Math 105 - Practice Problems for Final Exam

1. Differentiate the following functions. No simplifying. Take derivative and stop.

   a. $f(x) = 4^x - 3x^4 + \dfrac{2}{x^6} - e^{3x} + \log_4 x$

   b. $h(x) = x6^{\sqrt{x}}$

   c. $g(x) = 3e^{5x^2+4x-1}$

   d. $f(x) = \ln(5x + \sqrt{x} + e)$

   e. $h(x) = e^{2x} \ln x$

   f. $g(t) = \dfrac{e^t - \sqrt{t}}{t + 5^t}$

2. Use the **second derivative test** to identify the relative maxima and/or minima.
   $f(x) = x^4 - 2x^2 + 3$

3. Find all vertical and horizontal asymptotes using CALCULUS techniques (LIMITS).
   $f(x) = \dfrac{x+2}{x^2 + 5x + 6}$

4. Find the extrema (**absolute** maxima and minima) of the function.

   a. $f(x) = x^5 - 5x^4 + 1$ on the interval $0 \le x \le 5$

   b. $f(x) = \left(x^2 - 4\right)^5$ on the interval $-3 \le x \le 2$

5. Let $f(x) = \dfrac{x}{x^2+1}$
   a. Find the **x and y intercepts** of the function if they exist. Answer as ordered pairs.

   b. Find the **equations of any vertical and horizontal asymptotes**, if they exist. You may use precalculus reasoning, or calculus techniques.

   c. Using calculus (use first or second derivative test), identify **relative maxima and minima**, if they exist.

   d. Find the **inflection point(s),** if any, and the **intervals** where the function is concave up and concave down.

   e. Sketch a fairly accurate graph of *f(x)* based on the information in parts a-d.

6. Find the equation of the tangent line at the specific point.
   $f(x) = (x+1)e^{-2x}$; where $x = 0$

7. A carpenter has been asked to build an open box with a square base. You want to box to have a volume of 200 cubic meters. What are the dimensions of the box that will use the least amount of material? (Also study Sections 1.4 and 3.5 homework and lecture notes examples :)

8. Population Growth. It is projected that *t* years from now, the population of a certain town will be approximately P(t) thousand people, where $P(t) = \dfrac{100}{1+e^{-0.2t}}$
   a. What is the current population?
   b. At what rate will the population be changing 10 years from now?

9. Use calculus techniques learned in class to evaluate each of the following limits
   a. $\lim\limits_{x\to 3}\left(3x^2 - 5x + 2\right)$

   b. $\lim\limits_{x\to 5}\left(\dfrac{x^2-3x-10}{x-5}\right)$

   c. $\lim\limits_{x\to\infty}\left(\dfrac{1-2x^2}{x^2+3x+6}\right)$

   d. $\lim\limits_{x\to\infty}\left(\dfrac{1-2x^2}{x+3x+6}\right)$

   e. $\lim\limits_{x\to\infty}\left(\dfrac{1-2x}{x^2+3x+6}\right)$

10. Is the function continuous at the given point? Investigate using the definition of continuity.

$$f(x)=\frac{2x-4}{3x-2} \quad \text{at} \quad x=2$$

11. Differentiate the following functions. No simplifying. Take derivative and stop.

a. $f(x)=3x^5+4x^3-2x-5$

b. $h(x)=(2x+1)^{1.4}$

c. $h(x)=e^{2x}\ln x$

d. $f(t)=5^{3t+4}$

e. $g(t)=\dfrac{t-\sqrt{t}}{e^t+5t}$

f. $h(x)=3x^2\cdot\ln(5x)$

12. Differentiate the following functions implicitly.

a. $5x-7y=3$

b. $x^3-y^2=5$

c. $xy=4$

13. Evaluate the following indefinite integrals.

a. $\displaystyle\int\left(3x^2+5\sqrt{x}-\frac{2}{x}\right)dx$

b. $\displaystyle\int\frac{x^3-3x}{x^2}dx$

c. $\displaystyle\int(3-2x)^4dx$

14. Evaluate the definite integrals

a. $\int_1^3 \left(\frac{5}{x}+\sqrt{x}\right)dx$

b. $\int_0^4 \frac{1}{\sqrt{6x+1}}dx$

c. $\int_0^1 8x(x^2+1)^3 dx$

d. $\int_{-1}^3 \left(3x^2-1\right)e^{x^3-x}\,dx$

15. Sketch the function $f(x)=x^3+3x^2-2$ using the 7 step process (no x-int required)
Find y intercept, identify max/min points and intervals where *f(x)* is increasing and decreasing, inflection points and where *f(x)* is concave up and concave down and sketch.

16. An epidemiologist determines that a particular epidemic spreads in such a way that *t* weeks after the outbreak, *N* hundred new cases will be reported where

$$N(t)=\frac{5t}{12+t^2}$$

a. Find the instantaneous rate of change of the epidemic.
b. At what time is the epidemic at its worst? **At what time** is the number of cases maximal. (i.e. find absolute maximum) Find the **maximum number of new cases** also.
c. Health officials declare the epidemic to be under control when the **(instantaneous) rate of new infections is minimized**. When does this occur?

17. An environmental study of a certain community suggests that *t* years from now the level of carbon monoxide in the air, $L(t)$ will be changing at a rate $L'(t)=0.1t+0.1$ in parts per million per year. By how much will the pollution level change during the next 3 years

All material we have learned this semester is fair game for the final exam. You should also study old exams and quizzes.