{\kappa c}{\phi\gamma}\vec{Z}\right)$$

The Jacobian is

$$J = \begin{vmatrix} \gamma Z & 0 & -\gamma S + c & 0\\ \gamma Z & -\phi & \gamma S & 0\\ -\kappa Z & \phi & -\kappa S - c & \delta\\ \kappa Z & 0 & \kappa S & -\delta \end{vmatrix}$$

We now have

$$\begin{aligned}
det(J(\vec{H},\vec{I},\vec{Z},\vec{D}) = det &\begin{vmatrix} -\gamma\vec{Z} & 0 & 0 & 0\\ \gamma\vec{Z} & -\phi & c & 0\\ -\kappa\vec{Z} & \phi & -\frac{\kappa c}{\gamma} - c & \delta\\ \kappa\vec{Z} & 0 & \frac{\kappa c}{\gamma} & -\delta \end{vmatrix}\\
= -(\gamma\vec{Z} - \lambda) det &\begin{vmatrix} -\phi & c & 0\\ \phi & -\frac{\kappa c}{\gamma} - c & \delta\\ 0 & \frac{\kappa c}{\gamma} & -\delta \end{vmatrix}\\
= -(\gamma\vec{Z} - \lambda)&\left\{ -\lambda[\lambda^2 + (\phi + \frac{\kappa c}{\gamma} + c + \delta)\lambda + \frac{\phi\kappa c}{\gamma} + \phi\delta + c\delta] \right\}
\end{aligned}$$

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Since the quadratic expression has positive coefficients, there are no positive eigenvalues. Therefore, the coexistence equilibrium is stable. In this case, humans are not eradicated, but only survive in low numbers until they are able to defeat the majority of the zombies and can re-establish society.

6. Intelligent Resistance

Humans are the most intelligent creatures on earth, therefore it seems we would somehow be able to effectively fight back, or at the very least, learn how to fight better. For this model, we incorporated the idea that as time goes to infinity, the humans would actually become more and more effective against the zombies, killing them in larger numbers, with more ease. The model still includes the infected class, but the most significant change is that κ is now a positive function of time. As you can see, this model, (represented by Figure 11), is very similar to Figure 4.

In Figure 10, we see the result of the Mathematica plot with Intelligent Resistance added to the Latency model. One should notice the sort of pressure build up or tension between the existing human population and the newly formed zombie population. The side effect of this tension is a build up of the dead class.The eventual downfall of the human population is due to the overwhelming ability of the zombies to infect so many people. Soon, so many humans are infected that they can no longer fight back as effectively, despite their newly added intelligent resistance.

The following parameters apply to the system of differential equations where Humans begin to learn to fight more effectively with time, while latency is still present.

1. χ is the growth rate of humans, we used .01.
2. ζ is the natural death of humans, we used .00596.
3. δ is the rate at which humans re-animate from the dead class into zombies, we used .02.
4. γ is the rate at which zombies kill humans through random encounters, we used .0002.

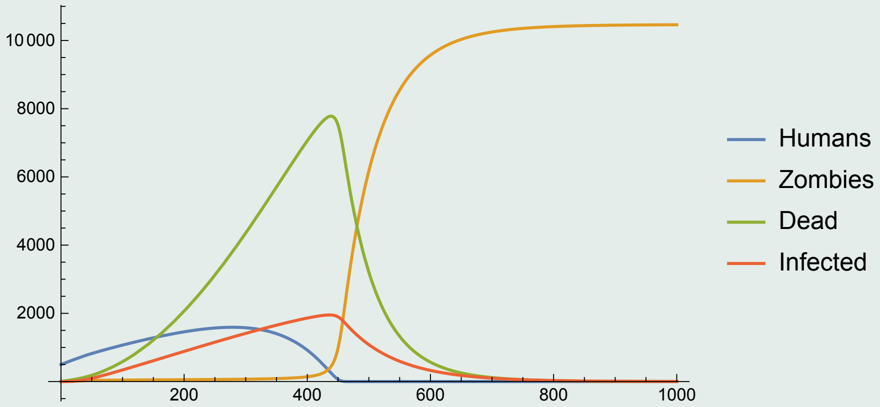

Figure 10: Model with latency and intelligent resistance

5. κ is the rate at which the humans will kill the zombies through any random encounter, we used $.0000028t$. As you can see it is now a function of time, this shows the humans learning to fight more effectively against the zombies.

6. ϕ is the new parameter for Latency, it represents the death rate of the infected population, we used .006.

$$
\begin{aligned}
H' &= \chi H - \gamma HZ - \zeta H \\
I' &= \gamma HZ - \phi I - \zeta I \\
Z' &= \phi I + \delta D - \kappa HZ \\
D' &= \zeta H + \zeta I + \kappa HZ - \delta D
\end{aligned}
$$

In Figure 12, we see the result of the Mathematica plot where intelligent resistance is added to the basic equation to show that humans, if able to learn quick enough, would be able to prevail

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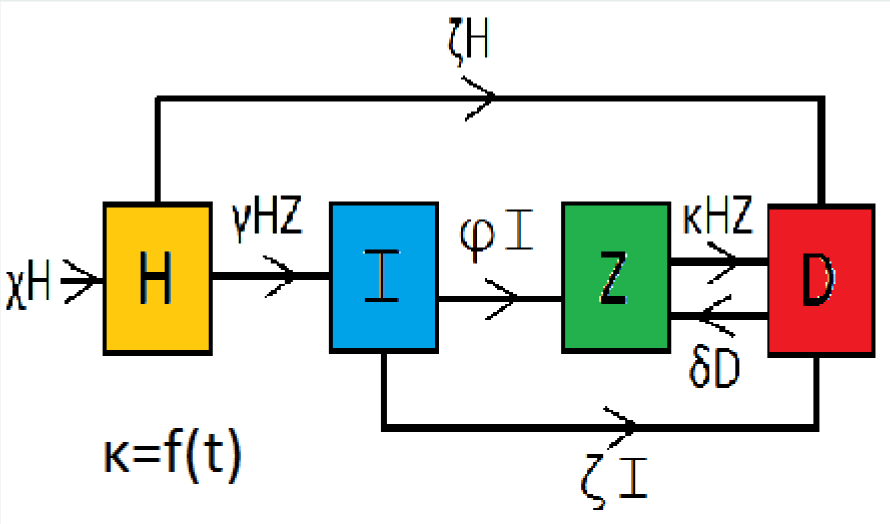

Figure 11: Resistance flowchart

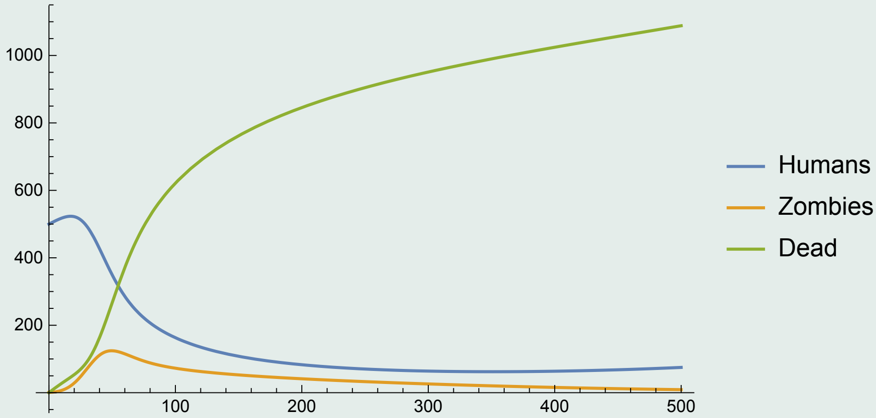

Figure 12: Basic equation with intelligent human resistance

in the given circumstance under the events of a zombie apocalypse. In this model, we see that the humans would eventually survive, although they take heavy losses. Especially at the start, the rate at which zombies kill humans is much faster, while simultaneously there is a larger dead class building up.

The following parameters apply to the basic system (our first model) where Humans learn to fight more effectively with time.

1. χ is the growth rate of humans, we used .01.

2. ζ is the natural death of humans, we used .00596.

3. δ is the rate at which humans re-animate from the dead class into zombies, we used .02.

4. γ is the rate at which zombies kill humans through random encounters, we used .0002.

5. κ is the rate at which the humans will kill the zombies through any random encounter, we used $.0000028t$. As you can see it is now a function of time, this shows the humans

learning to fight more effectively against the zombies.

$$
\begin{aligned}
H' &= \chi H - \gamma HZ - \zeta H \\
Z' &= \gamma HZ + \delta D - \kappa HZ \\
D' &= \zeta H + \kappa HZ - \delta D
\end{aligned}
$$

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7. Bibliography.

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