Introduction to Sensible Calculus: A Thematic Approach

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Day by Day Outline (Rev’d 6-25)

0. Sunday: Basic Themes Plus ...
   - Mapping Diagrams
   - Technology (Winplot and Geogebra)

I. Monday: Making Sense of the Derivative.

II. Tuesday: More on the Derivative

III. Wednesday: DE’s, Approximation and The Fundamental Theorem of Calculus

IV. Thursday: More on DE’s, Models and Estimations. Making Sense of Taylor Theory and the Calculus of Series.

V. Friday: Frontiers—Probability, Economics, ...
Daily Assignment
Submit on paper or electronically.

- **Create one exercise and one problem** that incorporates (and/or extends) something from the session content.
- **Pose one question** related to the class content that you would like explained further. [I will respond privately unless you grant permission for a public response.]
- Take one (or two) topics discussed in the session and **discuss how you can incorporate** its content or technology into your teaching.
- **Electronic submissions may be shared with the class through the course webpage with submitter’s permission.**
- **OPTIONAL: Complete any worksheet or problems suggested during class.**
Concept and Pedagogical Principles

• Themes of differential equations and estimation run throughout the first year of calculus, using modeling as a central motivation for applications of the calculus.
  - “...everything in a calculus course can be related to the study of differential equations. “
  - “...estimation is valuable for both numerical and conceptual development. “

• The consistent use of interpretations provides meaning for calculus concepts.
  - “... models serve as sources for concepts and interpretations as well as for applications.”
  - Present examples of models or arguments before more general applications and proofs.
• Habits of the mind
  - develop through informal understanding
  - form a foundation for later learning of concepts, language, and notation.
  - understand the specific and particular in experience and then unify, generalize, ..., abstract.
  - DON'T start with a general proposition or abstract proof and then apply the general and abstract to the particular.
  - Examples: Evolution of the derivative and integral

• A topic sensibly organized by itself and sensibly placed with regard to other topics, should remain a part of the course. But a topic failing to make sense, locally or globally, needs careful reassessment and revision.
Continuing from Last Class
Making Sense of Calculus: The Derivative Calculus

• Product Rule SC II.A
• Motivate with Linearity in Algebra
  – Linear Estimation
• Connect to Rate Interpretation
  – Rectangular Area
  – Mapping Diagram of Sides
  – Using a mapping diagram with a rectangle to visualize the 4 step method for finding the derivative of a product.
• Continuity and Differentiability Connection
Making Sense of Calculus: The Derivative Calculus

• Chain Rule SC II.B
• Motivate with Linearity in Algebra
  - Linear Estimation
• Connect to Rate Interpretation
  - Gas consumption, Motion, Time
  - Mapping Diagram for Composition
  - Visualizing the estimate of the quotients on mapping diagrams and some of the details if $\Delta x = 0$.
  - Pattern Recognition in the Leibnitz Notation
• Using the chain rule in implicit differentiation.
Making Sense of Calculus: Applications to Estimation

- Local Linearity and the Differential

III.A.1

- Linear Estimation Function:
  - Geometric Interpretation (Slope of Tangent line)
  - Motion Interpretation (Mapping Diagram, Magnification and Focus Point)

- Leibniz Notation and the Differential

- Using the second derivative (acceleration) to determine the quality of the differential estimate.
  - [Aristotle: The race track principle.]
Making Sense of the Calculus of Derivatives

• Finding derivatives from the definition can be tedious for more complicated elementary functions.
• The calculus is a systematic procedure for finding the derivatives of elementary functions.
• An elementary function is a function built from a list of core functions by applying addition, subtraction, multiplication, division, and composition to the core functions and their inverses.
• The Core Functions (Short list): $c, x^n, e^x, \sin(x)$
• (Others) $x^r, b^x, \ln(x), \cos(x), \tan(x), \sec(x)$
• Rules: Linearity, Product, Quotient, Chain
Making Sense of a Differential Equation and the Fundamental Theorem of Calculus

• Example: The following differential equations of the form $\frac{dy}{dx} = P(x)$ have solutions that cannot be expressed as an elementary function.

- $\frac{dy}{dx} = \sin(x^2)$
- $\frac{dy}{dx} = e^{-x^2}$

• The solutions to these are given by using the FT of C:

$$y = f(t) = \int_0^t P(x)\,dx$$

The Fundamental Theorem of Calculus says:

When $P(x)$ is continuous, then $\frac{dy}{dt} = P(t)$. 
The Fundamental Theorem of Calculus
Derivative Form

If $f$ is continuous and $G(t) = \int_a^t f(x) \, dx$ then $G$ is a differentiable function and $G'(t) = f(t)$.

Interpretation:

$f(x)$ is velocity of object at time $x$.

$G(t)$ is the net change in position of object from time $a$ to time $t$.

$G'(t) =$ velocity of object at time $t$. 
Making Sense of Calculus: Applications to Estimation

- Intermediate Value Theorem, Roots and Continuity.

**SC I.I.2.** Intermediate Values
  - Bisection Algorithm
    - Graphical
    - Mapping Diagrams
  - Spreadsheets
Making Sense of Calculus: Applications to Estimation

- Linearity and Estimating Roots
  III.A.2
  - Linear Estimation Function:
    - Geometric Interpretation (Slope of Tangent line)
    - Motion Interpretation (Mapping Diagram, Magnification and Focus Point)
  - Solving for roots in linear functions.
    - Brief excursion into inverses for linear functions.
    - More mapping diagrams!
Examples on Excel, Winplot, Geogebra

• Excel example(s):
  - Linear Mapping Diagram example
  - Newtons Method

• Winplot examples:
  - Linear Mapping Diagram-composition examples
  - Linear Graph Linked File-composition examples

• Geogebra examples:
  - IV Steps
  - Secant Tangent
  - Alternative Derivative for Sine.
End of Session II

Questions for next session?
Catch me between sessions or e-mail them to me:
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