

Related Rates

1. Let V be the volume of a cube of side x . Assume that x varies with time.
 - a) How are dV/dt and dx/dt related?
 - b) At a certain instant the sides are 4 feet long and growing at a rate of 3 ft/min. How fast is the volume growing at this instant?
 - c) Suppose the volume is decreasing at a rate of $2 \text{ ft}^3/\text{min}$. when the sides are 5 ft. long. How fast are the sides decreasing at this instant?
2. Suppose a sphere has a radius r that varies with time. How are dV/dt and dr/dt related?
3. A spherical balloon is expanding in the heat. If the radius is increasing at a rate of 2 inches/min., how fast is the volume increasing when $r = 5$ inches?
4. A spherical balloon is contracting in the cold so that the volume is decreasing at a rate of $4 \text{ in}^3/\text{min}$. How fast is the diameter of the balloon decreasing when the radius is 2 inches?
5. A rubbish heap in the shape of a cube is being compacted. If the volume decreases at a rate of $2 \text{ inch}^3/\text{min}$, how fast is an edge changing when the volume is 64 in^3 ?
6. A conical reservoir 100 ft. deep and 100 ft. across is being filled with water at a rate of $10 \text{ ft}^3/\text{min}$. How fast is the water level rising when $h = 10$ feet?
7. A 7 foot man is walking towards a 28 foot lamp post at a rate of 4 ft/sec.
 - a) How fast is his shadow shrinking?
 - b) How fast is the tip of his shadow moving?
8. A ladder 13 ft. long is leaning against a wall. If the base of the ladder is being pulled away from the wall at a rate of $(1/2) \text{ ft/sec}$., how fast is the top of the ladder being lowered when the base is 5 ft. from the wall?
9. Sand being dumped from a funnel forms a conical pile whose height is always $1/5$ the diameter of the base. If the sand is dumped at a rate of $3 \text{ meters}^3/\text{min}$., how fast is the pile rising when it is 2 meters deep?
10. An icicle is in the shape of a right circular cone. At a certain point in time the height is 12cm and its length increasing at a rate of $.5 \text{ cm/hr}$., while the radius of the base is 1cm and is decreasing at a rate of $.05 \text{ cm/hr}$. Is the volume of ice increasing or decreasing at that instant? At what rate?

11. A boat floating several feet away from a dock is pulled in by a rope that is being wound up by a windlass at a rate of 2ft/sec. If the windlass is 5 ft above the level of the boat, how fast is the boat moving through the water when it is 9 ft. from the dock?

12. A rocket is launched vertically and tracked by a radar station which is located on the ground 3 miles from the launch site. What is the vertical speed of the rocket at the instant when its distance from the radar station is 5 miles and this distance is increasing at a rate of 5000 miles/hr.?

13. A ship with a long anchor chain is anchored in 11 fathoms of water. The anchor chain is being wound in at a rate of (10)fathoms/min. causing the ship to move towards the spot directly over where the anchor is resting on the ocean's bottom. The point of contact between the ship and the chain is located 1 fathom above the water line. At what speed is the ship moving when there are exactly 13 fathoms of chain still out?

14. Two ships sail from the same island port, one going north at 24 knots (24 nautical miles per hour) and the other east at 30 knots. The northbound ship departed at 9:00 a.m. and the eastbound ship left at 11:00 a.m. How fast is the distance between them increasing at 2:00 p.m.?