Introduction to Sensible Calculus: A Thematic Approach

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Day by Day Outline (Rev'd 6-25)

0. Sunday: Basic Themes Plus …
   - Mapping Diagrams
   - Technology (Winplot and Geogebra)

I. Monday: Making Sense of the Derivative.

II. Tuesday: More on the Derivative

III. Wednesday: DE's, Approximation and The Fundamental Theorem of Calculus

IV. Thursday: More on DE's, Models and Estimations. Making Sense of Taylor Theory and the Calculus of Series.

V. Friday: Frontiers-Probability, Economics, …

Daily Assignment
Submit on paper or electronically.

- **Create one exercise and one problem** that incorporates (and/or extends) something from the session content.
- **Pose one question** related to the class content that you would like explained further. [I will respond privately unless you grant permission for a public response.]
- Take one (or two) topics discussed in the session and **discuss how you can incorporate** its content or technology into your teaching.
- **Electronic submissions may be shared with the class through the course webpage with submitter's permission.**
- **OPTIONAL: Complete any worksheet or problems suggested during class.**

Concept and Pedagogical Principles

- Themes of **differential equations and estimation** run throughout the first year of calculus, **using modeling** as a central motivation for applications of the calculus.
  - *"…everything in a calculus course can be related to the study of differential equations. "*
  - *"…estimation is valuable for both numerical and conceptual development. "*
- The consistent use of interpretations provides meaning for calculus concepts.
  - *"… models serve as sources for concepts and interpretations as well as for applications."*
  - Present examples of models or arguments before more general applications and proofs.

- Habits of the mind
  - develop through informal understanding
  - form a foundation for later learning of concepts, language, and notation.
  - understand the specific and particular in experience and then **unify**, **generalize**, …, **abstract**.
  - DON'T start with a *general* proposition or abstract proof and then apply the *general* and abstract to the particular.
  - Examples: Evolution of the derivative and integral
- A topic sensibly organized by itself and sensibly placed with regard to other topics, should remain a part of the course. But a topic failing to make sense, locally or globally, needs careful reassessment and revision.

Continuing from Last Class

Making Sense of Calculus: The Derivative Calculus

- Product Rule SC II.A
- Motivate with Linearity in Algebra
  - Linear Estimation
- Connect to Rate Interpretation
  - Rectangular Area
  - Mapping Diagram of Sides
  - Using a mapping diagram with a rectangle to visualize the 4 step method for finding the derivative of a product.
- Continuity and Differentiability Connection

Making Sense of Calculus: The Derivative Calculus

- Chain Rule SC II.B
- Motivate with Linearity in Algebra
  - Linear Estimation
- Connect to Rate Interpretation
  - Gas consumption, Motion, Time
  - Mapping Diagram for Composition
  - Visualizing the estimate of the quotients on mapping diagrams and some of the details if $\Delta x = 0$.
  - Pattern Recognition in the Leibnitz Notation
- Using the chain rule in implicit differentiation.

Making Sense of Calculus: Applications to Estimation

- Local Linearity and the Differential
  III.A.1
  - Linear Estimation Function:
    - Geometric Interpretation (Slope of Tangent line)
    - Motion Interpretation (Mapping Diagram, Magnification and Focus Point)
  - Leibniz Notation and the Differential
  - Using the second derivative (acceleration) to determine the quality of the differential estimate.
    - [Aristotle: The race track principle.]

Making Sense of the Calculus of Derivatives

- Finding derivatives from the definition can be tedious for more complicated elementary functions.
- The calculus is a systematic procedure for finding the derivatives of elementary functions.
- An elementary function is a function built from a list of core functions by applying addition, subtraction, multiplication, division, and composition to the core functions and their inverses.
- The Core Functions (Short list): $c, x^n, e^x, \sin(x)$
- (Others) $x^r,\ b^x,\ \ln(x), \cos(x), \tan(x), \sec(x)$
- Rules: Linearity, Product, Quotient, Chain

Making Sense of a Differential Equation and the Fundamental Theorem of Calculus

- Example: The following differential equations of the form $\frac{dy}{dx} = P(x)$ have solutions that cannot be expressed as an elementary function.
  - $\frac{dy}{dx} = \sin(x^2)$
  - $\frac{dy}{dx} = e^{-x^2}$
- The solutions to these are given by using the FT of C:

$$y = f(t) = \int_0^t P(x)dx$$

The Fundamental Theorem of Calculus says:

When $P(x)$ is continuous, then $\frac{dy}{dt} = P(t)$.

The Fundamental Theorem of Calculus
Derivative Form

If $\boldsymbol{f}$ is continuous and $\boldsymbol{G(t) = \int_a^t f(x)\,dx}$ then

$\boldsymbol{G}$ is a differentiable function and $\boldsymbol{G'(t) = f(t)}$.

Interpretation:

$f(x)$ is velocity of object at time x.

$G(t)$ is the net change in position of object from time a to time t.

$G'(t)$ = velocity of object at time t.

Making Sense of Calculus: Applications to Estimation

- Intermediate Value Theorem, Roots and Continuity. SC I.I.2. Intermediate Values
  - Bisection Algorithm
    - Graphical
    - Mapping Diagrams
  - Spreadsheets

Making Sense of Calculus: Applications to Estimation

- Linearity and Estimating Roots
  III.A.2
  - Linear Estimation Function:
    - Geometric Interpretation (Slope of Tangent line)
    - Motion Interpretation (Mapping Diagram, Magnification and Focus Point)
  - Solving for roots in linear functions.
    - Brief excursion into inverses for linear functions.
    - More mapping diagrams!
  - Newton's Method Algorithms. Estimation appications to error estimates.

Examples on Excel, Winplot, Geogebra

- **Excel example(s):**
  - **Linear Mapping Diagram example**
  - **Newtons Method**
- **Winplot examples:**
  - **Linear Mapping Diagram-composition examples**
  - **Linear Graph Linked File-composition examples**
- Geogebra examples:
  - IV Steps
  - Secant Tangent
  - Alternative Derivative for Sine.

End of Session II

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Questions for next session?
Catch me between sessions or e-mail them to me:
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- [FL1] Flashman, Martin. "Differential Equations: A Motivating Theme for A Sensible Calculus," in "Calculus for All Users" The Report of A Conference on Calculus and Its Applications Held at the University of Texas, San Antonio, NSF Calculus Reform Conference, October 5 - 8, 1990.
- [UMAP] Flashman, Martin. "*A Sensible Calculus.*," The UMAP Journal, Vol. 11, No. 2, Summer, 1990, pp. 93-96.
- [FL2] Flashman, Martin. "Using Computers to Make Integration More Visual with Tangent Fields," appearing in Proceedings of the Second Annual Conference on Technology in Collegiate Mathematics, Teaching and Learning with Technology of November 2-4, 1989, edited by Demana, Waits, and Harvey, Addison-Wesley, 1991.
- [FL3] Flashman, Martin. "Concepts to Drive Technology," in Proceedings of the Fifth Annual Conference on Technology in Collegiate Mathematics, November 12-15, 1992, edited by Lewis Lum, Addison-Wesley, 1994.
- [FL4] Flashman, Martin. "Historical Motivation for a Calculus Course: Barrow's Theorem," in Vita Mathematica: Historical Research and Integration with Teaching, edited by Ronald Calinger, MAA Notes, No. 40, 1996.