1. Complete the following tables for $P(x)$ to estimate $\int_{1}^{3} f'(t) \, dt$ using Euler’s Method and visualize the estimation with a mapping diagram.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\int_{1}^{x} f'(t) , dt$</th>
<th>$f'(t)$</th>
<th>$f'(t) \Delta t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td></td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Visualize the additive property of the definite integral, $\int_{a}^{c} P(x) \, dx + \int_{b}^{c} P(x) \, dx = \int_{a}^{b} P(x) \, dx$, with a mapping diagram with $P(x) = 2x$, $a = 1$, $b = 3$, $c = 2$.

[Use your knowledge of the FT of C to find the actual integrals. Adjust the scale for the target as needed to make the diagram fit on the given axes.]

\[
\int_{1}^{3} 2x \, dx = \quad \int_{2}^{3} 2x \, dx = \quad \int_{1}^{2} 2x \, dx = \quad \int_{1}^{3} 2x \, dx =
\]
3. Visualize the scalar multiplication property of the definite integral,
\[ \int_a^b aP(x)\,dx = a\int_a^b P(x)\,dx \, , \] with a mapping diagram with \( P(x) = 2x \), \( a = 0 \), \( b = 2 \), \( a = 3 \).

Explain your figure’s connection to the equation.

[Use your knowledge of the FT of C to find the two integrals.
Adjust the scale for the three axes as needed to make the diagram fit on the given axes.
Place the integral values on the middle axis.
Apply the linear function \( m(x) = 3x \) to the values on the middle axis.]

\[ \int_0^2 2x\,dx = \] 
\[ \int_0^2 3\cdot 2x\,dx = \]

4. Visualize the mean value property of the definite integral, \( \int_a^b P(x)\,dx = P(c)(b-a) \), with a mapping diagram with \( P(x) = 2x \), \( a = 0 \), \( b = 2 \). Explain your figure’s connection to the equation.

[Use your knowledge of the FT of C to find the integral.
Adjust the scale for the three axes as needed to make the diagram fit on the given axes.
Visualize the function \( P \) between the first and middle axes.
Find \( M \) and \( m \) - the max and min values for \( P \) on the middle axis.
Place the integral value on the third axis.
Apply the linear function \( m(x) = (b-a)\,x \) to the values on the middle axis.
Indicate where \( c \) lies on the first axis.]

\[ \int_0^2 2x\,dx = \]
5. Use mapping diagrams to visualize the sequences defined by $a_n = f(n)$, $n = 0, 1, 2, 3, \ldots$ for the following functions:

a. $f(n) = (-1)^n$

b. $f(n) = \frac{(-1)^n}{n+1}$

c. $f(n) = (-1)^n \frac{n}{n+1}$

d. $f(n) = \left(1 - \frac{1}{n+1}\right)^{n+1}$
6. Let $S_n(x) = \sum_{k=0}^{n} \frac{1}{2k+1} x^k$

   a. Use mapping diagrams to visualize the sequences $S_n(1)$ and $S_n(1/2)$ when $n = 0, 1, 2, \text{ and } 3$.

   b. Use a mapping diagrams to visualize the function $S_1(x)$ for $x \in [-2,2]$.

   c. Use a mapping diagrams to visualize the function $S_2(x)$ for $x \in [-2,2]$.

   d. Use a mapping diagrams to visualize the function $S_3(x)$ for $x \in [-2,2]$. 

   

   | 4.0 | 4.0 | 4.0 |
   | 3.0 | 3.0 | 3.0 |
   | 2.0 | 2.0 | 2.0 |
   | 1.0 | 1.0 | 1.0 |
   | 0.0 | 0.0 | 0.0 |
   | -1.0 | -1.0 | -1.0 |
   | -2.0 | -2.0 | -2.0 |
   | -3.0 | -3.0 | -3.0 |
   | -4.0 | -4.0 | -4.0 |
7. Let \( z = f(x, y) = 3x - 2y + 1 \).

a. Complete the following table for values of \( f \) in the appropriate cells.

\[
\begin{array}{c|ccc}
 y/x & -1 & 0 & 1 \\
\hline
 1 & f(-1,1) = -4 & & \\
 0 & & & \\
 -1 & & & \\
\end{array}
\]

b. Draw arrows as appropriate for \( f \) on the mapping diagram below for the information in the table where \( y = 1 \).

Discuss briefly the connection of the mapping diagram to \( \frac{\partial z}{\partial x} = f_x(x, y) = 3 \).

\[
\begin{array}{ccc}
 x & y & z \\
\end{array}
\]

\[
\begin{array}{ccc}
 x & y & z \\
\end{array}
\]

c. Draw arrows as appropriate for \( f \) on the mapping diagram for the information in the table where \( x = 1 \).

Discuss briefly the connection of the mapping diagram to \( \frac{\partial z}{\partial y} = f_y(x, y) = -2 \).

\[
\begin{array}{ccc}
 x & y & z \\
\end{array}
\]