Introduction to Sensible Calculus: A Thematic Approach

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Day by Day Outline (Rev'd 6-26)

0. Sunday: Basic Themes Plus …
   - Mapping Diagrams
   - Technology (Winplot and Geogebra)

I. Monday: Making Sense of the Derivative.

II. Tuesday: More on the Derivative

III. Wednesday: DE's, Approximation and The Fundamental Theorem of Calculus

IV. Thursday: More on the FT, DE's, Models and Estimations

V. Friday: Frontiers-Making Sense of Taylor Theory, Probability.

Brief Review:
Concept and Pedagogical Principles

- Themes of **differential equations and estimation**, **using modeling**.
  - "*…everything in a calculus course can be related to the study of differential equations.* "
  - "*…estimation is valuable for both numerical and conceptual development.* "
- The consistent use of interpretations provides meaning for calculus concepts and helps develop habits of the mind.
  - Present examples of models or arguments before more general applications and proofs.
  - Form a foundation for later learning of concepts, language, and notation.
  - understand the specific and particular in experience and then **unify, generalize, …, abstract**.

Continuing from Last Class

Sensible Calculus: Two Forms of the Fundamental Theorem of Calculus

Evaluation Form

If f is continuous and $F'(x) = f(x)$ for all x …. then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Derivative Form (Barrow's Theorem)

If f is continuous and $G(t) = \int_a^t f(x)\,dx$ then

G is a differentiable function and $G'(t) = f(t)$.

FT of Calculus
Objective & Key Ideas

Two Key Ideas:

- When x is close to a, $f(x)$ is approximately equal to a linear function,

$$f(a) + f'(a)(x - a).$$

(The Differential/Linear Estimator)

- As long as f is a sufficiently well behaved function there is some c between a and b where

$$f(b) - f(a) = f'(c)(b - a).$$

(The Mean Value Theorem - MVT)

Sensible Proofs of FT (Evaluation)

- Use an extended Euler Sum to estimate $\int_a^b f(x)\,dx$ then use the Mean Value Theorem for each subinterval and watch the sum telescope!
  - For each interval choose c_k where $f(c_k) * \Delta x = F'(c_k) * \Delta x = F(x_{k+1}) - F(x_k)$ SO the sum "telescopes" to
    $$F(b) - F(a).$$
  - Interpret with motion and geometry.

Sensible Proofs of FT (Evaluation)

Use FT of C Derivative Form to justify FT of C Evaluation.

- $G'(t) = F'(t)$ so $G(t) = F(t) + C \;\; for\ some\ C.$
- $G(a) = 0 = F(a) + C$ so $C = -F(a)$
- So

$$\int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{f}(\boldsymbol{x})\, \boldsymbol{dx} = G(b) = F(b) + C = F(b) - F(a)$$

Session V : Frontiers-Probability, Economics, …

We complete our introduction to making calculus sensible by a consideration of **some frontiers and results that provide both motivation and consolidation for the first year experience with calculus.**

Focus Themes for Taylor Theory: Estimation, Differential Equations, Models

Focus on **estimating a growth model with a differential equation**:

$$P'(x) = P(x), \qquad P(0) = 1.$$

- Solution is already treated
  - with estimation by Euler's method.
  - "Exactly":

$$P(x) = e^x$$

Focus Themes for Taylor Theory: Estimation, Differential Equations, Models

$$\boldsymbol{P'(x) = P(x), \qquad P(0) = 1.}$$

- Estimation of the solution: Use the polynomial of degree n that best matches the differential equation.
- Determine estimate of error for estimating
  - e
  - $\int_0^1 e^{-x^2}\,dx$

Sample Exercises

1. Use the Taylor polynomial for e^x of degree 4 to estimate the following: $(a)e^2$ $(b)e^3$ $(c)e^{0.5}$ $(d)e^{-1}$ (e) $e^{3.14}$. [Spreadsheet helper supplied.]
2. Estimate e using the Taylor polynomial of degree n where n is (a) 6 (b) 7 (c) 8 (d) 10.
   In each estimate discuss the size of the error term R_n. [Spreadsheet helper.]
3. What value of n should be used so that the Taylor polynomial of degree n will give an estimate of e that is within .000001 of the exact value of e? Explain your result.
4. Use the Taylor polynomial for e^x of degree 5 to estimate $\int_0^1 e^{-t^2} dt$. Discuss the error in this approximation.

Sensible Calculus: Evolving Taylor Theory

- IX.A Taylor Theory for e^x
- IX.B MacLaurin Polynomials and Taylor Theory
- IX.C MacLaurin Polynomials: How to Find Them
- IX.D Taylor Polynomials

Session V : Frontiers-Probability

What are **some frontiers for the first year experience with calculus?**

- **Probability and Calculus**
  - **Choose Darts, NOT Dice**
  - **Start with distributions, not density**
  - **Make sense of calculus with probability**
  - **Make sense of probability with calculus**

Session V : Frontiers-Economics

What are **some frontiers for the first year experience with calculus?**

- **Economics and Calculus**
  - **Choose Micro before Macro**
  - **Start with margins**
  - **Make sense of calculus with economics**
  - **Make sense of economics with calculus**

Session V : Frontiers-History

What are **some frontiers for the first year experience with calculus?**

- **History and Calculus**
  - **Choose (original) sources before biographical sketches**
  - **Start with pre-Newton/Leibniz**
    - **Euclid, Archimedes, Galileo, Kepler, Descartes, Fermat, Barrow, …**
  - **Make sense of calculus with history**
  - **Make sense of history with calculus**

Session V : Frontiers-Probability More Detail…

- **Probability and Calculus**
  - **Darts, NOT Dice**
  - **Start with distributions, not density**
  - **Make sense of calculus with probability**
  - **Make sense of probability with calculus**

End of Session V

- Questions?

Thanks
The End!

Still have questions?
Comments?
e-mail them to me:
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Thanks

The End!

Questions?

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