(1)

Practice Exam #2

1. Take the derivative and stop. DO NOT SIMPLIFY!

a. $H(x) = x^4 + \frac{2}{x^3} - \sqrt{x} + \pi$ $= x^4 + 2x^{-3} - x^{1/2} + \pi$ constant!

$\boxed{H'(x) = 4x^3 - 6x^{-4} - \frac{1}{2}x^{-1/2}}$

b. $f(x) = \left(4x^2 - 5x + \frac{1}{x}\right)^3$ $= (4x^2 - 5x + x^{-1})^3$

$\boxed{f'(x) = 3(4x^2 - 5x + x^{-1})^2 \cdot (8x - 5 - x^{-2})}$

c. $g(t) = \sqrt[3]{7t^6 + 5t^2 + 8}$ $= (7t^6 + 5t^2 + 8)^{1/3}$

$g'(t) = \frac{1}{3}(7t^6 + 5t^2 + 8)^{1/3 - 3/3}(42t^5 + 10t)$

$\boxed{g'(t) = \frac{1}{3}(7t^6 + 5t^2 + 8)^{-2/3}(42t^5 + 10t)}$

d. $h(x) = (4x^8 - x)(3x^3 + 2x + 1)$

Product Rule!

$\boxed{h'(x) = (4x^8 - x)(9x^2 + 2) + (3x^3 + 2x + 1)(32x^7 - 1)}$

e. $f(x) = 3x(7x+1)^5$

$\boxed{f'(x) = 3x[5(7x+1)^4 \cdot 7] + (7x+1)^5 \cdot 3}$

or

$\boxed{f'(x) = 105x(7x+1)^4 + 3(7x+1)^5}$

$21 \times 5 = 105$

f. $f(x) = \frac{3x - 1}{2x^2 - 5}$

$\boxed{f'(x) = \frac{(2x^2 - 5)(3) - (3x - 1)(4x)}{(2x^2 - 5)^2}}$

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2. Compute the derivative using implicit differentiation

a. $4x+8y=3$

$$\frac{d}{dx}4x+\frac{d}{dx}8y=\frac{d}{dx}3$$

$$4+8y'=0$$

$$8y'=-4$$

$$y'=\frac{-4}{8}$$

$$\boxed{y'=\frac{-1}{2}}$$

b. $\frac{2}{x}+\frac{1}{y}=6$

$$2x^{-1}+y^{-1}=6$$

$$\frac{d}{dx}2x^{-1}+\frac{d}{dx}y^{-1}=\frac{d}{dx}6$$

$$-2x^{-2}+(-1y^{-2})y'=0$$

$$\frac{-2}{x^2}-\frac{y'}{y^2}=0$$

$$-\frac{y'}{y^2}=\frac{2}{x^2}$$

$$\cancel{y^2}\,\frac{-y'}{\cancel{y^2}}=\frac{2}{x^2}y^2$$

$$-y'=\frac{2y^2}{x^2}$$

$$\boxed{y'=\frac{-2y^2}{x^2}}$$

★ Product Rule

c. $5x+x^2y^3=2$

$$\frac{d}{dx}5x+\frac{d}{dx}x^2y^3=\frac{d}{dx}2$$

$$\cancel{5}+(x^2 3y^2y'+y^3\cdot 2x)=0$$

$$-5 \qquad\qquad -5$$

$$3x^2y^2y'+\cancel{2xy^3}=-5$$

$$-\cancel{2xy^3} \qquad -2xy^3$$

$$\frac{3x^2y^2y'}{3x^2y^2}=\frac{-5-2xy^3}{3x^2y^2}$$

$$\boxed{y'=\frac{-5-2xy^3}{3x^2y^2}}$$

3. Compute the second derivative of the following function. SIMPLIFY COMPLETELY!

$f(x) = 7x + \frac{1}{x^2} = 7x + x^{-2}$

$f'(x) = 7 - 2x^{-3}$

$f''(x) = 6x^{-4}$

$\boxed{f''(x) = \frac{6}{x^4}}$

4. For the function $f(x) = x^4 + 4x^3 + 4x^2$

a. Find the **x and y intercepts** of the function if they exist. Answer as ordered pairs.

x int $\Rightarrow y = 0$

$0 = x^4 + 4x^3 + 4x^2$

$0 = x^2(x^2 + 4x + 4)$

$0 = x^2(x+2)^2$

$x^2 = 0$ $\quad$ $x = 0$

$(x+2)^2 = 0$ $\quad$ $x + 2 = 0$ $\quad$ $x = -2$

$\boxed{\begin{array}{c}\text{x-int} \\ (0,0) \\ (-2,0)\end{array}}$

y intercept $\Rightarrow$ $x = 0$

$f(0) = 0 + 0 + 0$

$= 0$

$\boxed{(0,0) \text{ y int}}$

b. Find the **equations of any vertical and horizontal asymptotes**, if they exist. You may use precalculus reasoning, or calculus techniques.

This is a polynomial so there are no vertical or horizontal asymptotes.

c. Using calculus, identify **relative maxima and minima**, if they exist.

$f'(x) = 4x^3 + 12x^2 + 8x$

$f'(x) = 4x(x^2 + 3x + 2)$

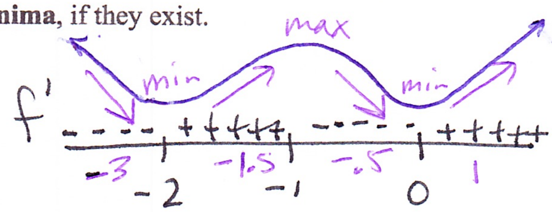
$f'(x) = 4x(x+1)(x+2)$

$0 = 4x(x+1)(x+2)$

$\frac{0}{4} = \frac{4x}{4}$ $\quad$ $x + 1 = 0$ $\quad$ $x + 2 = 0$

$0 = x$ min $\quad$ $x = -1$ max $\quad$ $x = -2$ min

Critical Points



$f'(-3) = 4(-3)(-3+1)(-3+2) = -$

$f'(-1.5) = 4(-1.5)(-1.5+1)(-1.5+2) = +$

$f'(-.5) = 4(-.5)(-.5+1)(-.5+2) = -$

$f'(1) = 4(1)(1+1)(1+2) = +$

f is increasing $(-2,-1) \cup (0,\infty)$

f is decreasing $(-\infty,-2) \cup (-1,0)$

$(0,0)$ and $(-2,0)$ are relative minimums

$(-1,1)$ is a relative max.

$f(-1) = (-1)^4 + 4(-1)^3 + 4(-1)^2$

$= 1 - 4 + 4 = 1$

$3x^2+6x+2$ $\frac{6}{6}$

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Recall, $f(x) = x^4 + 4x^3 + 4x^2$

d. Find the **inflection point(s),** if any, and the **intervals** where the function is concave up and concave down.

$f'(x) = 4x^3 + 12x^2 + 8x$

$f''(x) = 12x^2 + 24x + 8$

$f''(x) = 4(3x^2 + 6x + 2)$

need quadratic formula

$$\frac{0}{4} = \frac{4(3x^2+6x+2)}{4}$$

$0 = 3x^2 + 6x + 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{36 - 4(3)(2)}}{2(3)} = \frac{-6 \pm \sqrt{36-24}}{6} = \frac{-6 \pm \sqrt{12}}{6} = \frac{-6 \pm 2\sqrt{3}}{6} = \frac{-3 \pm \sqrt{3}}{3}$$

≈ -1.577

≈ -0.4226

f'': $+ + + + + +$ | $- - - - -$ | $+ + + + +$ on the number line -2, -1.5, -1, -0.4 $\left(\frac{-3+\sqrt{3}}{3}\right)$, 0

$f''(-2) = 4(3(-2)^2 + 6(-2) + 2) = 4 \cdot 2 = +$

$f''(-1) = 4(3(-1)^2 + 6(-1) + 2) = -$

$f''(0) = 4(3(0)^2 + 6(0) + 2) = +$

Concave up $\left(-\infty, \frac{-3-\sqrt{3}}{3}\right) \cup \left(\frac{-3+\sqrt{3}}{3}, \infty\right)$

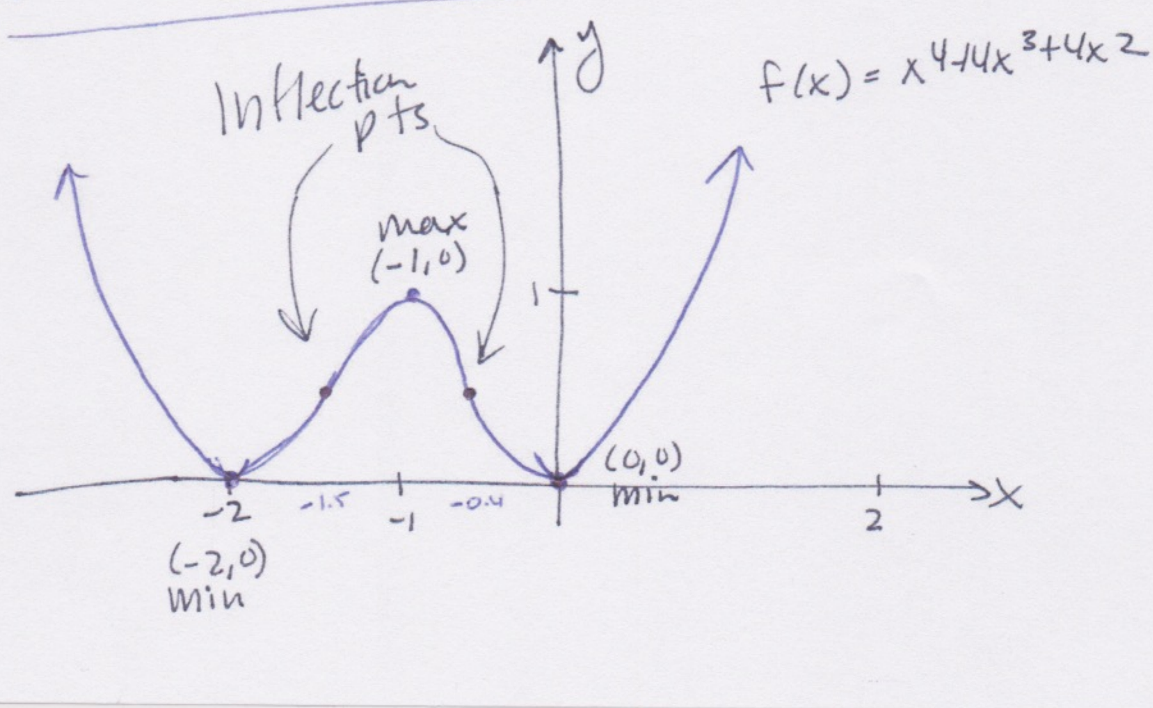
Concave down $\left(\frac{-3-\sqrt{3}}{3}, \frac{-3+\sqrt{3}}{3}\right)$

e. Sketch a fairly accurate graph of *f(x)* based on the information in parts a-d.

d) can't

★ You do not need to find y values on this one ... too ugly!!

$\boxed{\text{There are inflection points at } x = \frac{-3 \pm \sqrt{3}}{3}.}$



↙ Domain = $\mathbb{R}$

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5. For the function $f(x) = \dfrac{1}{x^2+x+1} = (x^2+x+1)^{-1}$

a. Find the **x and y intercepts** of the function if they exist. Answer as ordered pairs.

x-int $\Rightarrow y=0$ $\quad 0 = \dfrac{1}{x^2+x+1}$

$0 \neq 1$

No x int

y int $\Rightarrow x=0$ $\quad f(0) = \dfrac{1}{0+0+1} = 1$

$\boxed{(0,1)}$

y intercept

b. Find the **equations of any vertical and horizontal asymptotes**, if they exist. You may use precalculus reasoning, or calculus techniques.

Since degree of denom is bigger than degree of numerator, there is a horizontal asymptote @ $\boxed{y=0}$

vertical asymptote $\Rightarrow$ x values where denom $=0$

$x^2+x+1=0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-1 \pm \sqrt{1-4(1)(1)}}{2(1)} = \dfrac{-1 \pm \sqrt{-3}}{2}$ ← imaginary

No vertical asymptotes

(*) $f(-\tfrac{1}{2}) = \dfrac{1}{(-\tfrac{1}{2})^2 + (-\tfrac{1}{2}) + 1}$

$\dfrac{1}{\tfrac{1}{4} + (-\tfrac{1}{2}) + \tfrac{4}{4}}$

$1/(\tfrac{5}{4} - \tfrac{1}{2}(\tfrac{2}{2}))$

$1/(\tfrac{5}{4} - \tfrac{2}{4})$ $(\tfrac{4}{3})$

c. Using calculus, identify **relative maxima and minima**, if they exist.

$f'(x) = -1(x^2+x+1)^{-2}(2x+1)$

$f'(x) = \dfrac{-2x-1}{(x^2+x+1)^2}$

$0 = \dfrac{-2x-1}{(x^2+x+1)^2}$

cannot equal zero

$0 = -2x-1$

$+1 \qquad +1$

$1 = -2x$

$-\tfrac{1}{2} = x$

max

f': $+ + + + + +$ $\quad - - - - -$

$-1 \quad -\tfrac{1}{2} \quad 0$

$f'(-1) = (-2(-1)-1)/((-1)^2+(-1)+1)^2$

$= (2-1)/(1-1+1)^2$

$= \dfrac{2}{+1} = +1$

$f'(0) = \dfrac{-1}{1} = -1$

f is increasing $(-\infty, -\tfrac{1}{2})$ & decreasing $(-\tfrac{1}{2}, \infty)$

max at $(-\tfrac{1}{2}, \tfrac{4}{3})$

(*) work above

Recall, $f(x)=\frac{1}{x^2+x+1}$ $\quad f'(x)=\frac{-2x-1}{(x^2+x+1)^2}$

d. Find the **inflection point(s),** if any, and the **intervals** where the function is concave up and concave down.

$$f''(x)=\frac{(x^2+x+1)^2(-2)-(-2x-1)2(x^2+x+1)\cdot(2x+1)}{(x^2+x+1)^{\cancel{2}4}}$$

$$f''(x)=\frac{\cancel{(x^2+x+1)}\big((x^2+x+1)(-2)-(-2x-1)2(2x+1)\big)}{(x^2+x+1)^{\cancel{2}3}}$$

$$f''(x)=\frac{-2x^2-2x-2+(4x+2)(2x+1)}{(x^2+x+1)^3}$$

$$f''(x)=\frac{-2x^2-2x-2+8x^2+8x+2}{(x^2+x+1)^3}$$

$$f''(x)=\frac{6x^2+6x}{(x^2+x+1)^3}$$

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e. ~~Sketch a fairly accurate graph of $f(x)$ based on the information in parts a-d.~~

d con't) $0=6x^2+6x$

$0=6x(x+1)$

$\frac{0}{6}=\frac{6x}{6}$ $\quad x+1=0$

$0=x$ $\quad x=-1$

f'': ++ / −− / ++

+++++ | −−−−− | +++++

$-2 \quad -1 \quad -\frac{1}{2} \quad 0 \quad 1$

$$f''(-2)=\frac{6(-2)^2+6(-2)}{((-2)^2+(-2)+1)^3}$$

$$=\frac{24-12}{(4-2+1)^3}=\frac{+}{+}$$

$f''(-\frac{1}{2}) = -$

$f''(1) = +$

$f(x)$ is concave up $(-\infty,-1)\cup(0,\infty)$ and concave down $(-1,0)$

$(-1,1)$ & $(0,1)$ are inflection pts

$$f(-1)=\frac{1}{((-1)^2+(-1)+1)^3}$$

$$=\frac{1}{(1-1+1)^3}$$

$$=1$$

5. e) Sketch use all information

y int $(0,1)$ (No x intercepts)

$y=0$ horizontal asymptote

$\left(-\frac{1}{2}, \frac{4}{3}\right)$ maximum

$(-1,1)$ $(0,1)$ inflection pts