Math 40 - Practice Problems

Solutions

1. a) $4y + x(5-x) + x^3$; Evaluate at $x = 2$ & $y = 3$

$4(3) + (2)(5-(2)) + (2)^3$

$= 12 + 2(5-2) + 8$ $\quad 2 \cdot 2 \cdot 2 = 8$

$= 12 + 2(3) + 8$

$= 12 + 6 + 8$

$= 18 + 8$

$= \boxed{26}$

b) $\dfrac{3(\frac{1}{2} - b) + \frac{5}{4}}{17 + 3a}$; $a = -5$ & $b = \frac{1}{3}$

$= \dfrac{3\left(\frac{1}{2} - \left(\frac{1}{3}\right)\right) + \frac{5}{4}}{17 + 3(-5)}$

$= \dfrac{3\left(\frac{1}{2}\frac{3}{3} - \frac{1}{3}\frac{2}{2}\right) + \frac{5}{4}}{17 - 15}$

$= \dfrac{3\left(\frac{3}{6} - \frac{2}{6}\right) + \frac{5}{4}}{2}$

$= \dfrac{\frac{3}{1}\left(\frac{1}{6}\right) + \frac{5}{4}}{2}$

$= \dfrac{\frac{1}{2}\frac{2}{2} + \frac{5}{4}}{2} = \dfrac{\frac{7}{4}}{2} = \dfrac{7}{4} \div \dfrac{2}{1} = \dfrac{7}{4} \cdot \dfrac{1}{2} = \boxed{\dfrac{7}{8}}$

2. Solve

a) $\frac{x+3}{5} = 14$ LCD $= \frac{5}{1}$

$\frac{5}{1}\cdot\frac{x+3}{5} = \frac{14}{1}\cdot\frac{5}{1}$

$14 \times 5 = 70$

$x+3 = 70$

$x = 70-3$

$\boxed{x = 67}$

b) $3.1(x-2) = 1.3x + 2.8$

$3.1x - 6.2 = 1.3x + 2.8$ clear decimals

$3.1 \times 2 = 6.2$

$10(3.1x - 6.2) = 10(1.3x + 2.8)$

$31x - 62 = 13x + 28$

$31 - 13 = 18$ $\quad 62 + 28 = 90$ $\quad 18 \times 5 = 90$

$31x - 13x = 28 + 62$

$18x = 90$

$x = \frac{90}{18}$

$\boxed{x = 5}$

c) $2(5x-3) + 4x = 1 - 6(2x+5)$

$10x - 6 + 4x = 1 - 12x - 30$

$14x - 6 = -12x - 29$

$29 - 6 = -23$

$14x + 12x = 6 - 29$

$26x = -23$

$\boxed{x = -\frac{23}{26}}$

3. Solve the following system

$$\begin{cases} 3y = x+6 \\ y+2 = -x \end{cases}$$

a) using substitution

(4) $y+2=-x$

$y=-x-2$

$y=-(-3)-2$

$y=3-2$

$y=1$

(B) $3y=x+6$

$3(-x-2)=x+6$

$-3x-6=x+6$

$-x \quad +6 \quad -x \quad +6$

$-4x=12$

$x=\frac{12}{-4}$

$x=-3$

$(-3,1)$
consistent
independent

b) using elimination (Get into $Ax+By=C$ form)

$$\begin{cases} 3y = x+6 \\ y+2 = -x \end{cases}$$

$$\begin{cases} -x+3y=6 \\ x+y=-2 \end{cases}$$

$+$

$4y=4$

$y=1$

$y+2=-x$

$1+2=-x$

$3=-x$

$-3=x$

$(-3,1)$
Consistent
Independent

4. Simplify

a) $5x^2y^2 + 2(xy)^2 - (3x^2)y^2$

$= 5x^2y^2 + 2x^2y^2 - 3x^2y^2$ ← like terms

$= 7x^2y^2 - 3x^2y^2$

$= \boxed{4x^2y^2}$

b) $3(x^3 + 9x^2 - 3x + 7) - 2(11x^2 - 5x + 9)$

$= 3x^3 + 27x^2 - 9x + 21 - 22x^2 + 10x - 18$

$= \boxed{3x^3 + 5x^2 + x + 3}$

c) $\frac{-2x^3}{y}\left(\frac{3x^2y}{4} - \frac{5y}{x^3}\right)$

$= -\frac{2x^3}{y} \cdot \frac{3x^2y}{4_2} + \frac{2x^3}{y} \cdot \frac{5y}{x^3}$

$= \boxed{-3x^5 + 10}$

5. Simplify

a) see 4a

b) see 4b

oops !! sorry!

c) $-2x^3(3x^2 - x)$

$= -2x^3 \cdot 3x^2 + 2x^3 \cdot x$

$= \boxed{-6x^5 + 2x^4}$

5. d) $(5x-3z)(4x+6z)$

$= 20x^2 + 30xz - 12xz - 18z^2$

$= \boxed{20x^2 + 18xz - 18z^2}$

$30 - 12 = 18$

e) $(4t+3)(t^2+2t+3)$

$= 4t^3+8t^2+12t$

$+ \quad +3t^2+6t+9$

$= \boxed{4t^3+11t^2+18t+9}$

f) $(2x+y)(11xy^2+2x^2y-6)$

$= 22x^2y^2+4x^3y-12x$

$+ 2x^2y^2 \quad +11xy^3-6y$

$\boxed{24x^2y^2+4x^3y-12x+11xy^3-6y}$

6. Set up equations to solve the applications. Use the 5-step Process (DESCS)

a) (1) Let l = length, w = width

(2) Area $= l \cdot w$

(3) $33 = l \cdot w$ $\quad l = 2w+5$

$33 = (2w+5)w$

$33 = 2w^2+5w$

$0 = 2w^2+5w-33$

$0 = 2w^2-6w+11w-33$

$0 = 2w(w-3)+11(w-3)$

$0 = (2w+11)(w-3)$

-66, -6, 11, 5

$0 = 2w+11$ $\quad 0 = w-3$

$\frac{-11}{2} = w$ $\quad 3 = w$

No

$l = 5+2w = 5+2(3) = 11$

(4) Check it!

(5) The length is 11ft and the width is 3ft.

6. b) ① Let x = # of dimes
y = # of quarters

Type of coins	# of coins	Value per coin	Total value
Dimes	x	10	$10x$
	+		+
Quarters	y	25	$25y$
	=		=
Total	19	↑ in cents!	295

② $\begin{cases} x+y=19 \\ 10x+25y=295 \end{cases}$

③ $\begin{cases} -10(x+y)=19(-10) \\ 10x+25y=295 \end{cases}$

$\begin{cases} -10x-10y=-190 \\ 10x+25y=295 \end{cases}$

$295 - 190 = 105$

$15 \times 9 = 135$

$15 \times 7 = 105$

$25 \times 7 = 175$

$15y = 105$

$y = \frac{105}{15}$

$y = 7$

$x+y=19$

$x+7=19$

$x=19-7$

$x=12$

④ Check: $x=12$ $y=7$

$x+y=19$

$12+7 \stackrel{?}{=} 19$

$19=19$ ✓

$10x+25y=295$

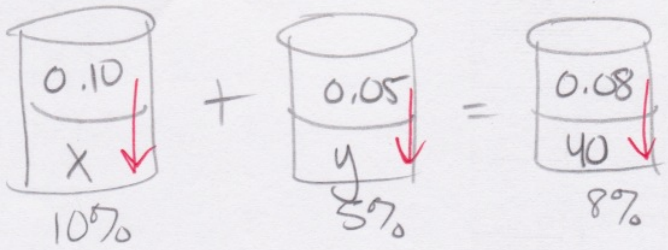
$10(12)+25(7) \stackrel{?}{=} 295$

$120+175 \stackrel{?}{=} 295$

$295=295$ ✓

⑤ Maria has 12 dimes and 7 quarters.

6c)



① Let x = liters of 10% solution
y = liters of 5% solution

② $\begin{cases} x+y=40 \\ 0.1x+0.05y=0.08(40) \end{cases}$

40 × 0.08 = 3.20

③ $\begin{cases} x+y=40 \\ 0.1x+0.05y=3.2 \end{cases}$ ← mult. by 100

$\begin{cases} x+y=40 \\ 10x+5y=320 \end{cases}$ ← mult by −5

$\begin{cases} -5x-5y=-200 \\ 10x+5y=320 \end{cases}$

$5x=120$

$x=24$

$x+y=40$

$y=40-24$

$y=16$

$120 \div 5 = 24$

$16 \times 5 = 80$, $240+80=320$

④ Check:

$x=24 \quad y=16$

$x+y=40$: $24+16 \stackrel{?}{=} 40$, $40=40$ ✓

$10x+5y=320$: $10(24)+5(16) \stackrel{?}{=} 320$, $240+80 \stackrel{?}{=} 320$, $320=320$ ✓

⑤ 24 liters of 10% solution and 16 liters of 5% solution should be mixed.

6d) ① Let x & y be numbers

② $x+y=15 \rightarrow y=15-x$

③ $x \cdot y = 54$

$x(15-x)=54$

$15x-x^2=54$

$0=x^2-15x+54$

$0=(x-9)(x-6)$

$-9 \cdot -6 = 54$, $-9-6=-15$

$x=9, 6$

④ check it!

⑤ The numbers are 6 and 9.

6. Solve the following word problems. **Be sure to declare all variables, state an equation, solve that equation, and answer in a complete sentence with units (The five step process as we did in class). You may want to check your answer too.**

a. **Area.** The length of a rectangular garden is 5 feet longer than twice its width. Find the dimensions of the garden if the area is 33 feet.

b. **Coins.** Coin Problem. Maria has $2.95 in dimes and quarters. She has a total of 19 coins. How many of each does she have?

c. **Mixtures.** How many liters of 10% alcohol solution and 5% alcohol solution must be mixed to obtain 40 liters of a 8% alcohol solution?

d. **Number Problem.** The sum of two numbers is 15 and their product is 54? Find the numbers.

7. Graph the following linear equation using one of the three methods learned in class.

(1) Plot pts
(2) Plot intercepts
(3) use slope & y intercept

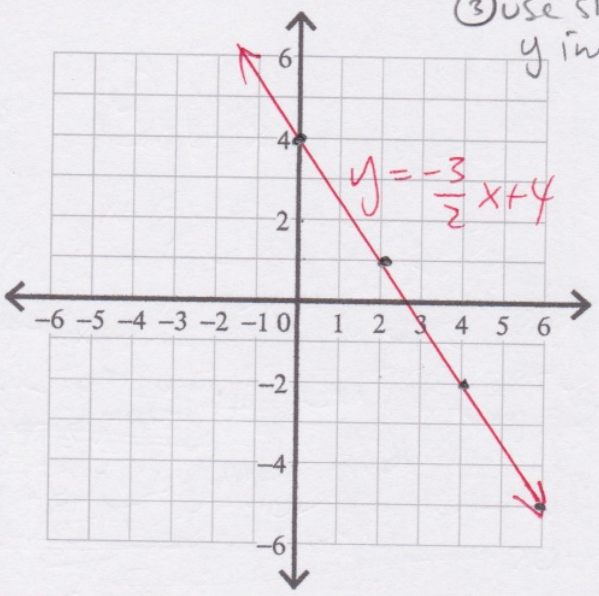
$2y = -3x + 8$

$y = -\frac{3}{2}x + \frac{8}{2}$

$y = -\frac{3}{2}x + 4$

$m = -\frac{3}{2}$

$(0, 4)$



8. Graph the following two-variable inequality. Shade the graph in the appropriate area to represent the solution. Pick a test point to check your answer.

$3x - 2y > 4$

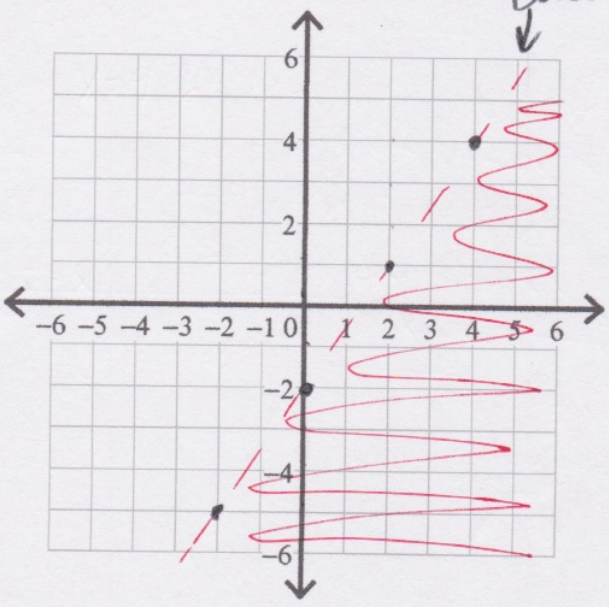
$-2y > -3x + 4$

$y < -\frac{3}{-2}x + \frac{4}{-2}$

$y < \frac{3}{2}x - 2$

↑ Shade below dashed line

Dashed!



9. Given the two points

a. Using algebra, determine the whole **equation** of the line in **slope-intercept** form that passes through the points $(-3,7)$ and $(6,1)$.

$x_1\ y_1 \qquad x_2\ y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{1-7}{6-(-3)}$

$= \frac{-6}{9} = \frac{-2}{3}$

$y - y_1 = m(x - x_1)$

$y - 7 = -\frac{2}{3}(x - (-3))$

$y - 7 = -\frac{2}{3}(x+3)$

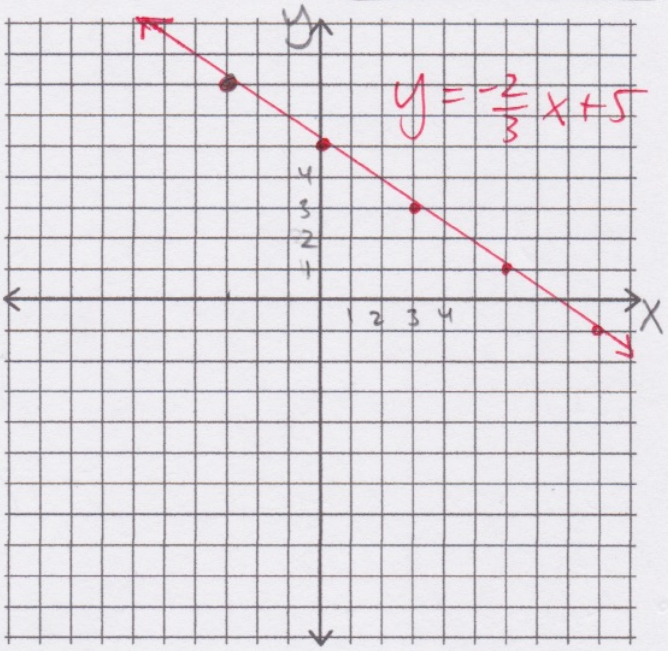
$y - 7 = -\frac{2}{3}x - 2$

$+7 \qquad +7$

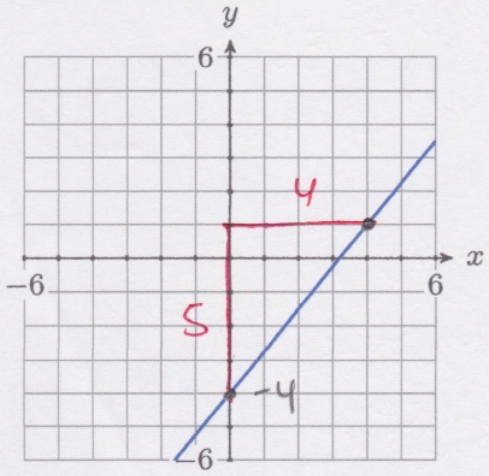
$y = -\frac{2}{3}x + 5$

b. Accurately graph the equation found in part a. above by using the slope and y intercept or plotting points or plotting both of the intercepts **(Be sure to label and scale the axes).**

Use slope & yint and check given points are on graph!



10. State the equation of the line in slope intercept form.



$y = mx + b$ m = slope $(0,b)$ ← yint

$m = \frac{5}{4}$ $(0,1)$

$y = \frac{5}{4}x + 1$

11. Write without negative exponents

a) $x^3 \cdot x^4 \cdot x$

$= x^{3+4+1}$

$= \boxed{x^8}$

b) $\dfrac{(-2a^5b^{-6})^3}{a^4b^2}$

$= \dfrac{(-2)^3(a^5)^3(b^{-6})^3}{a^4b^2} = \dfrac{-8a^{15}b^{-18}}{a^4b^2} = \dfrac{-8a^{11}}{b^{18}b^2}$

$= \boxed{\dfrac{-8a^{11}}{b^{20}}}$

c) $(x^6)^0 = x^0 = \boxed{1}$

d) $\left(\dfrac{x^5y^6}{z}\right)^2 = \boxed{\dfrac{x^{10}y^{12}}{z^2}}$

12. Factor completely

a) $x^5y^2 + 2x^3y^2 - 3x^2y^2$

$= \boxed{x^2y^2(x^3+2x-3)}$ ← GCF only

b) $2ax + 6x - 5a - 15$

$= 2x(a+3) - 5(a+3)$

$= \boxed{(2x-5)(a+3)}$

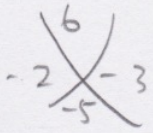
12. cont'd

c) $-12x - 2x^3 + 10x^2$ ← Put in descending order

$= -2x^3 + 10x^2 - 12x$

$= -2x(x^2 - 5x + 6)$

$= \boxed{-2x(x-2)(x-3)}$



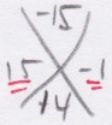
d) $3x^2y + 14xy^2 - 5y^3$

$= y(3x^2 + 14xy - 5y^2)$

$= y[3x^2 + 15xy - 1xy - 5y^2]$

$= y[3x(x+5y) - y(x+5y)]$ same

$= \boxed{y(x+5y)(3x-y)}$



e) $3x^6 - 300x^4$

$= 3x^4(x^2 - 100)$

$= \boxed{3x^4(x+10)(x-10)}$

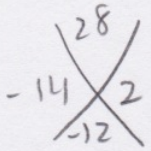
f) $35x^3z^4 - 60x^2z^4 - 20xz^4$

$= 5xz^4(7x^2 - 12x - 4)$

$= 5xz^4[7x^2 - 14x + 2x - 4]$

$= 5xz^4[7x(x-2) + 2(x-2)]$

$= \boxed{5xz^4(x-2)(7x+2)}$



13. Solve by factoring
① Get zero on one side
② Factor completely
③ Use zero factor property to solve

a) $(5x-6)(3x+4)=0$

$5x-6=0$ $\quad 3x+4=0$

$5x=6$ $\quad 3x=-4$

$x=\frac{6}{5}$ $\quad x=-\frac{4}{3}$

$x=\frac{6}{5}, -\frac{4}{3}$

b) $x^2+7x=8$

$x^2+7x-8=0$

$(x+8)(x-1)=0$

$8 \cdot -1 = -8$, $8 + -1 = 7$

$x+8=0$ $\quad x-1=0$

$x=-8$ $\quad x=1$

$x=-8, 1$

c) $4t^3-36t=0$

$4t(t^2-9)=0$

$4t(t+3)(t-3)=0$

$\frac{4t}{4}=\frac{0}{4}$ $\quad t+3=0$ $\quad t-3=0$

$t=0$ $\quad t=-3$ $\quad t=3$

$t=0, -3, 3$

d) $8x^3-10x=-16x^2$

$8x^3+16x^2-10x=0$

$2x(4x^2+8x-5)=0$

$2x[4x^2+10x-2x-5]=0$

$2x[2x(2x+5)-1(2x+5)]=0$

$2x(2x+5)(2x-1)=0$

$10 \cdot -2 = -20$, $10 + -2 = 8$

$\frac{2x}{2}=\frac{0}{2}$ $\quad 2x+5=0$

$x=0$ $\quad 2x=-5$

$x=-\frac{5}{2}$

$2x-1=0$

$2x=1$

$x=\frac{1}{2}$

$x=0, \frac{1}{2}, -\frac{5}{2}$