Introduction to Sensible Calculus: A Thematic Approach

The Anja S. Greer Conference on Mathematics, Science and Technology

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Day by Day Outline

0. **Sunday: Basic Themes Plus …**
   - **Mapping Diagrams**
   - **Technology (Winplot and Geogebra)**

I. Monday: Making Sense of the Derivative

II. Tuesday: DE's, Approximation and The Fundamental Theorem of Calculus

III. Wednesday: More on DE's, Models and Estimations

IV. Thursday: Making Sense of Taylor Theory and the Calculus of Series

V. Friday: Frontiers-Probability, Economics, …

Daily Assignment
Submit on paper or electronically.

- **Create one exercise and one problem** that incorporates (and/or extends) something from the session content.
- **Pose one question** related to the class content that you would like explained further. [I will respond privately unless you grant permission for a public response.]
- Take one (or two) topics discussed in the session and **discuss how you can incorporate** its content or technology into your teaching.
- **Electronic submissions may be shared with the class through the course webpage with submitter's permission.**
- **OPTIONAL: Complete any worksheet or problems suggested during class.**

What's Happening Now in The First Calculus Course

- Differential Calculus
- Integral Calculus THEN...
- Sequences and Sums of Numbers: the Theory and Tests of Convergence
- Power series
- Taylor / MacLaurin Series and Applications:
  - Estimation of
    - Numbers: $e, \pi, \sqrt{2}, ln(2)$ ...
    - Definite Integrals.
    - Solutions of Differential Equations.

What's Happening Now in The First Calculus Course

- **Critique**
  - **Little motivation from previous work**
  - **Delayed Connection with the previous progress**
  - **Unclear statement of what is fundamental.**

Making Sense of The First Calculus Course

- Experience- Spring 1978, 1979
- Two years of teaching 2nd semester of Calculus with students who had 1st semester with another instructor.
- Need for a better approach for all of first year.

Making Sense of The First Calculus Course

- “The key to solving the 'calculus problem'.... make sense in our calculus instruction ....
- internally and in context,
- to ourselves as instructors and
- to our students as learners.
- This criterion will provide the knife for cutting and the thread for reassembling the calculus curriculum of the next 40 to 50 years.”

Quote was from my Editorial that appeared in The UMAP Journal in 1990.

"A Sensible Calculus"

Making Sense of The First Calculus Course

Three themes…for reviewing and revising the calculus curriculum, namely,

- **Differential Equations,**
- **Estimation, and**
- **Mathematical Modeling**

The Sensible Calculus Program

- Primary Creator: Martin Flashman
  - Since 1981.
  - Associate: Tami Matsumoto (since 2006)
- New main web page (2002- latest technology revisions starting 2013)
- Many materials currently available on-line through this LINK for Current Materials for Exeter
- Key content and pedagogical concepts will be the focus of this week.

Concept and Pedagogical Principles

- Themes of **differential equations and estimation** run throughout the first year of calculus, **using modeling** as a central motivation for applications of the calculus.
  - *"…everything in a calculus course can be related to the study of differential equations. "*
  - *"…estimation is valuable for both numerical and conceptual development. "*
- The consistent use of interpretations provides meaning for calculus concepts.
  - *"… models serve as sources for concepts and interpretations as well as for applications."*
  - Present examples of models or arguments before more general applications and proofs.

- Habits of the mind
  - develop through informal understanding
  - form a foundation for later learning of concepts, language, and notation.
  - understand the specific and particular in experience and then **unify**, **generalize**, …, **abstract**.
  - DON'T start with a general proposition or abstract proof and then apply the general and abstract to the particular.
  - Examples: Evolution of the derivative and integral
- A topic sensibly organized by itself and sensibly placed with regard to other topics, should remain a part of the course. But a topic failing to make sense, locally or globally, needs careful reassessment and revision.

A Sensible Tool: Mapping Diagrams

We begin our introduction with

- A useful tool- **mapping diagrams-**
- And a key to calculus- **linear functions**

$$y = f(x) = mx + b$$

Sensible Calculus and Linear Functions

Why are linear functions important?

- A linear model is the simplest model of change :
  - Initial values: $f(0) = b$
  - Constant rates: m
- A linear model is the simplest model of accumulation :
  - $\Delta\, distance = rate * \Delta\, time; \Delta\, distance = \Delta\, rate * time$
  - $\Delta\, cost = price * \Delta\, quantity; \Delta\, cost = \Delta\, price * quantity$
  - $\Delta\, pay = rate * \Delta time\, ; \Delta\, pay = \Delta rate * time$

Mapping Diagrams And Linear Functions

A.k.a.
Function diagrams
Transformation Figures
Dynagraphs

Mapping Diagrams by Hand

MD Worksheet

Linear Functions: Tables

x	5 x - 7
3	
2	
1	
0	
-1	
-2	
-3	

Complete the table.

$x = 3,2,1,0,-1,-2,-3$

$f(x) = 5x - 7$

$f(0) =$ ____?

For which x is $f(x) > 0$?

Linear Functions: Tables

x	5 x – 7
3	8
2	3
1	-2
0	-7
-1	-12
-2	-17
-3	-22

Complete the table.

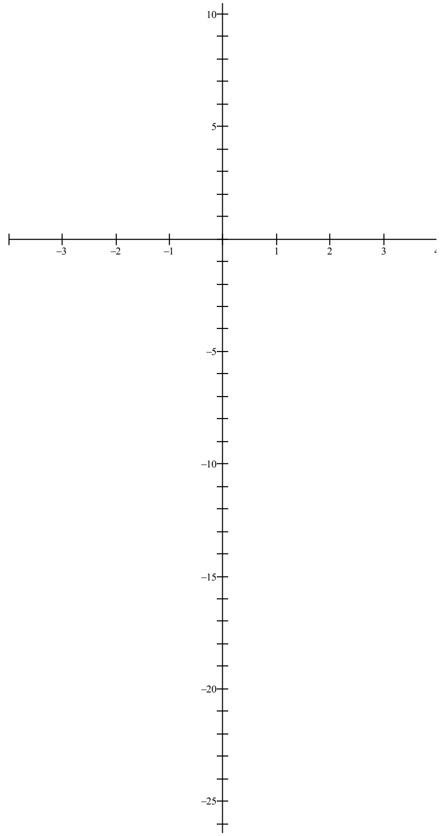
$x = 3,2,1,0,-1,-2,-3$

$f(x) = 5x - 7$

$f(0) =$ ____?

For which x is $f(x) > 0$?

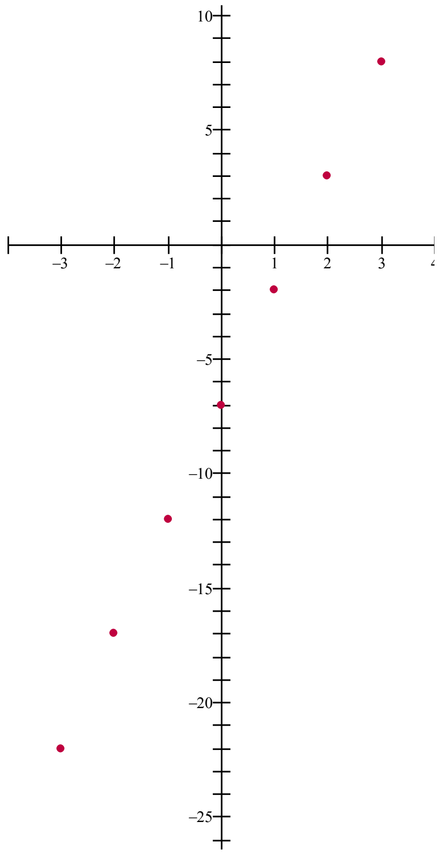
Linear Functions: On Graph



Plot Points (x , 5x - 7):

X	5 x – 7
3	8
2	3
1	-2
0	-7
-1	-12
-2	-17
-3	-22

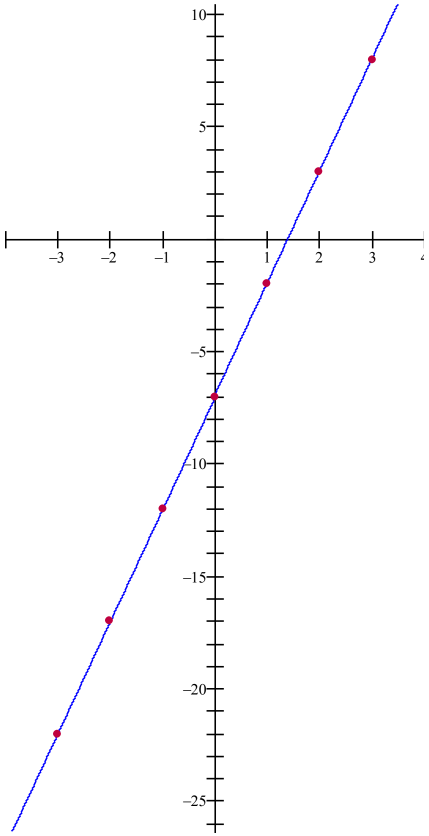
Linear Functions: On Graph



Connect Points
$(x\ ,\ 5x - 7)$:

X	5 x - 7
3	8
2	3
1	-2
0	-7
-1	-12
-2	-17
-3	-22

Linear Functions: On Graph



Connect the Points

X	5 x – 7
3	8
2	3
1	-2
0	-7
-1	-12
-2	-17
-3	-22

Linear Functions: Mapping Figures

- Connect point x on left axis to the point 5x – 7 on the right axis.

X	5 x – 7
3	8
2	3
1	-2
0	-7
-1	-12
-2	-17
-3	-22

Linear Functions: Mapping Figures

X	5 x – 7
3	8
2	3
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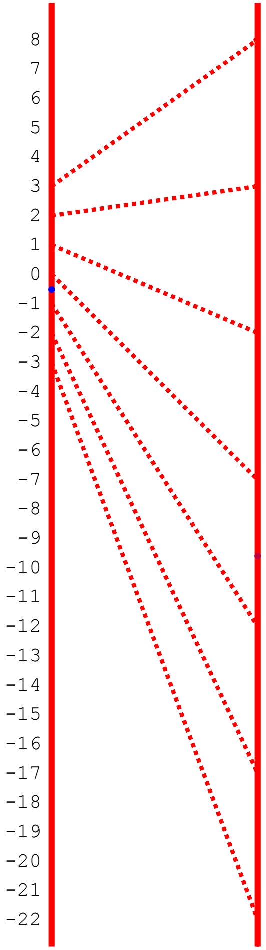


Sensible Calculus and Mapping Diagram Resources

- **Sensible Calculus Visualizations and Mapping Diagrams**
- **Mapping Diagrams from A(lgebra) B(asics) to C(alculus) and D(ifferential) E(quation)s.**
  A Reference and Resource Book on Function Visualizations Using Mapping Diagrams

Examples with Technology

LINK for Current Materials

- Excel examples
- Winplot examples
- Geogebra examples
- SketchPad examples

Simple Examples are important!

$\boldsymbol{f(x) = mx + b}$ with a mapping diagram –

Five examples:

- Example 1: $m = -2$; $b = 1$: $f(x) = -2x + 1$
- Example 2: $m = 2$; $b = 1$: $f(x) = 2x + 1$
- Example 3: $m = ½$; $b = 1$: $f(x) = ½\, x + 1$
- Example 4: $m = 0$; $b = 1$: $f(x) = 0\, x + 1$
- Example 5: $m = 1$; $b = 1$: $f(x) = x + 1$

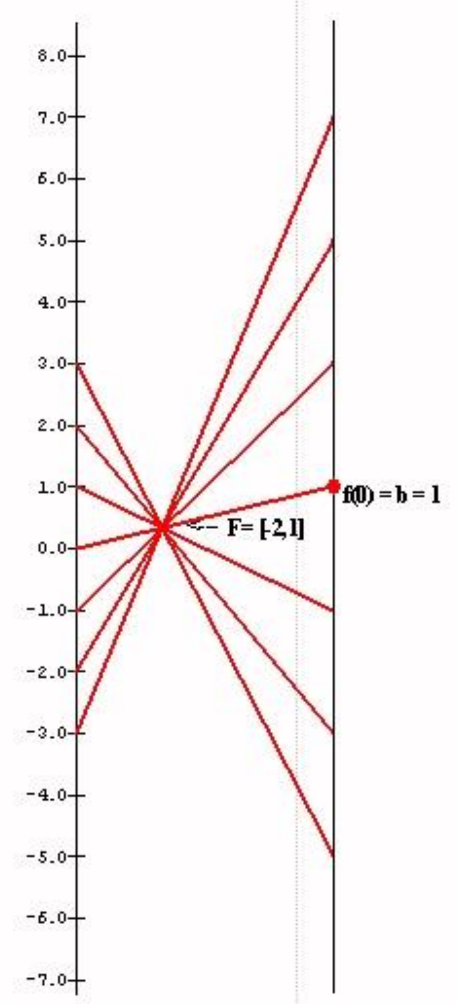
Visualizing $f(x) = \boldsymbol{mx} + \boldsymbol{b}$ with a mapping diagram -- Five examples

Example 1: $\boldsymbol{m} = -\mathbf{2};\ \boldsymbol{b} = \mathbf{1}$

$$f(x) = -\mathbf{2}x + \mathbf{1}$$

- Each arrow passes through a single point, which is labeled $\boldsymbol{F} = [-\mathbf{2}, \mathbf{1}]$.
- The point $\boldsymbol{F}$ completely determines the function f.
  - **given a point** / number, x, on the source line,
  - there is a **unique arrow passing through** $\boldsymbol{F}$
  - **meeting** the target line at **a unique point** / number, -$2x + 1$,

  which corresponds to the linear function's value for the point/number, x.



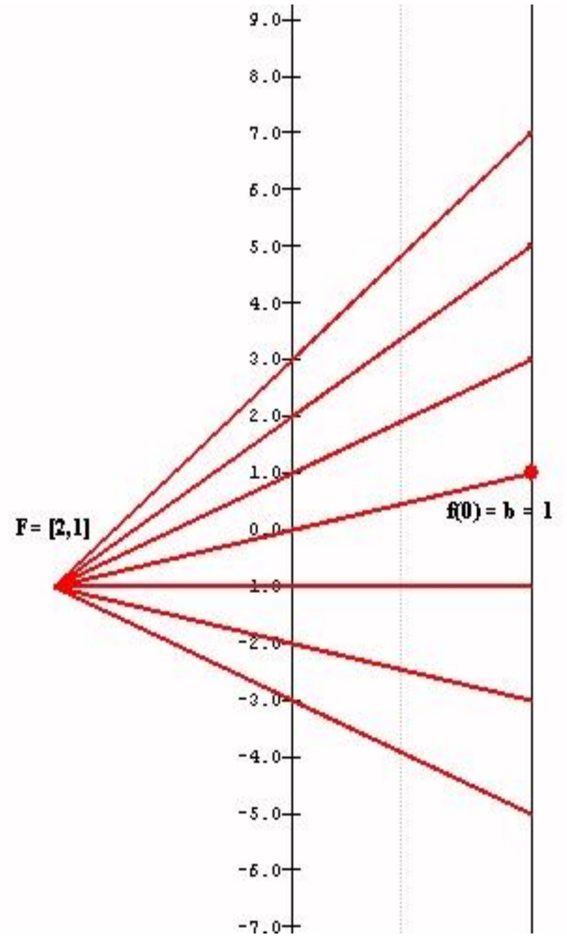
Visualizing $f\,(x) = \boldsymbol{m}x + \boldsymbol{b}$ with a mapping diagram -- Five examples:

Example 2: $\boldsymbol{m} = 2$; $\boldsymbol{b} = \mathbf{1}$

$$f\,(x) = \mathbf{2}x + \mathbf{1}$$

- Each arrow passes through a single point, which is labeled $\boldsymbol{F} = [\,\mathbf{2}, \mathbf{1}]$.
- The point $\boldsymbol{F}$ completely determines the function f.
  - **given a point** / number, x, on the source line,
  - there is a **unique arrow passing through** $\boldsymbol{F}$
  - **meeting** the target line at **a unique point** / number, $2x + 1$,

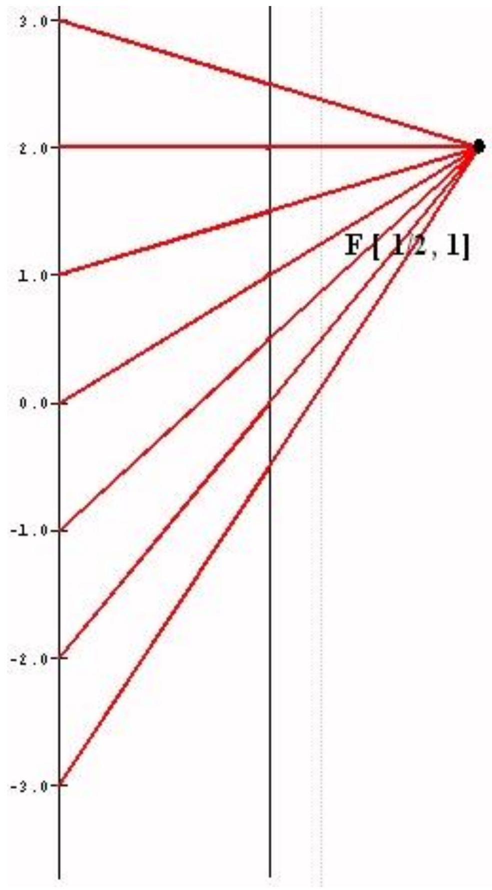
  which corresponds to the linear function's value for the point/number, x.



Visualizing $f(x) = \boldsymbol{m}x + \boldsymbol{b}$ with a mapping figure -- Five examples:

Example 3: $\boldsymbol{m} = \frac{1}{2}$; $\boldsymbol{b} = \mathbf{1}$

$$\boldsymbol{f}(\boldsymbol{x}) = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{x} + \mathbf{1}$$



- Each arrow passes through a single point, which is labeled $\boldsymbol{F} = \left[\frac{1}{2}, \mathbf{1}\right]$.
- The point $\boldsymbol{F}$ completely determines the function f.
  - **given a point** / number, x, on the source line,
  - there is a **unique arrow passing through** $\boldsymbol{F}$
  - **meeting** the target line at **a unique point** / number, $\frac{1}{2}x + 1$,

  which corresponds to the linear function's value for the point/number, x.

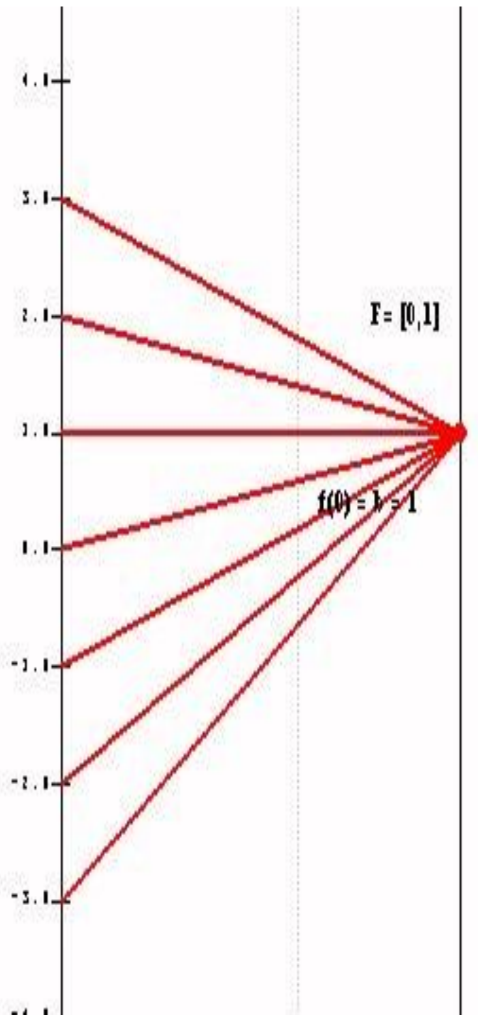
Visualizing $f(x) = \boldsymbol{m}x + \boldsymbol{b}$ with a mapping figure -- Five examples:

Example 4: $\boldsymbol{m} = \mathbf{0}$; $\boldsymbol{b} = \mathbf{1}$

$$f(x) = \mathbf{0}x + \mathbf{1}$$

- Each arrow passes through a single point, which is labeled $\boldsymbol{F} = [\mathbf{0}, \mathbf{1}]$.
- The point $\boldsymbol{F}$ completely determines the function f.
  - **given a point** / number, x, on the source line,
  - there is a **unique arrow passing through** $\boldsymbol{F}$
  - **meeting** the target line at **a unique point** / number, $0x + 1$,

  which corresponds to the linear function's value for the point/number, x.

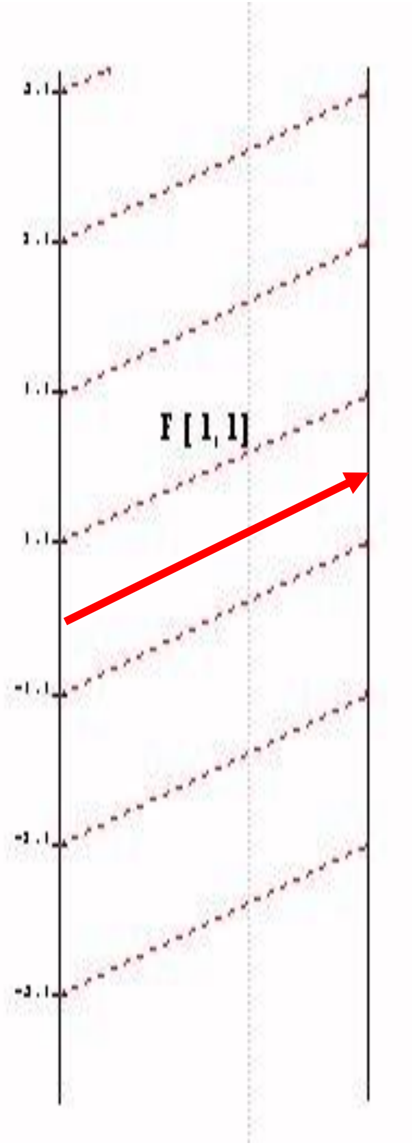


Visualizing $f(x) = \boldsymbol{mx} + \boldsymbol{b}$ with a mapping figure -- Five examples:

Example 5: $\boldsymbol{m} = \mathbf{1}$; $\boldsymbol{b} = \mathbf{1}$

$$f(x) = 1x + \mathbf{1}$$

- Unlike the previous examples, in this case it is not a single point that determines the mapping figure, but the single arrow from 0 to 1, which we designate as $F[1,1]$
- It can also be shown that this single arrow completely determines the function. Thus, given a point / number, x, on the source line, there is a unique arrow passing through $\boldsymbol{x}$ ***parallel to*** $F[1,1]$ meeting the target line a unique point / number, $x + 1$, which corresponds to the linear function's value for the point/number, x.
  - The single arrow completely determines the function f.
    - **given a point** / number, x, on the source line,
    - there is a **unique arrow** through $\boldsymbol{x}$ ***parallel to*** $F[1,1]$
    - **meeting** the target line at **a unique point** / number, $x + 1$, which corresponds to the linear function's value for the point/number, x.



End of Session 0

- Questions?

Thanks
The End!

☺

Questions for next session?
Catch me between sessions or
e-mail them to me:
flashman@humboldt.edu

- [FL1] Flashman, Martin. "Differential Equations: A Motivating Theme for A Sensible Calculus," in "Calculus for All Users" The Report of A Conference on Calculus and Its Applications Held at the University of Texas, San Antonio, NSF Calculus Reform Conference, October 5 - 8, 1990.
- [UMAP] Flashman, Martin. "A Sensible Calculus.," The UMAP Journal, Vol. 11, No. 2, Summer, 1990, pp. 93-96.
- [FL2] Flashman, Martin. "Using Computers to Make Integration More Visual with Tangent Fields," appearing in Proceedings of the Second Annual Conference on Technology in Collegiate Mathematics, Teaching and Learning with Technology of November 2-4, 1989, edited by Demana, Waits, and Harvey, Addison-Wesley, 1991.
- [FL3] Flashman, Martin. "Concepts to Drive Technology," in Proceedings of the Fifth Annual Conference on Technology in Collegiate Mathematics, November 12-15, 1992, edited by Lewis Lum, Addison-Wesley, 1994.
- [FL4] Flashman, Martin. "Historical Motivation for a Calculus Course: Barrow's Theorem," in Vita Mathematica: Historical Research and Integration with Teaching, edited by Ronald Calinger, MAA Notes, No. 40, 1996.