Math 105 Related Rates Solutions

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1) I did this in class

2) $V = \frac{4}{3}\pi r^3$

$$\boxed{\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}}$$

3) $\frac{dr}{dt} = 2 \text{ in/min}$

$r = 5 \text{ in}$

$$\frac{dV}{dt} = 4\pi(5)^2 \cdot 2 \frac{\text{in}^3}{\text{min}}$$

$$= 200\pi \frac{\text{in}^3}{\text{min}} \approx 628 \frac{\text{in}^3}{\text{min}}$$

4) $\frac{dV}{dt} = -4 \frac{\text{in}^3}{\text{min}}$ $r = 2 \text{ in}$

$$-4 = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{-4}{4\pi(2)^2} \frac{\text{in}}{\text{min}} = \frac{-1}{4\pi} \frac{\text{in}}{\text{min}}$$

$d = 2r$ (diameter = 2 · radius)

$$\frac{dd}{dt} = 2\frac{dr}{dt}$$

$$\frac{dd}{dt} = \frac{-1}{2\pi} \text{ in/min} \approx -0.0398 \text{ in/min}$$

5) $Vol = V$
$side = x$

$$V = x^3$$

$$\frac{dV}{dt} = 2 \frac{in^3}{min} \qquad V = 64 \text{ in}^3$$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

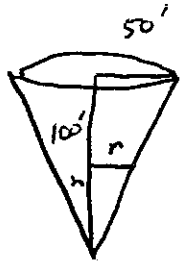
$x = ?$

$$64 = x^3$$
$$x = \sqrt[3]{64} = 4$$

$$2 = 3(4)^2 \cdot \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{2}{3(4)^2} \frac{in}{min} \approx 0.0417 \frac{in}{min}$$

6.



$$\frac{dV}{dt} = 10 \frac{ft^3}{min}$$

$$h = 10 \text{ ft}$$

$$V = \frac{1}{3} \pi r^2 h$$

We need r

Use proportional triangles

$$\frac{h}{r} = \frac{100}{50}$$

$$\frac{h}{r} = 2$$

$$h = 2r$$

$$r = \frac{h}{2}$$

So

$$V = \frac{1}{3} \pi \left(\frac{h}{2}\right)^2 h = \frac{1}{12} \pi h^3$$

$$\frac{dV}{dt} = \frac{1}{12\pi}\,\pi\,(3h^2)\,\frac{dh}{dt}$$

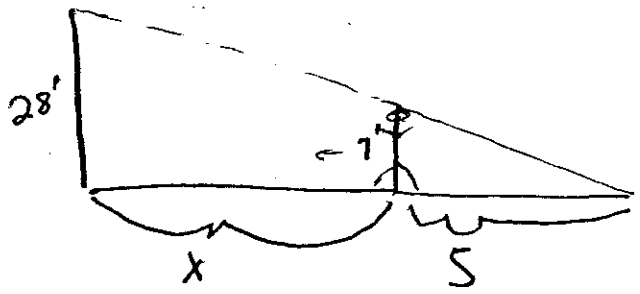
$$10 = \frac{1}{12\pi}\,\pi \cdot 3 \cdot (10)^2\,\frac{dh}{dt}$$

$$10 = \frac{100\pi}{4}\,\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{100\pi} \cdot 10\ \frac{ft}{min}$$

$$= \frac{2}{5\pi}\ \frac{ft}{min} \approx 0,0637\ \frac{ft}{min}$$

7)



$$\frac{dx}{dt} = -4\ \frac{ft}{sec}$$

4) To relate x & s use proportional triangles

$$\frac{28}{x+s} = \frac{7}{s}$$

$$28s = 7(x+s) = 7x + 7s$$

$$28s - 7s = 7x$$

$$\frac{21s}{7} = \frac{7x}{7}$$

$$x = 3s$$

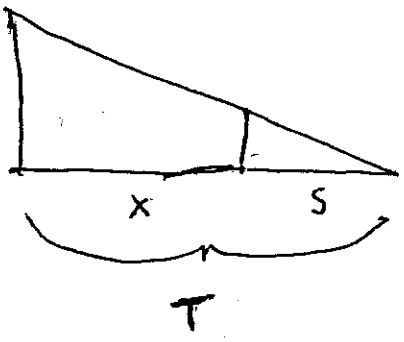
$$\frac{dx}{dt} = 3\,\frac{ds}{dt}$$

$$-4 = 3\,\frac{ds}{dt}$$

$$\frac{ds}{dt} = -\frac{4}{3}\,\frac{ft}{s}$$

The shadow is shrinking @ a rate of $\frac{4}{3}\,\frac{ft}{sec.}$

b)



The tip of the shadow is moving at a rate $\frac{dT}{dt}$

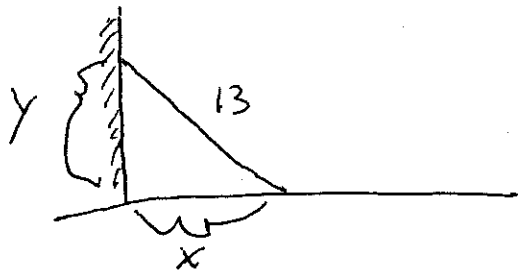
$$T = x + s$$

$$\frac{dT}{dt} = \frac{dx}{dt} + \frac{ds}{dt} = \left(-4 + -\frac{4}{3}\right)\frac{ft}{sec}$$

$$= -\frac{16}{3}\ \frac{ft}{sec}$$

The tip of shadow is moving with speed $\frac{16}{3}\ \frac{ft}{sec}$.

8)



$\frac{dx}{dt} = \frac{1}{2} \frac{ft}{sec}$ $\quad x = 5$ ft

$x^2 + y^2 = (13)^2$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$x = 5$ ft, we need y

$5^2 + y^2 = 13^2$

$y^2 = 169 - 25 = 144$

$y = 12$ ft

$2 \cdot 5 \cdot \frac{1}{2} + 2 \cdot 12 \frac{dy}{dt} = 0$

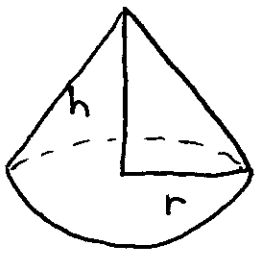
$$5 + 24\frac{dy}{dt} = 0$$

$$24\frac{dy}{dt} = -5$$

$$\frac{dy}{dt} = \frac{-5}{24}\ \frac{ft}{sec}$$

The ladder is being lowered at a rate of $\frac{5}{24}$ ft/sec.

9)



$$\frac{dV}{dt} = 3\ m^3/min$$

$$h = 2\ m$$

$$h = \frac{1}{5}d = \frac{1}{5}(2r) = \frac{2}{5}r$$

$$r = \frac{5}{2}h$$

4

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \left(\frac{5}{2}h\right)^2 h$$

$$= \frac{1}{3}\pi \frac{25}{4} h^3 = \pi \frac{25}{12} h^3$$

$$\frac{dV}{dt} = \pi \frac{25}{12} 3h^2 \cdot \frac{dh}{dt}$$

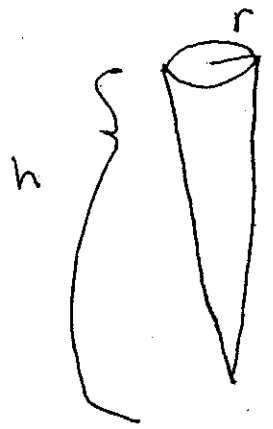
$$3 = \pi \frac{25}{12} 3(2)^2 \frac{dh}{dt}$$

$$3 = \pi \frac{25}{\cancel{12}} \cancel{12} \frac{dh}{dt}$$

$$3 = 25\pi \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{3}{25\pi} \frac{m}{min} \approx 0.038 \frac{m}{min}$$

10.



$h = 12$, $r = 1$

$\frac{dh}{dt} = 0.5 \ \frac{cm}{hr}$

$\frac{dr}{dt} = -0.05 \ \frac{cm}{hr}$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \text{~~}\frac{1}{3}\pi\left[2r\,\frac{dr}{dt}\,h + r^2\,\frac{dh}{dt}\right]\text{~~} \quad \text{product rule}$$

$$= \frac{1}{3}\pi\left[2r\,\frac{dr}{dt}\,h + r^2\,\frac{dh}{dt}\right]$$

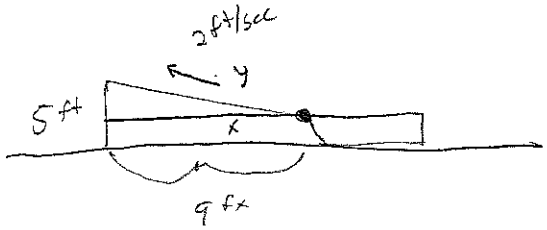
$$= \frac{1}{3}\pi\left[2(1)(-0.05)12 + (1)^2\ 0.5\right]$$

$$= \frac{1}{3}\pi\left[-1.2 + 0.5\right] = -\frac{0.7\,\pi}{3}$$

$$\approx -.733 \ \frac{cm^3}{hr}$$

Thus the Volume is decreasing

11)



Know

$$\frac{dy}{dt} = 2 \frac{ft}{sec}$$

Want

$$\frac{dx}{dt}$$

$$x^2 + 5^2 = y^2$$

$$x^2 + 25 = y^2$$

$$9^2 + 25 = y^2$$

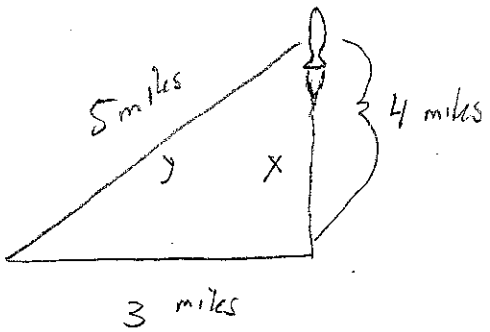
$$y = \sqrt{106}$$

$$2x \frac{dx}{dt} = 2y \frac{dy}{dt}$$

$$2(9) \frac{dx}{dt} = 2(\sqrt{106})\, 2$$

$$\frac{dx}{dt} = \frac{2\sqrt{106}}{9} \approx 2.29 \text{ ft/sec}$$

12)



Know

$\frac{dy}{dt} = 5000 \frac{mi}{hr}$

Want

$\frac{dx}{dt}$

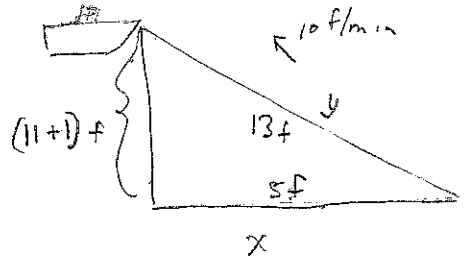
$3^2 + x^2 = y^2$

$2x \frac{dx}{dt} = 2y \frac{dy}{dt}$

$2(4) \frac{dx}{dt} = 2(5)\ 5000$

$\frac{dx}{dt} = 6250 \frac{mi}{hr}$

13)



$$12^2 + x^2 = 13^2$$

$$x^2 = 25$$

$$x = 5$$

Want	Know
$\frac{dx}{dt}$	$\frac{dy}{dt} = 10$

$$x^2 + 12^2 = y^2$$

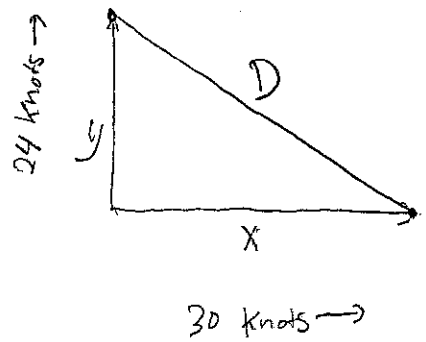
$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

$$2(5)\frac{dx}{dt} = 2(13)(10)$$

$$10\frac{dx}{dt} = 260$$

$$\frac{dx}{dt} = 26 \ \frac{\text{fathoms}}{\text{min}}$$

14)



Northbound ship left 9:00 am, 5 hrs before 2:00 pm.

Thus it has traveled $5 \cdot 24 = 120$ nautical miles.

Eastbound ship left at 11:00 am, 3 hrs before 2:00 pm. Thus it has traveled $3 \cdot 30 = 90$ nautical miles

Know

$\frac{dx}{dt} = 30$

$\frac{dy}{dt} = 24$

Want

$\frac{dD}{dt}$

$x = 90$, $y = 120$

$D^2 = 90^2 + 120^2 \Rightarrow D = 150$

$x^2 + y^2 = D^2$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2D \frac{dD}{dt}$

$2(90) \cdot 30 + 2(120)\,24 = 2(150) \frac{dD}{dt} \Rightarrow \frac{dD}{dt} = 37.2$ knots