Math 105 Practice Problems Sol'ns

1. a) $\lim_{x\to 3} 3x^2-5x+2$

$= 3(3)^2-5(3)+2$

$= 3(9)-15+2$

$= 27-15+2$

$= \boxed{14}$

b) $\lim_{x\to 5} \frac{x^2-3x-10}{x-5}$

$= \lim_{x\to 5} \frac{\cancel{(x-5)}(x+2)}{\cancel{x-5}}$

$= \lim_{x\to 5} x+2$

$= 5+2$

$= \boxed{7}$

c) $\lim_{x\to\infty} \frac{1-2x^2}{x^2+3x+6}$

$= \lim_{x\to\infty} \frac{\frac{1}{x^2}-\frac{2x^2}{x^2}}{\frac{x^2}{x^2}+\frac{3x}{x^2}+\frac{6}{x^2}}$

$= \lim_{x\to\infty} \frac{\cancel{\frac{1}{x^2}}^{\to 0}-2}{1+\cancel{\frac{3}{x}}^{\to 0}+\cancel{\frac{6}{x^2}}^{\to 0}} = \frac{-2}{1} = \boxed{-2}$

d) $\lim_{x\to\infty} \frac{1-2x^2}{x+3x+6}$

$= \lim_{x\to\infty} \frac{\frac{1}{x} - \frac{2x^2}{x}}{\frac{x}{x} + \frac{3x}{x} + \frac{6}{x}}$

$= \lim_{x\to\infty} \frac{\cancel{\frac{1}{x}}^{\to 0} - 2x}{1+3+\cancel{\frac{6}{x}}^{\to 0}}$

$= \lim_{x\to\infty} \frac{-2x}{4}$

$= \frac{-2(\infty)}{4}$

$= \boxed{-\infty}$

or

DNE

e) $\lim_{x\to\infty} \frac{1-2x}{x^2+3x+6}$

$= \lim_{x\to\infty} \frac{\frac{1}{x^2} - \frac{2x}{x^2}}{\frac{x^2}{x^2} + \frac{3x}{x^2} + \frac{6}{x^2}}$

$= \lim_{x\to\infty} \frac{\cancel{\frac{1}{x^2}}^{\to 0} - \cancel{\frac{2}{x}}^{\to 0}}{1 + \cancel{\frac{3}{x}}^{\to 0} + \cancel{\frac{6}{x^2}}^{\to 0}}$

$= \frac{0}{1} = \boxed{0}$

2. $f(x) = \frac{2x-4}{3x-2}$ @ $x=2$

① Is $f(2)$ defined?

$f(2) = \frac{2(2)-4}{3(2)-2}$

$= \frac{4-4}{6-2} = \frac{0}{4} = 0$ ✓

② Does $\lim_{x \to 2} f(x)$ exist?

$\lim_{x \to 2} \frac{2x-4}{3x-2} = \frac{2(2)-4}{3(2)-2} = 0$ ✓

Need to take limit!

③ Does $f(2) = \lim_{x \to 2} f(x)$?

$\lim_{x \to 2} f(x) = 0 = f(2)$ ✓

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3. a) $f(x) = 3x^5 + 4x^3 - 2x - 5$

$\boxed{f'(x) = 15x^4 + 12x^2 - 2}$

Take Derivative and Stop!

b) $h(x) = (2x+1)^{1.4}$

"Stuff"

$\boxed{h'(x) = 1.4(2x+1)^{0.4}(2)}$

No Simplifying.

c) $h(x) = e^{2x} \cdot \ln x$ ← product rule

$\boxed{h'(x) = e^{2x}\frac{1}{x} + \ln x \, e^{2x} \cdot 2}$

3. d. $f(t) = 5^{3t+4}$

$$\boxed{f'(t) = \ln 5 \cdot 5^{3t+4} \cdot 3}$$

e) $g(t) = \dfrac{t - \sqrt{t}}{e^t + 5t}$ ← quotient rule

$$\boxed{g'(t) = \frac{(e^t+5t)\left(1 - \frac{1}{2\sqrt{t}}\right) - (t - \sqrt{t})(e^t+5)}{(e^t+5t)^2}}$$

f) $h(x) = 3x^2 \cdot \ln(5x)$ ← oops... it's on practice exam... sorry!

$$\boxed{h'(x) = 3x^2\left(\frac{1}{5x} \cdot 5\right) + \ln(5x) \cdot 6x}$$

Differentiate Implicitly

4. a) $5x - 7y = 3$

$5 - 7y' = 0$

$-7y' = -5$

$$\boxed{y' = \frac{5}{7}}$$

b) $x^3 - y^2 = 5$

$3x^2 - 2y \cdot y' = 0$

$-2yy' = -3x^2$

$$\boxed{y' = \frac{3x^2}{2y}}$$

c) $xy = 4$ ← product rule

$x \cdot y' + y \cdot 1 = 0$

$xy' = -y$

$$\boxed{y' = \frac{-y}{x}}$$

5. a) $\int \left(3x^2 + 5\sqrt{x} - \frac{2}{x}\right) dx = \int \left(3x^2 + 5x^{1/2} - 2\cdot\frac{1}{x}\right) dx$ 5

$= \cancel{3}\frac{x^3}{\cancel{3}} + 5\frac{x^{1/2+1}}{1/2+1} - 2\ln|x| + C$

$= x^3 + 5\frac{x^{3/2}}{3/2} - 2\ln|x| + C$

$= x^3 + 5\cdot\frac{2}{3}x^{3/2} - 2\ln|x| + C$

$= x^3 + \frac{10}{3}x^{3/2} - 2\ln|x| + C$

$= \boxed{x^3 + \frac{10}{3}\sqrt[3]{x^2} - 2\ln|x| + C}$

Break apart!

b) $\int \frac{x^3 - 3x}{x^2} dx = \int \left(\frac{x^3}{x^2} - \frac{3x}{x^2}\right) dx$

$= \int \left(x - \frac{3}{x}\right) dx$

$= \frac{x^2}{2} - 3\ln|x| + C$

Use u-substitution

$= \boxed{\frac{1}{2}x^2 - 3\ln|x| + C}$

c) $\int (3-2x)^4 dx$

Let $u = 3-2x$

$du = -2dx$

$-\frac{1}{2}du = dx$

$= -\frac{1}{2}\int u^4 du$

$= -\frac{1}{2}\cdot\frac{u^5}{5} + C$

$= \boxed{-\frac{1}{10}(3-2x)^5 + C}$

ba) $\int_1^3 \left(\frac{5}{x} + \sqrt{x}\right) dx$

$= \int_1^3 \left(5 \cdot \frac{1}{x} + x^{1/2}\right) dx$ Integrate

$= 5\ln|x| + \frac{2}{3}x^{3/2} \Big|_1^3$

$= 5\ln|3| + \frac{2}{3}(3)^{3/2} - \left[5\underbrace{\ln|1|}_{0} + \frac{2}{3}(1)^{3/2}\right]$

$= 5\ln 3 + \frac{2}{3} \cdot 3^{3/2} - \left\{0 + \frac{2}{3}\right\}$

$= 5\ln 3 + 2 \cdot \frac{3^{3/2}}{3} - \frac{2}{3}$

$= 5\ln 3 + 2 \cdot 3^{3/2-1} - \frac{2}{3}$

$= 5\ln 3 + 2 \cdot 3^{1/2} - \frac{2}{3}$

$= \boxed{5\ln 3 + 2\sqrt{3} - \frac{2}{3}}$

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let $u = 6x+1$
$du = 6dx$
$\frac{1}{6}du = dx$

If $x=0$
$u=1$
If $x=4$
$u = 6(4)+$
$u = 25$

b) $\int_0^4 \frac{1}{\sqrt{6x+1}} dx$

$= \frac{1}{6}\int_1^{25} \frac{1}{\sqrt{u}} du = \frac{1}{6}\int_1^{25} u^{-1/2} du$

$= \frac{1}{6} \frac{u^{-1/2+2/2}}{-1/2+2/2} \Big|_1^{25} = \frac{1}{6}\frac{2}{1}u^{1/2}\Big|_1^{25} = \frac{1}{3}u^{1/2}\Big|_1^{25} = \frac{1}{3}\left[25^{1/2} - 1^{1/2}\right]$

$= \frac{1}{3}\left\{\sqrt{25} - \sqrt{1}\right\}$

$= \frac{1}{3}\left\{5 - 1\right\}$

$= \boxed{\frac{4}{\ }}$

6c) $\int_0^1 8x(x^2+1)^3\,dx$

Let $u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

If $x=0$, $u=1$

If $x=1$, $u=2$

$= \int_1^2 4\cancel{8x}(u)^3 \frac{du}{\cancel{2x}}$

$= 4\int_1^2 u^3\,du$

$= \cancel{4}\frac{u^4}{\cancel{4}}\Big|_1^2$

$= u^4\Big|_1^2 = 2^4 - 1^4 = 16-1 = \boxed{15}$

6d. $\int_{-1}^3 (3x^2-1)e^{x^3-x}\,dx$

Let $u = x^3 - x$

$du = 3x^2-1\,dx$

$\frac{du}{3x^2-1} = dx$

$= \int_0^{24} \cancel{(3x^2-1)}e^u \frac{du}{\cancel{3x^2-1}}$

If $x=-1$

$u = (-1)^3-(-1)$

$= -1+1$

$= 0$

If $x=3$

$u = 3^3-3$

$= 27-3$

$= 24$

$= \int_0^{24} e^u\,du$

$= e^u\Big|_0^{24} = e^{24} - e^0$

$= \boxed{e^{24}-1}$

7. $f(x) = x^3 + 3x^2 - 2$

① Domain $\mathbb{R}$

Polynomial

② x/y intercepts

y-int $\Rightarrow x = 0$

$f(0) = -2$

$(0,-2)$ yint

③ $\boxed{\text{No Vertical or horizontal asymptotes since this is a polynomial.}}$

④ $f'(x) = 3x^2 + 6x$

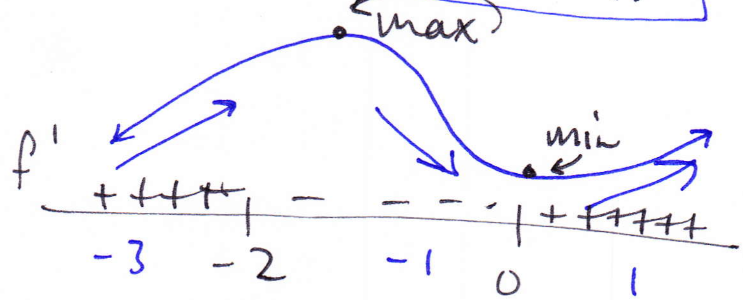
$0 = 3x^2 + 6x$

$0 = 3x(x+2)$

$3x = 0 \qquad x+2 = 0$

$x = 0 \qquad x = -2$

C.ps.



$f'(x) = 3x(x+2)$

$f'(-3) = 3(-3)(-3+2) = (-)(-) = +$

$f'(-1) = 3(-1)(-1+2) = (-)(+) = -$

$f'(1) = 3(1)(1+2) = (+)(+) = +$

Increasing $(-\infty,-2) \cup (0,\infty)$
Decreasing $(-2,0)$

⑤ Maxima/minima

$f(-2) = (-2)^3 + 3(-2)^2 - 2$
$= -8 + 3(4) - 2$
$= -8 + 12 - 2$
$= -10 + 12$
$= 2$

$\boxed{(-2,2) \text{ is a relative maximum.}}$

$f(0) = -2$

$\boxed{(0,-2) \text{ is a relative minimum.}}$

7 con't)

(6) Inflection Pts & concavity

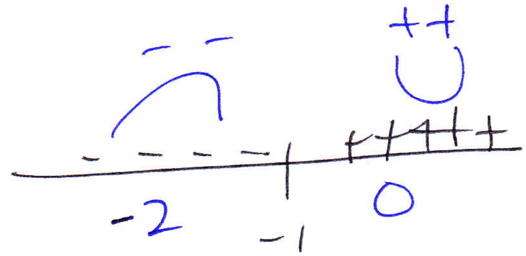
$f(x) = x^3 + 3x^2 - 2$

$f'(x) = 3x^2 + 6x$

$f''(x) = 6x + 6 = 6(x+1)$

$0 = 6(x+1)$

$x = -1$ ← possible I.P.



$f''(-2) = 6(-2+1) = 6(-) = -$

$f''(0) = 6(0+1) = 6 = +$

$\boxed{\begin{array}{l} f(x) \text{ is} \\ \text{Concave up } (-1, \infty) \\ \text{Concave down } (-\infty, -1) \end{array}}$

I.P.

$x = -1$

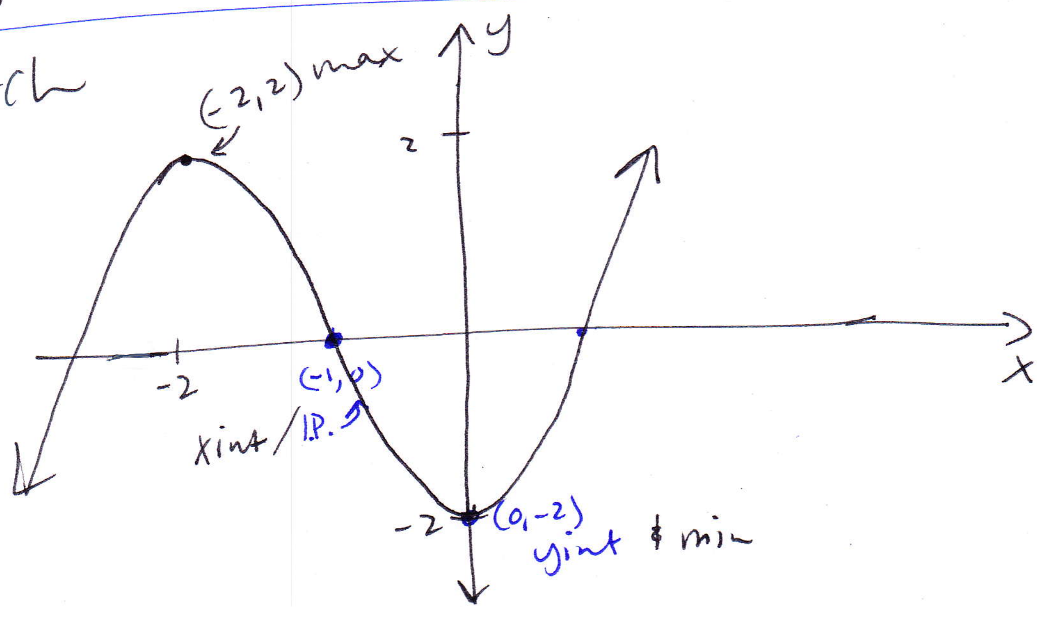
$f(-1) = (-1)^3 + 3(-1)^2 - 2$

$= -1 + 3 - 2$

$= 0$

$\boxed{(-1, 0) \text{ is an inflection point.}}$

(7) Sketch



8. $N(t) = \frac{5t}{12+t^2}$

Domain $= \mathbb{R}$

a) Rate of change $\Rightarrow$ 1st derivative!

$$N'(t) = \frac{(12+t^2)(5) - 5t(2t)}{(12+t^2)^2}$$

$$= \frac{60 + 5t^2 - 10t^2}{(12+t^2)^2}$$

$$\boxed{N'(t) = \frac{-5t^2+60}{(12+t^2)^2}}$$

b) Epidemic at its worst $\Rightarrow$ Find max new cases... max of $N(t)$.

$N'(t) = 0$

$0 = \frac{-5t^2+60}{(12+t^2)^2}$

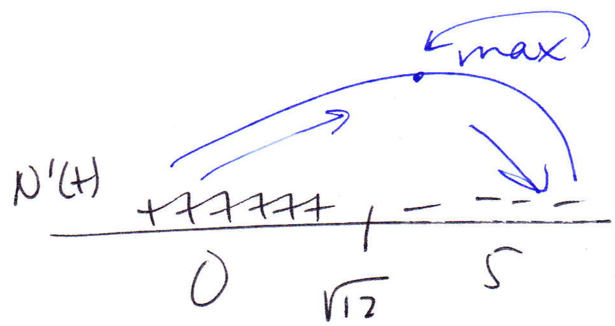
$0 = -5t^2 + 60$

$5t^2 = 60$

$t^2 = 12$

$t = \pm\sqrt{12}$

$t = +\sqrt{12} = 2\sqrt{3}$



$N'(0) = \frac{0+60}{+} = +$

$N'(5) = \frac{-5(5)^2+60}{+} = \frac{-}{+}$

The maximum number of cases occurs after $2\sqrt{3}$ or about 3.46 weeks.

Max # new cases $\Rightarrow N(\sqrt{12})$ y-value

There are $5\sqrt{3}/12$ new cases.

$N(\sqrt{12}) = 5(\sqrt{12}) / [12 + (\sqrt{12})^2] = 5\sqrt{12}/12+12 = \frac{5\sqrt{12}}{24} = \frac{5 \cdot 2 \cdot \sqrt{3}}{2 \cdot 12} = \frac{5\sqrt{\ }}{12}$

8. c) Minimum of instantaneous rate of change $\Rightarrow$ minimize N' ... so find $N''(t)=0$

$$N'(t) = \frac{-5t^2+60}{(12+t^2)^2}$$

$$N''(t) = \frac{(12+t^2)^2(-10t) - (-5t^2+60)(2(12+t^2)\cdot 2t)}{(12+t^2)^4}$$

$$= \frac{\cancel{(12+t^2)}\left[(12+t^2)(-10t) - 4t(-5t^2+60)\right]}{(12+t^2)^{\cancel{4}\,3}}$$

$$= \frac{-120t - 10t^3 + 20t^3 - 240t}{(12+t^2)^3}$$

$$\begin{array}{r} 240 \\ +120 \\ \hline 360 \end{array}$$

$$= \frac{10t^3 - 360t}{(12+t^2)^3}$$

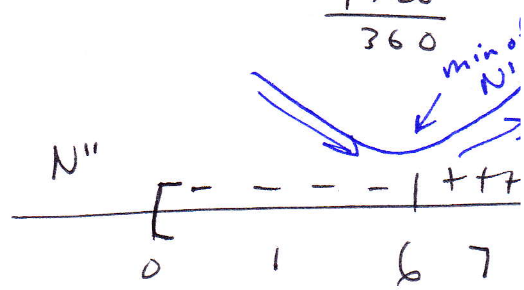
$$N''(t) = \frac{10t(t^2-36)}{(12+t^2)^3}$$

$$N''(t) = \frac{10t(t+6)(t-6)}{(t^2+12)^3}$$

$$0 = 10t(t+6)(t-6)$$

$t=0 \quad t=\pm 6$

$t=6$ ← check it's a min



$$N''(1) = \frac{10\overset{+}{(1)}\,\overset{+}{(1+6)}\,\overset{-}{(1-6)}}{+} =$$

$$N''(7) = \frac{10(7)(7+6)(7-6)}{+} =$$

$\boxed{\text{After 6 weeks the rate of spread is minimized.}}$

4. Given $L'(t) = 0.1t + 0.1$ ...

question about $L \Rightarrow$ we need to integrat

$$\int_0^3 0.1t + 0.1\,dt$$

$$= \frac{0.1t^2}{2} + 0.1t \Big|_0^3$$

$$= 0.05t^2 + 0.1t \Big|_0^3$$

$$= \underline{0.05(3)^2} + 0.1(3) - \overset{0}{\cancel{\left[0.05(0)^2 - 0.1(0)\right]}}$$

$$= 0.45 + 0.3 - 0$$

$$= 0.75$$

The pollution will change by 0.75 parts per million over next three years.