

# *Making Sense of Solving Linear and Quadratic Equations with Mapping Diagrams*

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<http://users.humboldt.edu/flashman>

- Mapping diagrams provide a valuable tool for visualizing functions and connects function concepts to solving equations in many contexts.
- In this presentation both linear and quadratic equations will be solved using mapping diagrams to make sense visually of the functions and steps used in common algebraic approaches to these problems.
- GeoGebra will be used as a dynamic tool to connect the concepts with technology.

Equations, Functions, and  
Mapping Diagrams in Common Core  
Links:

<http://users.humboldt.edu/flashman/Presentations/CMC/CMC3.MD.LINKS.html>

**Mapping  
Diagram Sheets**

**[Mapping Diagram blanks](#)  
(2 axis diagrams)**

**[Mapping Diagram blanks](#)  
(2 and 3 axes)**

**Work/Spreadsh  
eets**

**[Worksheet.pdf](#)**

**[Spreadsheet Template](#) (Linear  
Functions)**

**Section from  
MD from A B to  
C and DE  
(Drafts)**

**[Visualizing Functions](#): An  
Overview**

**[Linear Functions](#) (LF)  
[Quadratic Functions\(QF\)](#)**

**GeoGebra**

**[Sketch to Visualize Solving a  
Linear Equation using Mapping  
Diagrams](#)**

**[Mapping Diagrams for Solving a  
Quadratic Equation](#)**

**YouTube Videos**

**[Using Mapping Diagrams to  
Visualize Linear Functions \(10  
Minutes\)](#)**

**[Solving Linear Equations  
Visualized with Mapping  
Diagrams. \(10 Minutes\)](#)**

# Background Questions

- Are you familiar with Mapping Diagrams to visualize functions?
- Have you used Mapping Diagrams to teach functions?
- Have you used Mapping Diagrams to teach content besides function definitions?

# Main Resource

- Mapping Diagrams from  $A(\text{Igebra})$   $B(\text{asics})$  to  $C(\text{alculus})$  and  $D(\text{ifferential})$   $E(\text{quation})\text{s}$ . A Reference and Resource Book on Function Visualizations Using Mapping Diagrams (Preliminary Sections- NOT YET FOR publication)
- <http://users.humboldt.edu/flashman/MD/section-1.1VF.html>

# Mapping Diagram Prelim

- Examples of mapping diagrams
  - Worksheet 1.a
  - Make tables for  $m(x) = 2x$  and  $s(x) = x+1$

$x$	$m(x) = 2x$
2	
1	
0	
-1	
-2	

$x$	$s(x) = x+1$
2	
1	
0	
-1	
-2	

# Mapping Diagram Prelim

- Examples of mapping diagrams
  - Worksheet 1.b
  - On separate diagrams sketch mapping diagrams for  $m(x) = 2x$  and  $s(x) = x+1$

$x$	$m(x) = 2x$
2	4
1	2
0	0
-1	-2
-2	-4

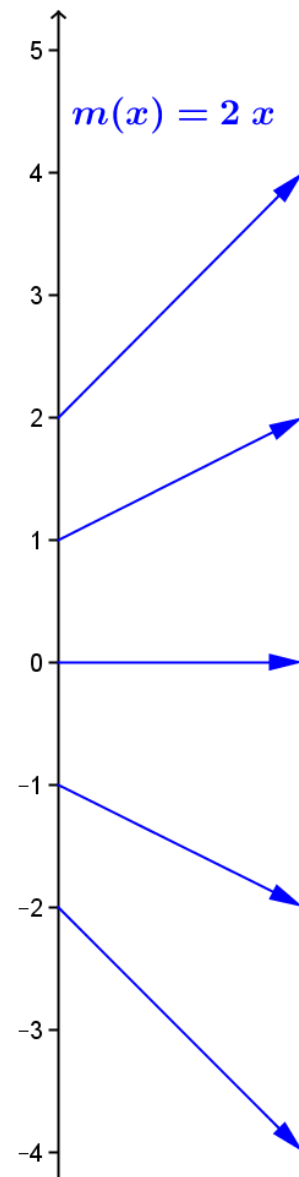
$x$	$s(x) = x+1$
2	3
1	2
0	1
-1	0
-2	-1



# Worksheet 1.b Mapping Diagram:

$m(x) = 2x$

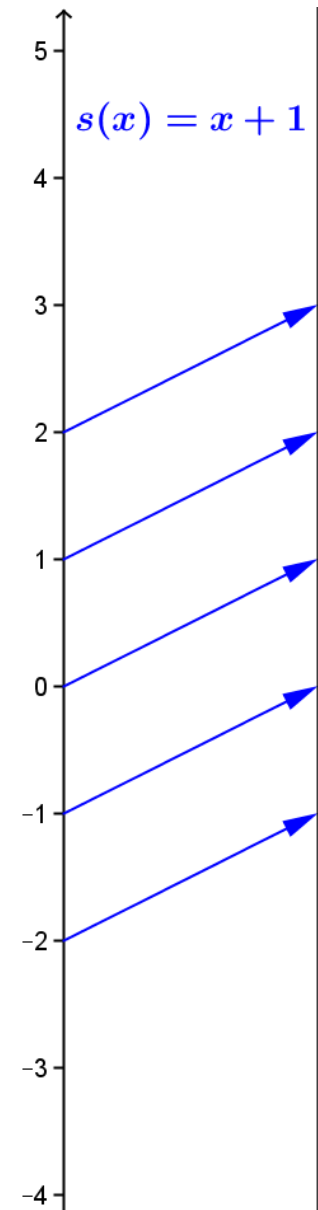
$x$	$m(x) = 2x$
2	4
1	2
0	0
-1	-2
-2	-4



# Worksheet 1.b Mapping Diagram:

$s(x) = x + 1$

$x$	$s(x)=x+1$
2	3
1	2
0	1
-1	0
-2	-1



# Mapping Diagram Prelim

- Examples of mapping diagrams
  - Worksheet 2
  - a. First make table for  $q(x) = x^2$ .

$x$	$q(x) = x^2$
2	
1	
0	
-1	
-2	

# Mapping Diagram Prelim

- Examples of mapping diagrams
  - Worksheet 2
  - a. First make table for  $q$ .

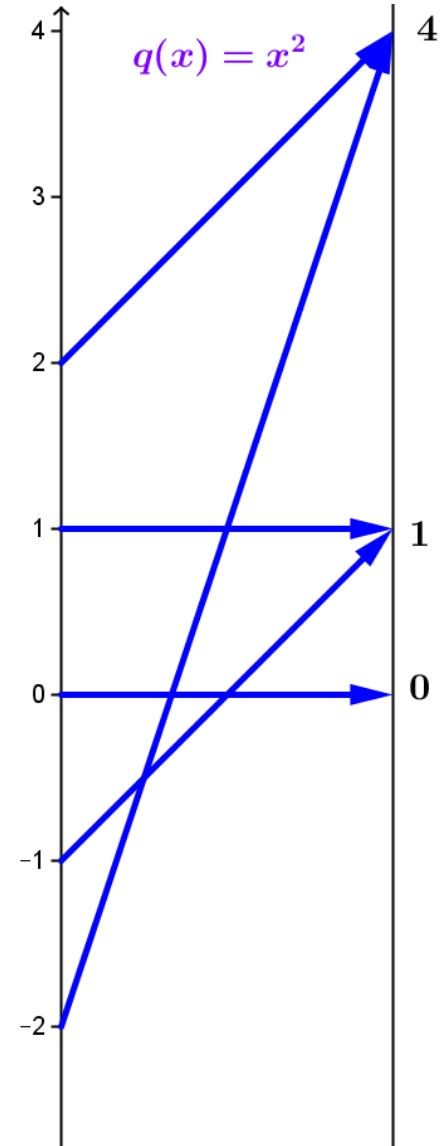
$x$	$q(x) = x^2$
2	4
1	1
0	0
-1	1
-2	4

- b. Sketch a mapping diagram for  $q(x) = x^2$ .

# Mapping Diagram Prelim

## Worksheet 2.b. Mapping Diagram for $q(x) = x^2$

$x$	$q(x) = x^2$
2	4
1	1
0	0
-1	1
-2	4



Worksheet 3.a. Complete the following table for the composite function  $f(x) = s(m(x)) = 2x + 1$

$x$	$m(x)$	$f(x)=s(m(x))$
2		
1		
0		
-1		
-2		



Worksheet 3.a. Complete the following table for the composite function  $f(x) = s(m(x)) = 2x + 1$

$x$	$m(x)$	$f(x)=s(m(x))$
2	4	5
1	2	3
0	0	1
-1	-2	-1
-2	-4	-3



# Mapping Diagram Prelim

- Worksheet 3.b
- Use the table 3.a and the previous sketches of 1.b to draw a composite sketch of the mapping diagram with 3 axes for the composite function

$$\underline{f(x) = h(g(x)) = 2x + 1}$$



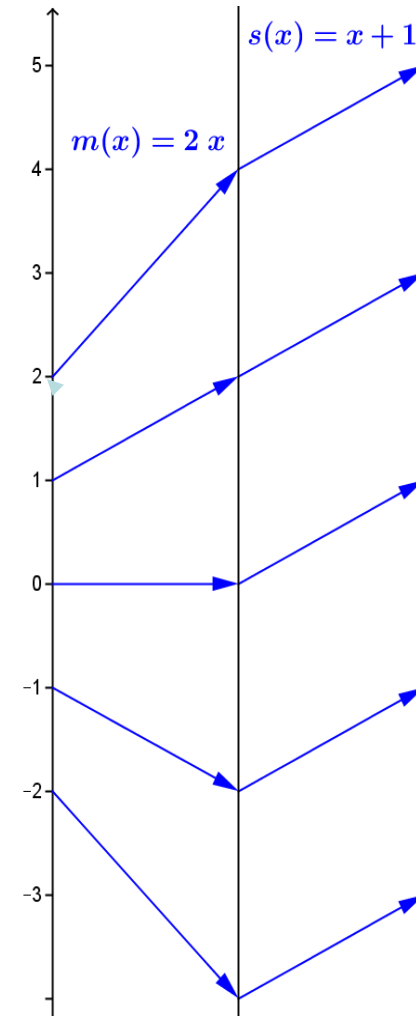
Worksheet 3.b Draw a sketch for the mapping diagram with 3 axes of  $f(x) = 2x + 1$ .

$x$	$m(x)$	$f(x)=s(m(x))$
2	4	5
1	2	3
0	0	1
-1	-2	-1
-2	-4	-3



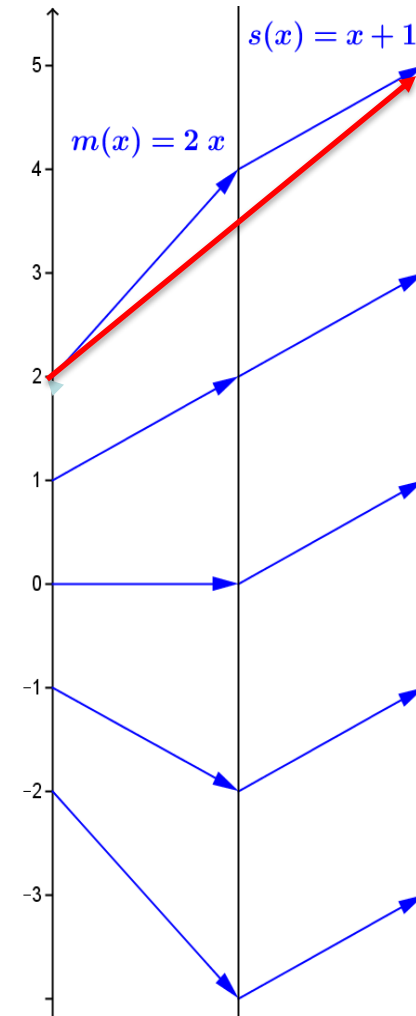
# Worksheet 3.b Draw a sketch for the mapping diagram with 3 axes of $f(x) = 2x + 1$ .

$x$	$m(x)$	$f(x)=s(m(x))$
2	4	5
1	2	3
0	0	1
-1	-2	-1
-2	-4	-3



# Worksheet 3.c Draw a sketch for the mapping diagram with 2 axes of $f(x) = 2x + 1$ .

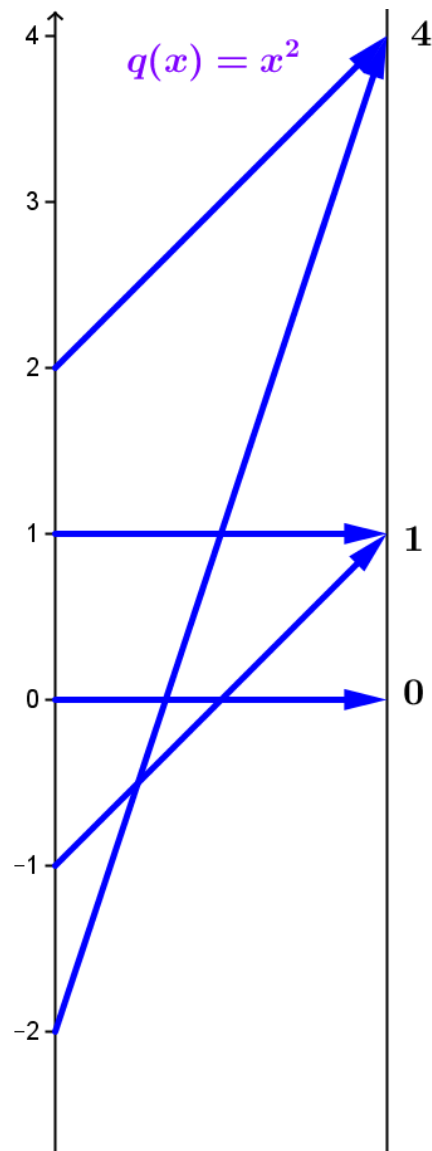
$x$	$m(x)$	$f(x)=s(m(x))$
2	4	5
1	2	3
0	0	1
-1	-2	-1
-2	-4	-3



# Worksheet 4 Mapping Diagram:

$q(x) = x^2$

$x$	$q(x) = x^2$
2	4
1	1
0	0
-1	1
-2	4



# Worksheet 4.a

Complete the following tables for  $q(x) = x^2$   
and  $R(x) = s(q(x)) = x^2 + 1$

$x$	$q(x)$	$R(x)=s(q(x))$
2		
1		
0		
-1		
-2		

# Worksheet 4.a

Complete the following tables for  $q(x) = x^2$   
and  $R(x) = s(q(x)) = x^2 + 1$

x	q(x)	R(x)=s(q(x))
2	4	5
1	1	2
0	0	1
-1	1	2
-2	4	5

# Worksheet 4.b

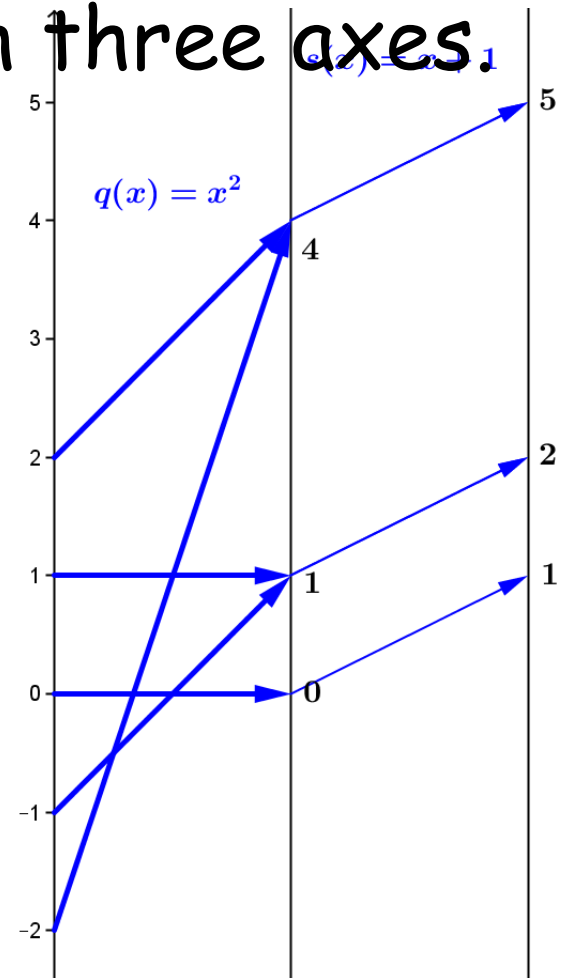
- 4.b Using the data from part a), sketch mapping diagrams for the composition  $R(x) = s(q(x)) = x^2 + 1$  with three axes.

$x$	$q(x)$	$R(x)=s(q(x))$
2	4	5
1	1	2
0	0	1
-1	1	2
-2	4	5

# Worksheet 4.b

- 4.b Using the data from part a), sketch mapping diagrams for the composition  $R(x) = s(q(x)) = x^2 + 1$  with three axes.

$x$	$q(x)$	$R(x)=s(q(x))$
2	4	5
1	1	2
0	0	1
-1	1	2
-2	4	5





# Worksheet 4.b

- 4.b Using the data from part a), sketch mapping diagrams for the composition  $R(x) = s(q(x)) = x^2 + 1$  with two axes.

$x$	$q(x)$	$R(x)=s(q(x))$
2	4	5
1	1	2
0	0	1
-1	1	2
-2	4	5

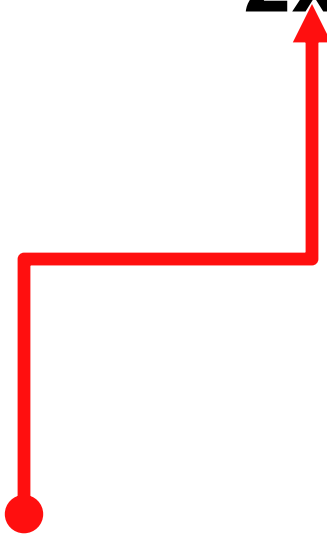


# An Old Friend: Solving A Linear Equation

- Worksheet 5.a Solve a linear equation:

$$2x + 1 = 5$$

Find x.





# An Old Friend: Solving A Linear Equation

Worksheet 5.a Solve a linear equation:

$$2x + 1 = 5$$

$$\begin{array}{r} -1 = -1 \\ \hline 2x = 4 \end{array}$$



# An Old Friend: Solving A Linear Equation

Worksheet 5.a Solve a linear equation:

$$2x + 1 = 5$$

$$\underline{-1 = -1}$$

$$2x = 4$$

$$\underline{1/2(2x) = 1/2(4)}$$

$$x = 2$$





# An Old Friend: Solving A Linear Equation

Worksheet 5.a Solve a linear equation:

$$2x + 1 = 5$$

$$\underline{-1 = -1}$$

$$2x = 4$$

$$\underline{1/2(2x) = 1/2(4)}$$

$$x = 2$$

Check!

$$2x+1 = 2*2 + 1 \stackrel{!}{=} 5$$





# Linear Equations Use Linear Functions!

## Linear Equations

$$2x + 1 = 5$$

$$\underline{-1 = -1}$$

$$2x = 4$$

$$\underline{1/2(2x) = 1/2(4)}$$

$$x = 2$$

Check:

$$\underline{2x + 1 = 2*2 + 1 = 5}$$

## Linear Functions

$$f(x) = 2x + 1$$



So, we meet again!



# Linear Equations

## Use Linear Functions!

### Linear Equations

$$2x + 1 = 5$$

$$\underline{-1 = -1}$$

$$2x = 4$$

$$\underline{1/2(2x) = 1/2(4)}$$

$$x = 2$$

Check:

$$\underline{2x + 1 = 2*2 + 1 = 5}$$

### Linear Functions

$$f(x) = 2x + 1$$



$$\underline{m(x) = 2x; s(x) = x + 1}$$

$$f(x) = s(m(x))$$

# Worksheet 5.b Solving $2x + 1 = 5$ visualized with a mapping diagram

Algebra:

$$2x + 1 = 5$$

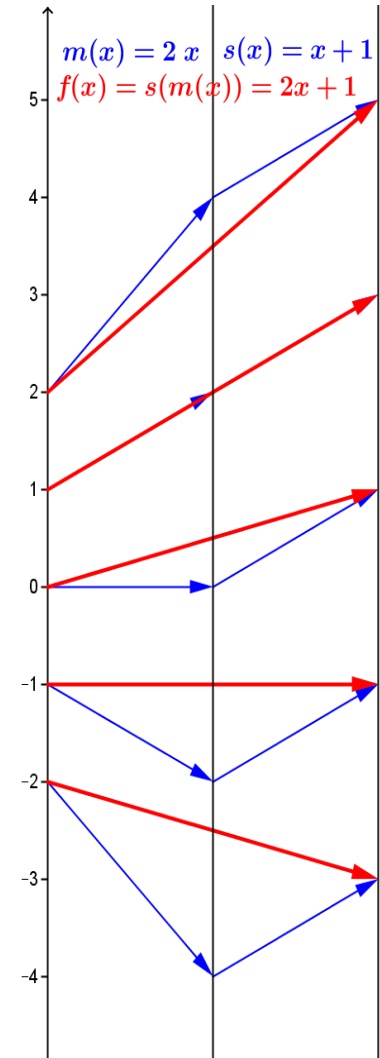
$$\underline{-1 = -1}$$

$$2x = 4$$

$$\underline{\frac{1}{2}(2x) = \frac{1}{2}(4)}$$

$$x = 2$$

How does the  
MD for the  
function  
**VISUALIZE**  
the algebra?





# Worksheet 5.b Solving $2x + 1 = 5$ visualized with a mapping diagram

Algebra:

$$2x + 1 = 5$$

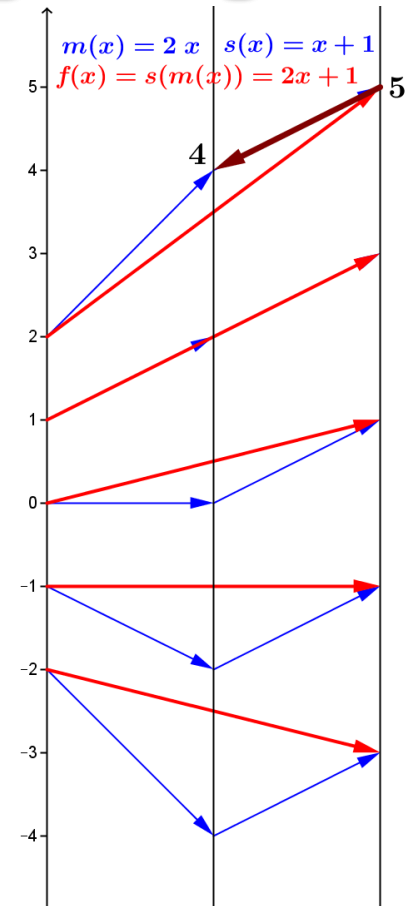
$$\begin{array}{r} -1 = -1 \\ \hline 2x = 4 \end{array}$$

Function:

$$f(x) = s(m(x)) = 5$$

"Undo  $s$ "

$$m(x) = 4$$



# Worksheet 5.b Solving $2x + 1 = 5$ visualized with a mapping diagram

Algebra:

$$2x + 1 = 5$$

$$\underline{-1 = -1}$$

$$2x = 4$$

$$\underline{\frac{1}{2}(2x) = \frac{1}{2}(4)}$$

$$x = 2$$

Function:

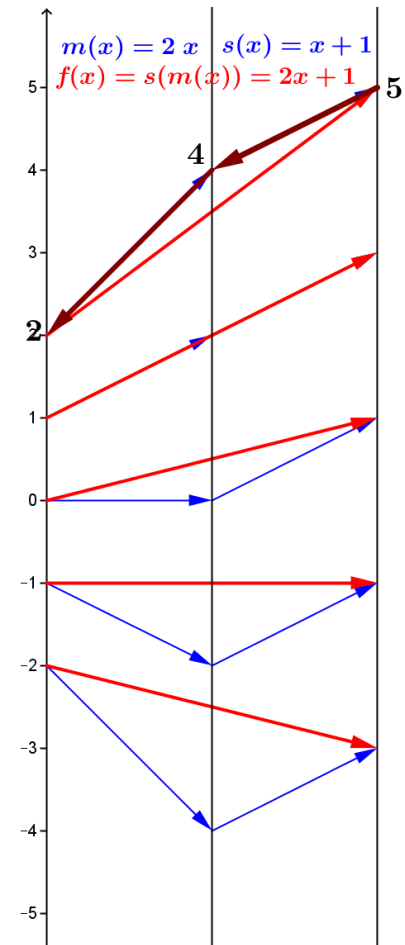
$$f(x) = s(m(x)) = 5$$

"Undo  $s$ "

$$m(x) = 4$$

"Undo  $m$ "

$$x = 2$$



# Worksheet 5.b Solving $2x + 1 = 5$ visualized with a mapping diagram

Algebra:

$$2x + 1 = 5$$

$$\underline{-1 = -1}$$

$$2x = 4$$

$$\underline{\frac{1}{2}(2x) = \frac{1}{2}(4)}$$

$$x = 2$$

Function:

$$f(x) = s(m(x)) = 5$$

"Undo  $s$ "

$$m(x) = 4$$

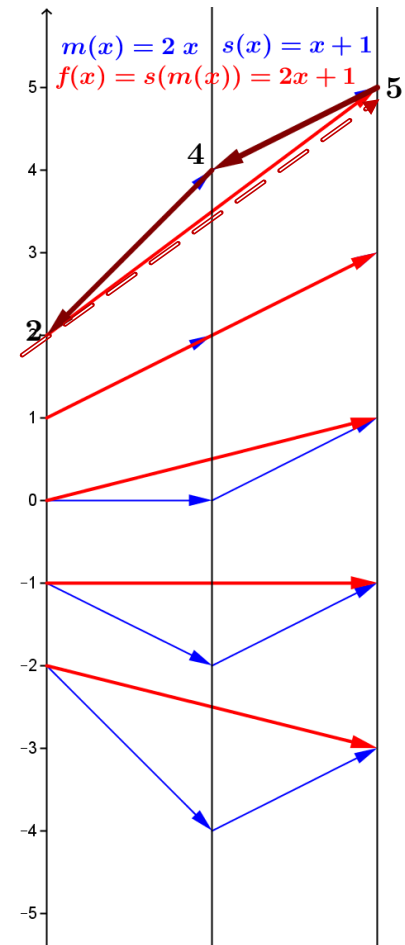
"Undo  $m$ "

$$x = 2$$



CHECK! 😊

$$f(2) = 5$$



# Worksheet 5.b Solving $2x + 1 = 5$ visualized on GeoGebra

Algebra:

$$2x + 1 = 5$$

$$\underline{-1 = -1}$$

$$2x = 4$$

$$\underline{\frac{1}{2}(2x) = \frac{1}{2}(4)}$$

$$x = 2$$

Function:

$$f(x) = s(m(x)) = 5$$

"Undo s"

$$m(x) = 4$$

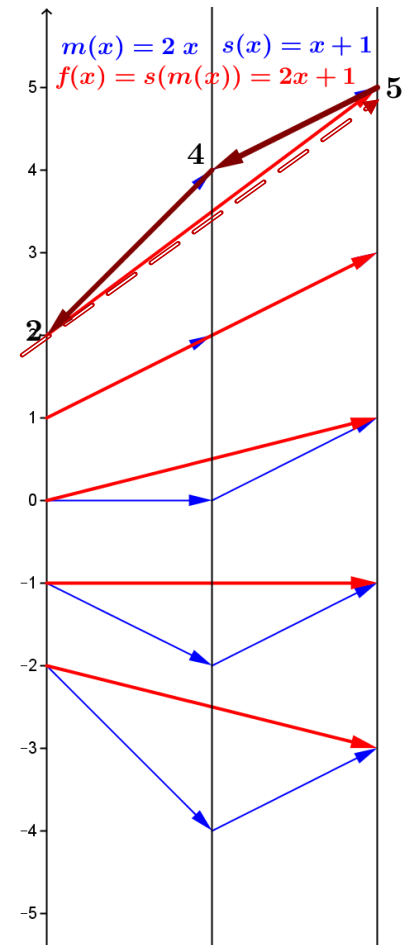
"Undo m"

$$x = 2$$

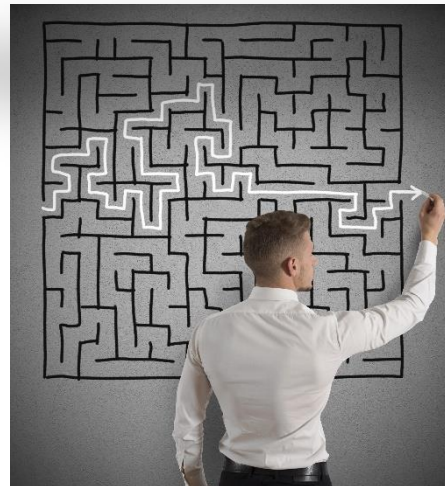


CHECK! 😊

$$f(2) = 5$$



Challenge: Solve  $2(x-3)^2 + 1 = 9$   
with a mapping diagram



# Worksheet 6.a Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

## **Understand the problem**

- $2(x-3)^2 + 1$  is a function of  $x$ .
  - $P(x) = 2(x-3)^2 + 1$
- Find any and all  $x$  where  $P(x) = 9$ .
- $2(x-3)^2 + 1$  is a composition of functions
  - $P(x) = s(m(q(z(x))))$  where
  - $z(x) =$
  - $q(x) =$
  - $m(x) =$
  - $s(x) =$

# Worksheet 6.a Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

## **Understand the problem**

- $2(x-3)^2 + 1$  is a function of  $x$ .
  - $P(x) = 2(x-3)^2 + 1$
- Find any and all  $x$  where  $P(x) = 9$ .
- $2(x-3)^2 + 1$  is a composition of functions
  - $P(x) = s(m(q(z(x))))$  where
  - $z(x) = x-3$ ;
  - $q(x) = x^2$  ;
  - $m(x) = 2x$ ;
  - $s(x) = x+1$ .

Worksheet 6.a Solve  $2(x-3)^2 + 1 = 9$   
with a mapping diagram.

### **Make a plan**

- Find any and all  $x$  where  $P(x) = 9$ .
- Construct mapping diagram for  $P$  as a composition of function :  
$$P(x) = s(m(q(z(x))))$$
- Undo  $P(x) = 9$  by undoing each step of  $P$ 
  - Undo  $s(x) = x+1$
  - Undo  $m(x) = 2x$
  - Undo  $q(x) = x^2$
  - Undo  $z(x) = x-3$
- Check results to see that  $P(x) = 9$



Worksheet 6.b Solve  $2(x-3)^2 + 1 = 9$   
with a mapping diagram.

Execute the **plan**

- Construct mapping diagram for  $P$  as a composition of function :

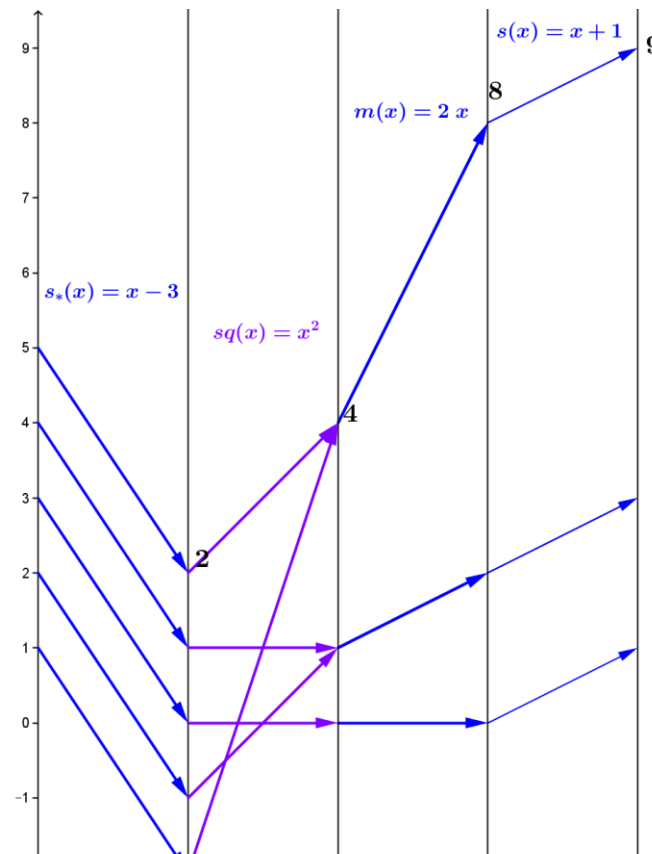
$$P(x) = s(m(q(z(x))))$$

Worksheet 6.b Solve  $2(x-3)^2 + 1 = 9$   
with a mapping diagram.

Execute the **plan**

- Construct mapping diagram for  $P$  as a composition of function :

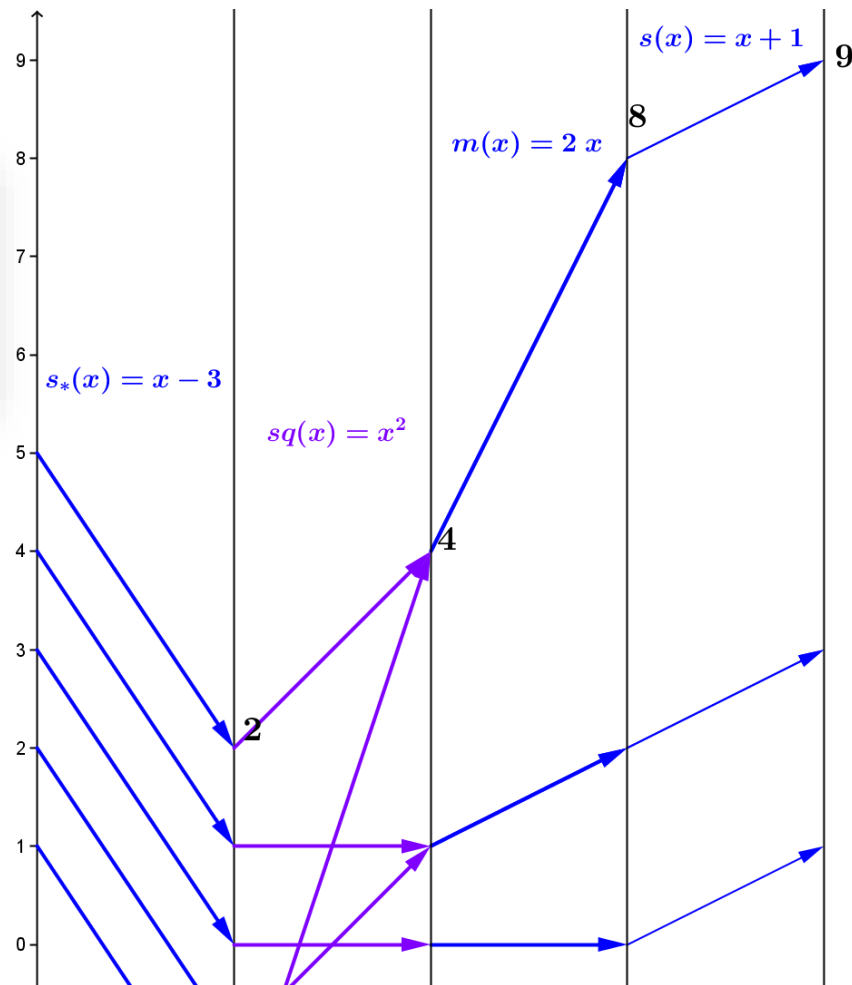
$$P(x) = s(m(q(z(x))))$$



# Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

## Execute the plan

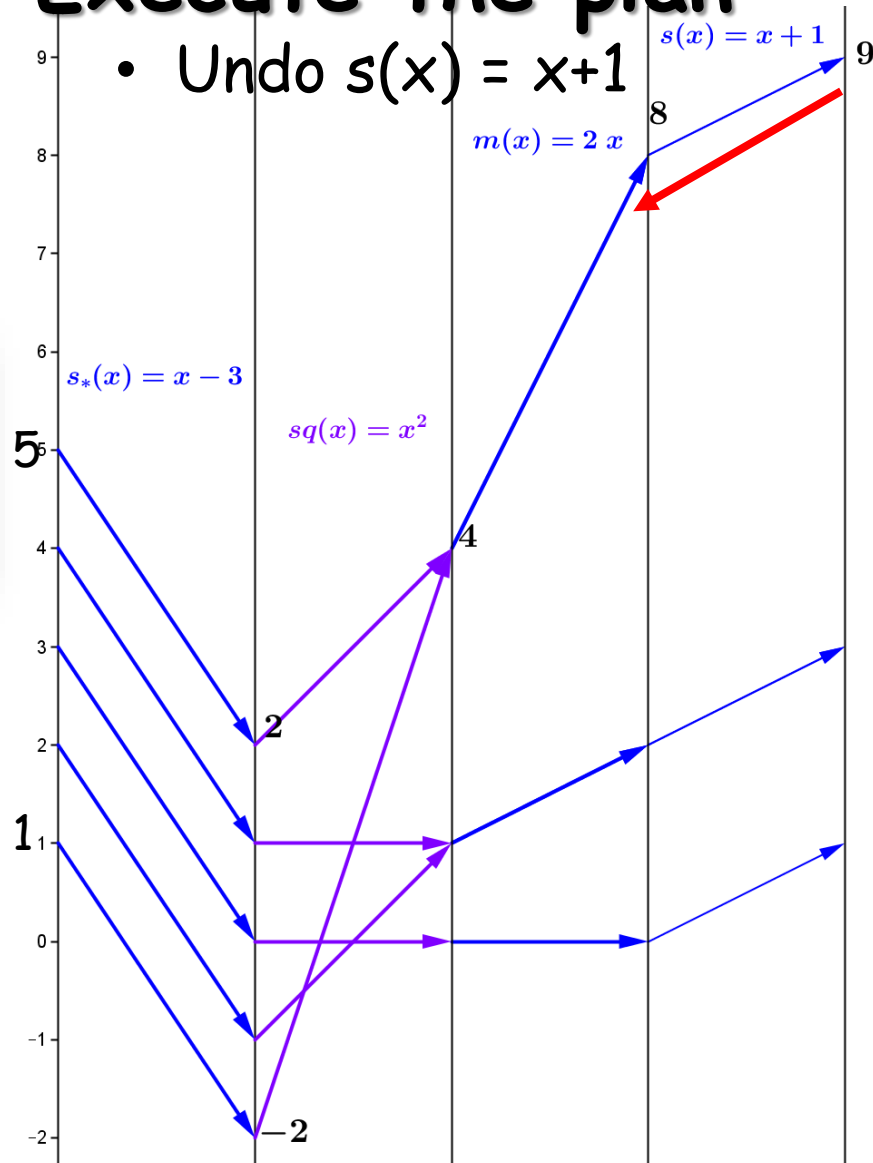
- Find any and all  $x$  where  $P(x) = 9$ .



# Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

## Execute the plan

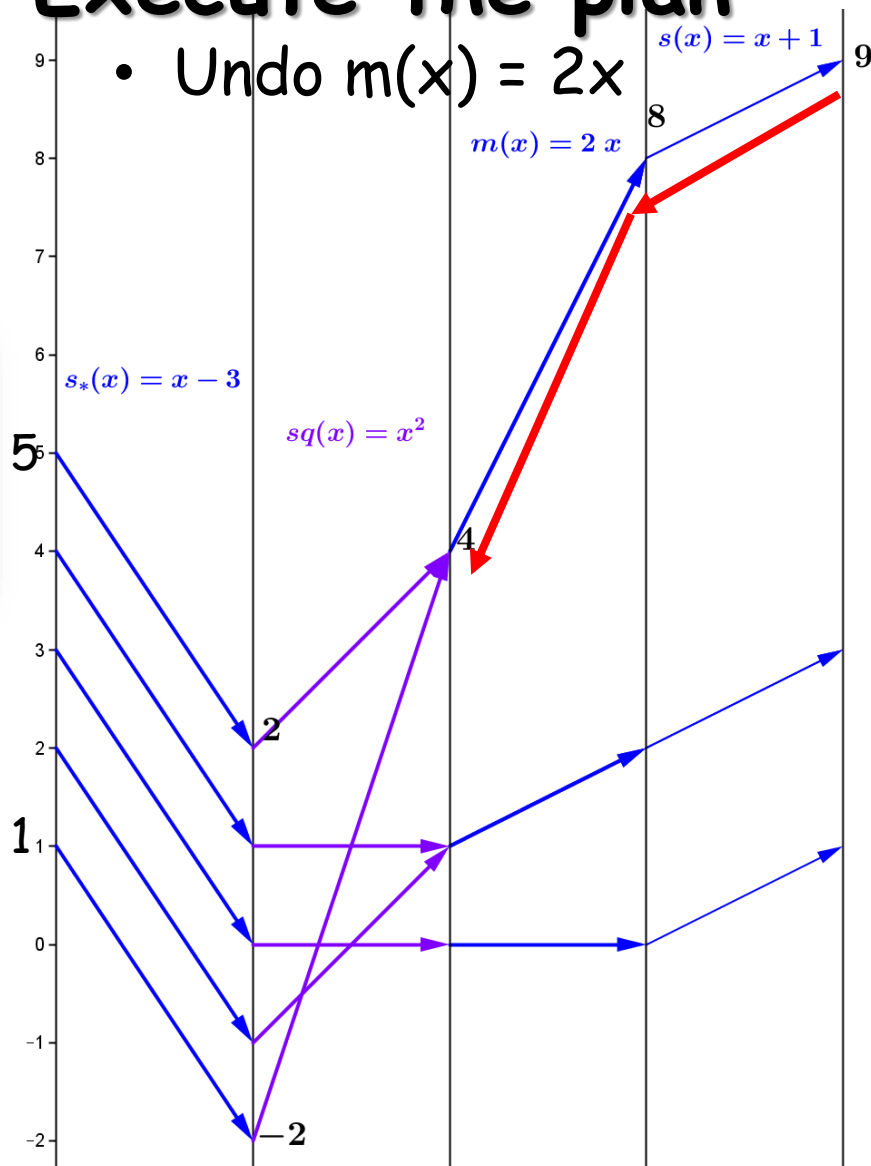
- Undo  $s(x) = x+1$



# Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

## Execute the plan

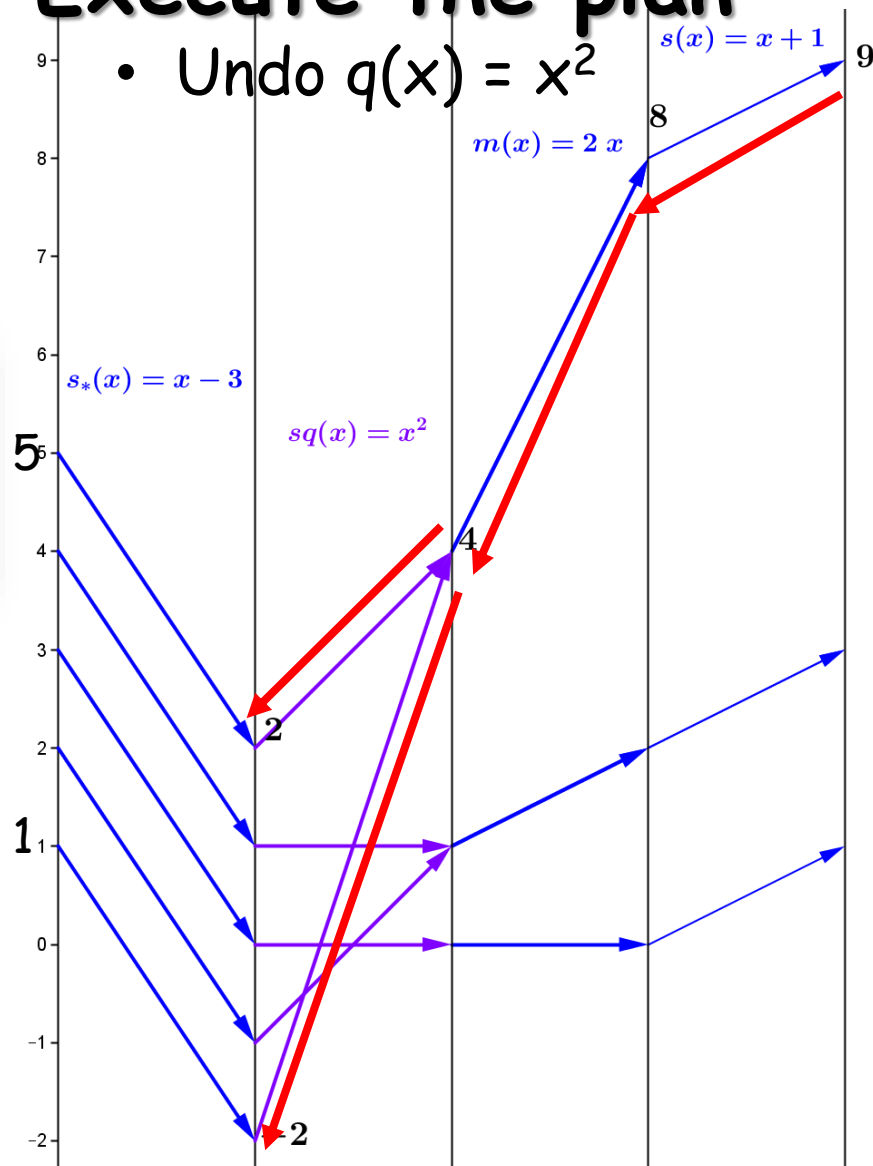
- Undo  $m(x) = 2x$



# Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

## Execute the plan

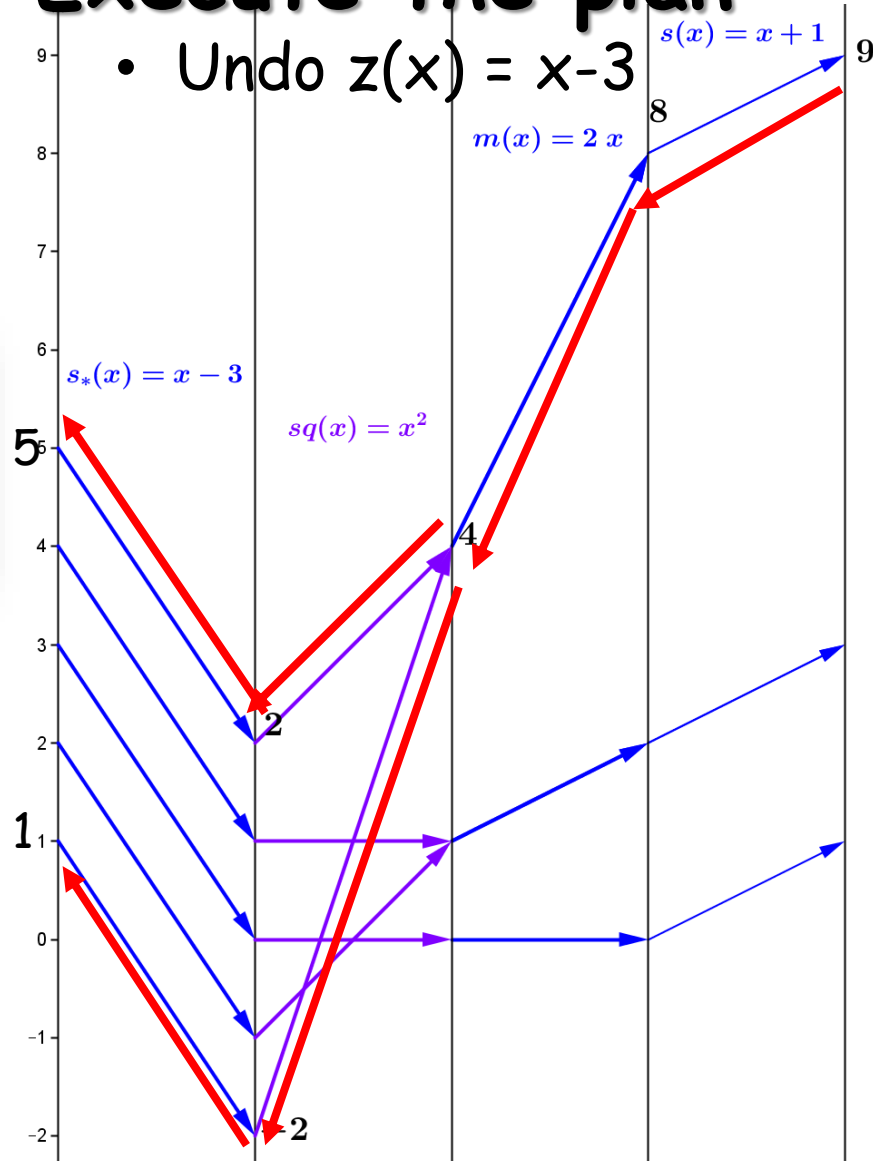
- Undo  $q(x) = x^2$



# Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

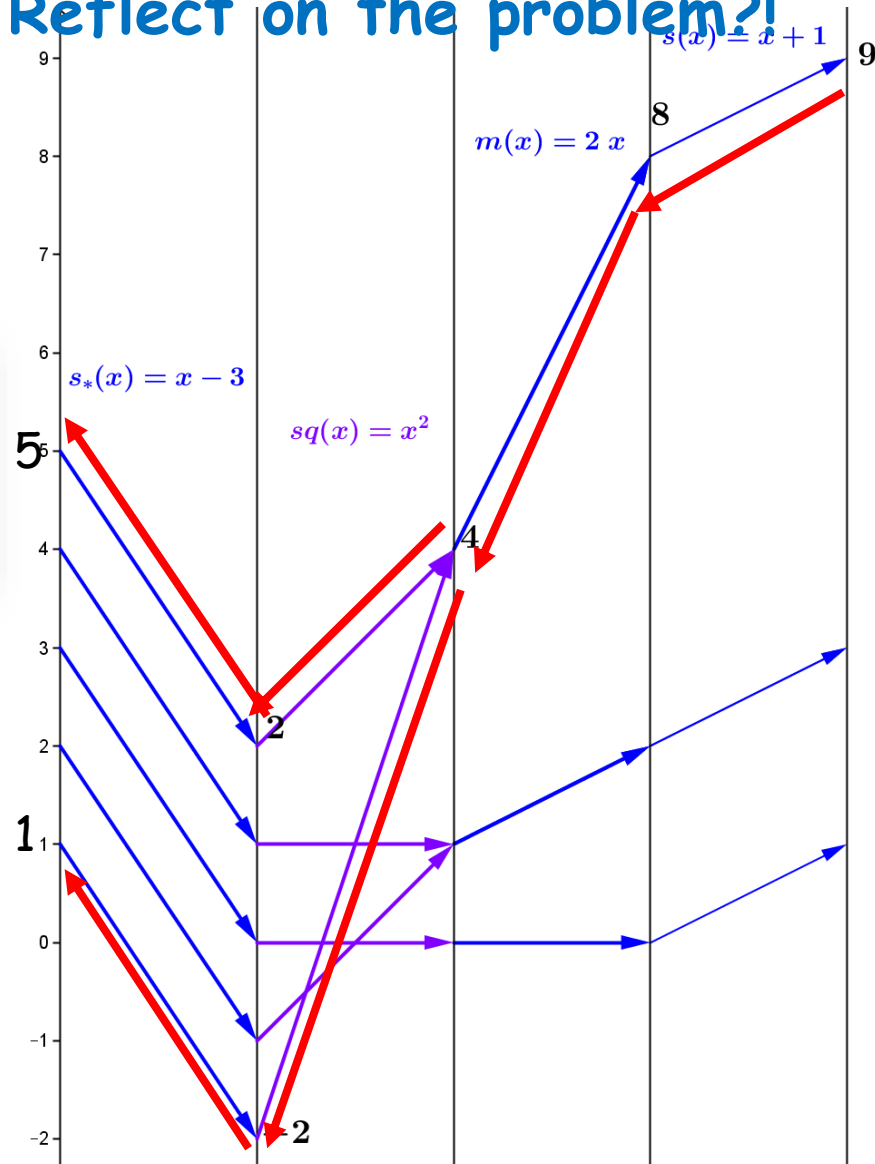
## Execute the plan

- Undo  $z(x) = x-3$



# Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Reflect on the problem?!



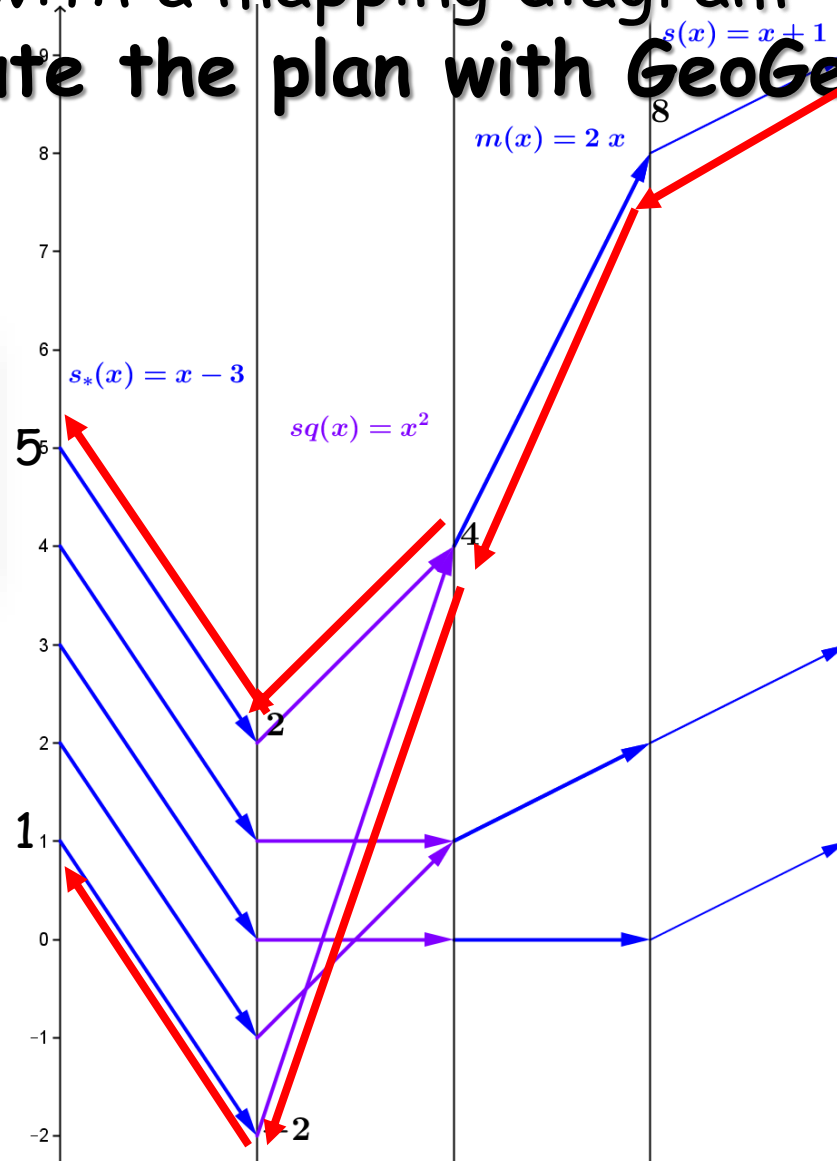


# Challenge: Worksheet 6.c

$$\text{Solve } 2(x-3)^2 + 1 = 9$$

with a mapping diagram

Execute the plan with GeoGebra



# Technology Examples

- Excel examples
- Geogebra examples

# Overtime?

Simple Examples are important!

- $f(x) = x + C$  Added value:  $C$
- $f(x) = mx$  Scalar Multiple:  $m$

Interpretations of  $m$ :

- slope
- rate
- Magnification factor
- $m > 0$  : Increasing function
- $m < 0$  : Decreasing function
- $m = 0$  : Constant function

# Simple Examples are important!

$f(x) = mx + b$  with a mapping diagram --

Five examples:

Back to Worksheet Problem #7

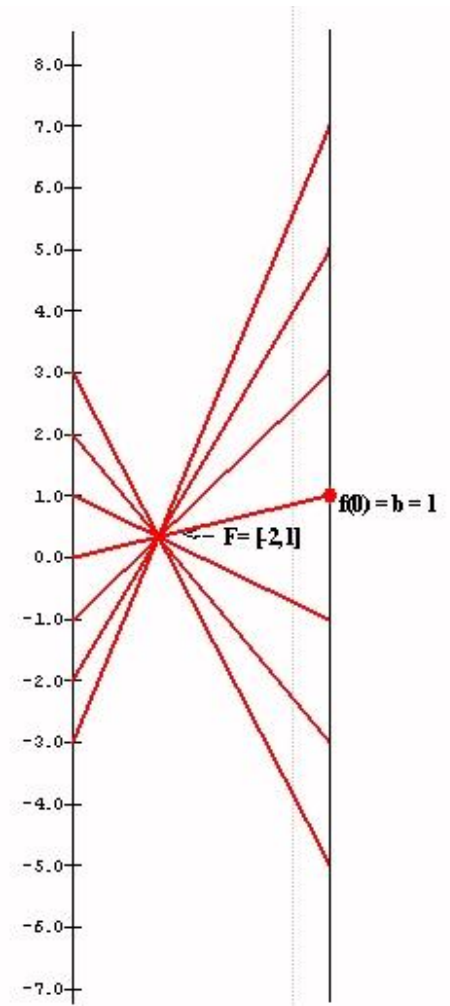
- Example 1:  $m = -2$ ;  $b = 1$ :  $f(x) = -2x + 1$
- Example 2:  $m = 2$ ;  $b = 1$ :  $f(x) = 2x + 1$
- Example 3:  $m = \frac{1}{2}$ ;  $b = 1$ :  $f(x) = \frac{1}{2}x + 1$
- Example 4:  $m = 0$ ;  $b = 1$ :  $f(x) = 0x + 1$
- Example 5:  $m = 1$ ;  $b = 1$ :  $f(x) = x + 1$

# Visualizing $f(x) = mx + b$ with a mapping diagram -- Five examples:

**Example 1:  $m = -2$ ;  $b = 1$**

$$f(x) = -2x + 1$$

- Each arrow passes through a single point, which is labeled  $F = [-2, 1]$ .
    - The point  $F$  completely determines the function  $f$ .
      - **given** a point / number,  $x$ , on the source line,
      - there is a **unique** arrow passing through  $F$
      - **meeting** the target line at a **unique** point / number,  $-2x + 1$ ,
- which corresponds to the linear function's value for the point/number,  $x$ .



# Visualizing $f(x) = mx + b$ with a mapping diagram -- Five examples:

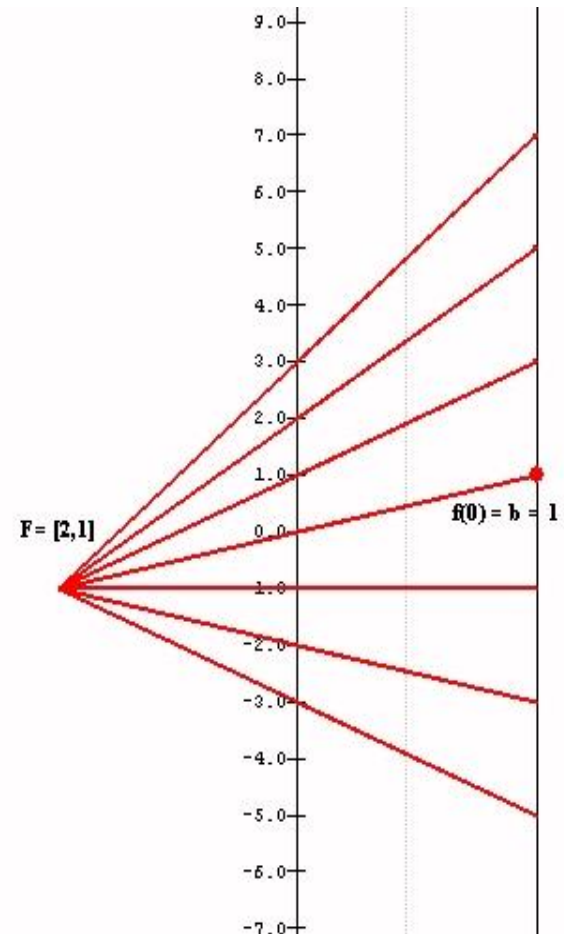
Example 2:  $m = 2; b = 1$

$$f(x) = 2x + 1$$

Each arrow passes through a single point, which is labeled

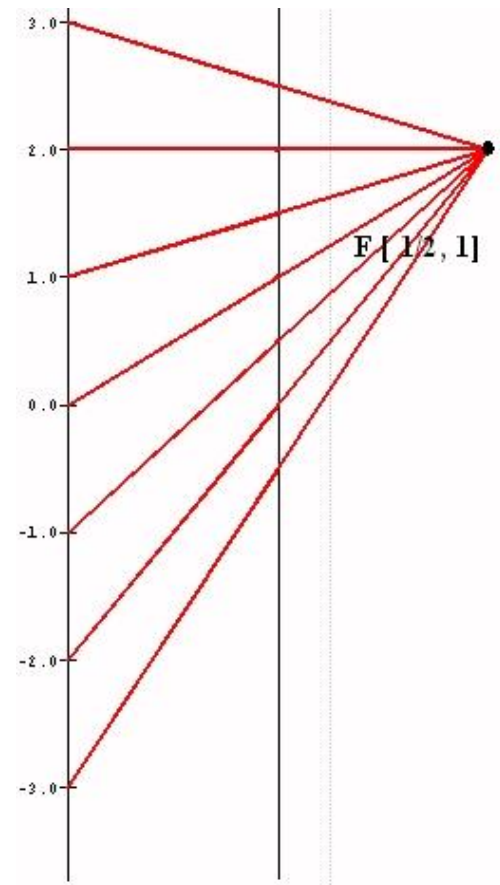
$$F = [2, 1].$$

- The point  $F$  completely determines the function  $f$ .
  - given a point / number,  $x$ , on the source line,
  - there is a **unique** arrow passing through  $F$
  - **meeting** the target line at a **unique** point / number,  $2x + 1$ ,which corresponds to the linear function's value for the point/number,  $x$ .



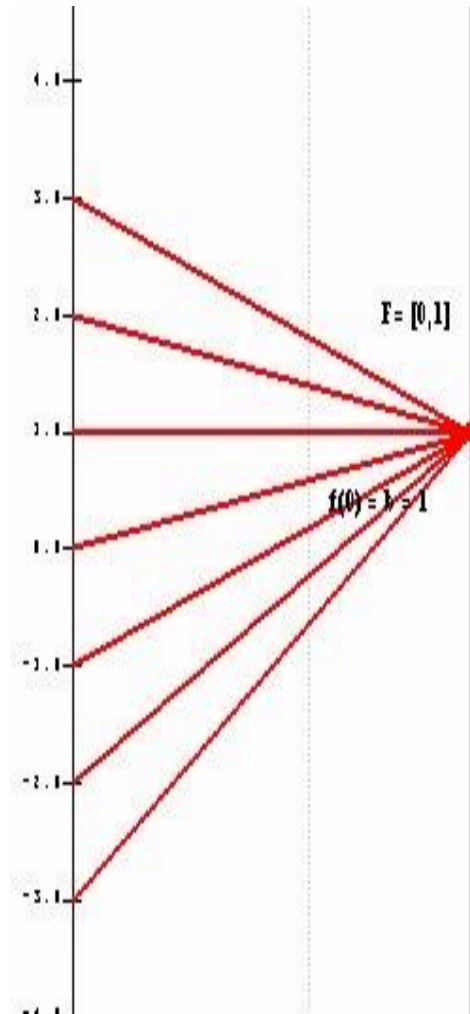
# Visualizing $f(x) = mx + b$ with a mapping diagram -- Five examples:

- **Example 3:  $m = 1/2$ ;  $b = 1$**   
 $f(x) = \frac{1}{2}x + 1$
  - Each arrow passes through a single point, which is labeled  $F = [1/2, 1]$ .
    - The point  $F$  completely determines the function  $f$ .
      - **given a point / number,  $x$ , on the source line,**
      - **there is a unique arrow passing through  $F$**
      - **meeting the target line at a unique point / number,  $\frac{1}{2}x + 1$ ,**
- which corresponds to the linear function's value for the point/number,  $x$ .



Visualizing  $f(x) = mx + b$  with a mapping diagram -- Five examples:

- **Example 4:  $m = 0$ ;  $b = 1$**   
 $f(x) = 0x + 1$
- Each arrow passes through a single point, which is labeled  $F = [0, 1]$ .
  - The point  $F$  completely determines the function  $f$ .
    - given a point / number,  $x$ , on the source line,
    - there is a **unique** arrow passing through  $F$
    - **meeting** the target line at a **unique** point / number,  $f(x)=1$ ,  
which corresponds to the linear function's value for the point/number,  $x$ .



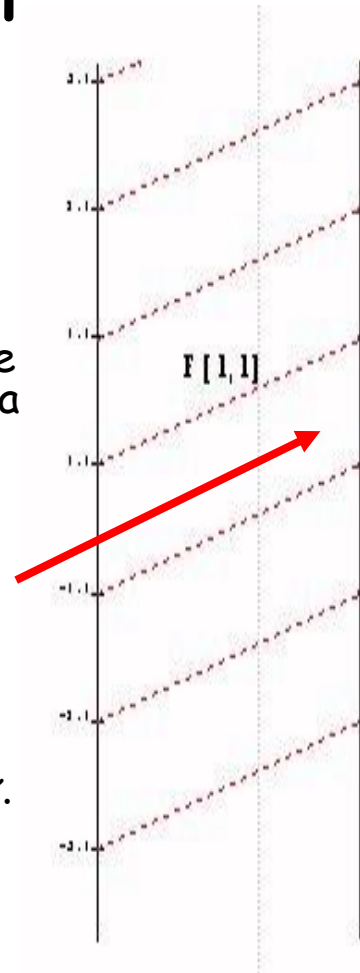


# Visualizing $f(x) = mx + b$ with a mapping diagram -- Five examples

Example 5:  $m = 1; b = 1$

$$f(x) = x + 1$$

- Unlike the previous examples, in this case it is not a single point that determines the mapping diagram, but the single arrow from 0 to 1, which we designate as  $F[1,1]$
  - It can also be shown that this single arrow completely determines the function. Thus, given a point / number,  $x$ , on the source line, there is a unique arrow passing through  $x$  **parallel to**  $F[1,1]$  meeting the target line a unique point / number,  $x + 1$ , which corresponds to the linear function's value for the point/number,  $x$ .
    - The single arrow completely determines the function  $f$ .
      - given a point / number,  $x$ , on the source line,
      - there is a **unique arrow** through  $x$  **parallel to**  $F[1,1]$
      - **meeting** the target line at a **unique point** / number,  $x + 1$ ,
- which corresponds to the linear function's value for the point/number,  $x$ .



# Simple Examples are important!

- $f(x) = x + C$  Added value:  $C$
- $f(x) = mx$  Scalar Multiple:  $m$

Interpretations of  $m$ :

- slope
- rate
- Magnification factor
- $m > 0$  : Increasing function
- $m < 0$  : Decreasing function
- $m = 0$  : Constant function

# Function-Equation Questions

## with linear focus points (Problem 8.a)

- Use a focus point in the mapping diagram to solve a linear equation:

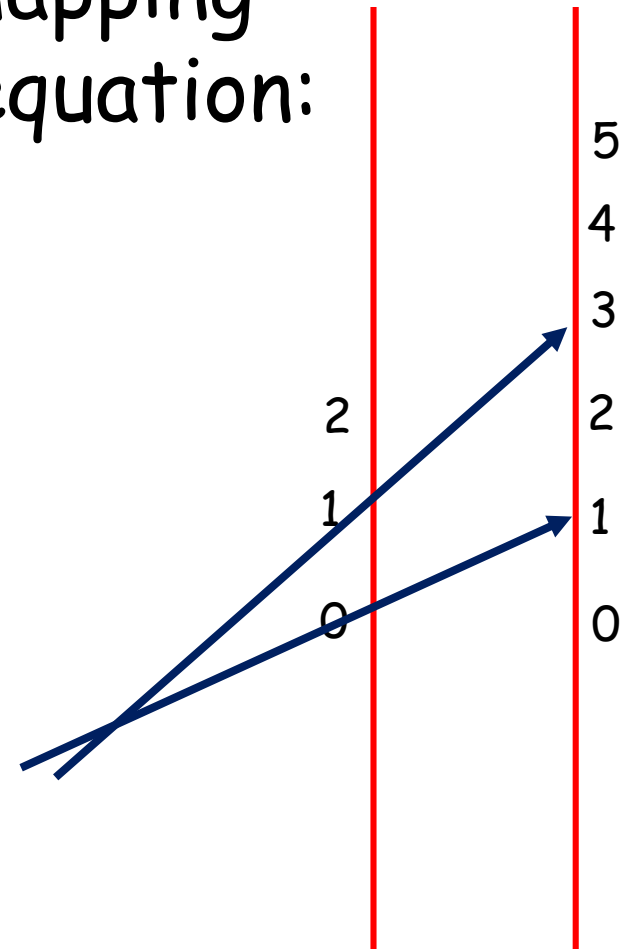
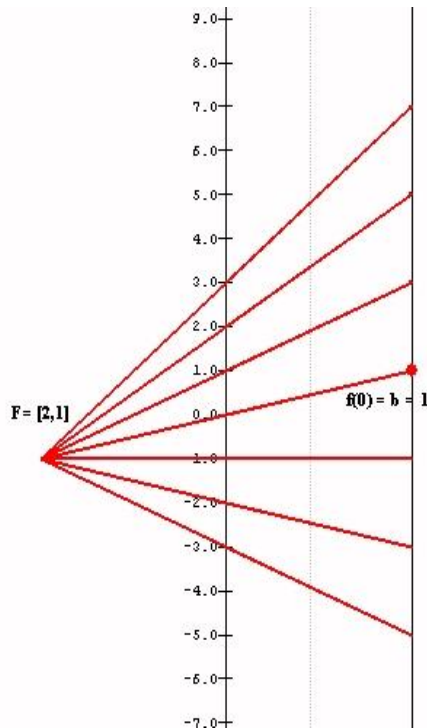
$$2x+1 = 5$$

# Function-Equation Questions

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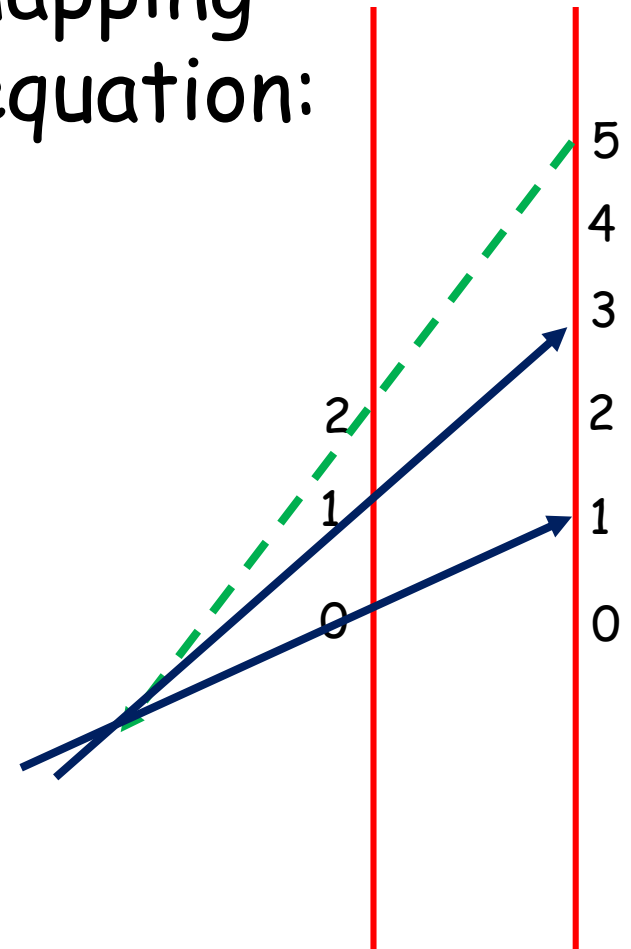
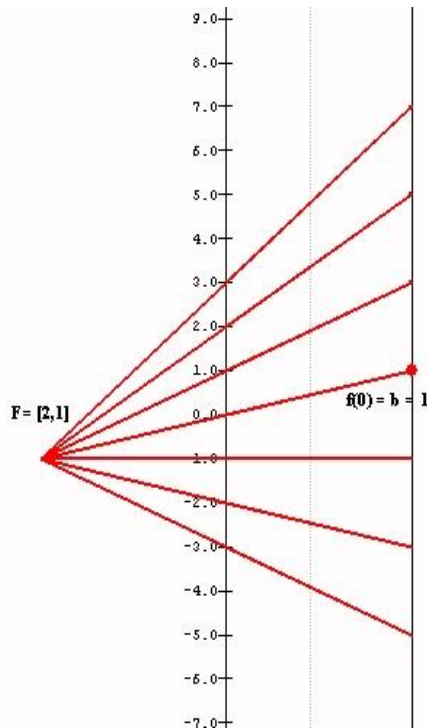


# Function-Equation Questions

## with linear focus points (Problem 8.a)

- Use a focus point in the mapping diagram to solve a linear equation:

$$2x+1 = 5$$



# Function-Equation Questions

## with linear focus points (Problem 8)

Suppose  $f$  is a linear function  
with  $f(1) = 3$  and  $f(3) = -1$ .

Without algebra

- 8.b Use a focus point to find  $f(0)$ .
- 8.c Use a focus point to find  $x$   
where  $f(x) = 0$ .

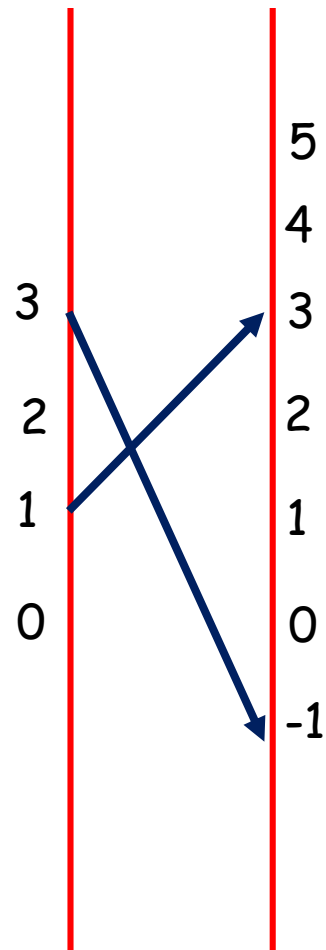
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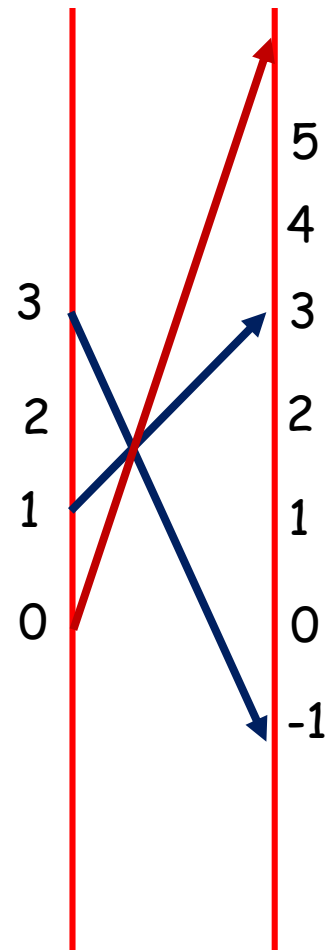
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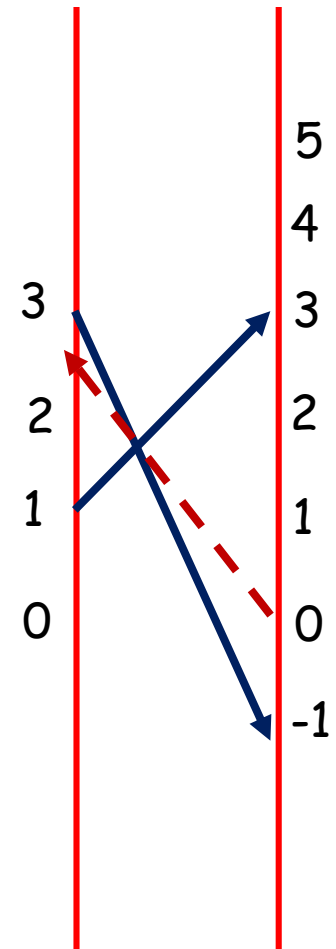
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Thanks  
The End!



Questions?

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# References

- Solving Linear Equations Visualized with Mapping Diagrams (YouTube) by M. Flashman
- Function Diagrams by Henri Picciotto  
Excellent Resources!
  - Henri Picciotto's Math Education Page
  - Some rights reserved
- Mapping Diagrams from A(lgebra) B(asics) to C(alculus) and D(ifferential) E(quation)s. A Reference and Resource Book on Function Visualizations Using Mapping Diagrams (Preliminary Sections- NOT YET FOR publication)  
<http://users.humboldt.edu/flashman/MD/section-1.1VF.html>
- Mapping Diagrams and Graphs... Visualizing linear functions using mapping diagrams and graphs. [tube.geogebra.org](http://tube.geogebra.org) Martin Flashman

Thanks  
The End! REALLY!



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