Exam 2

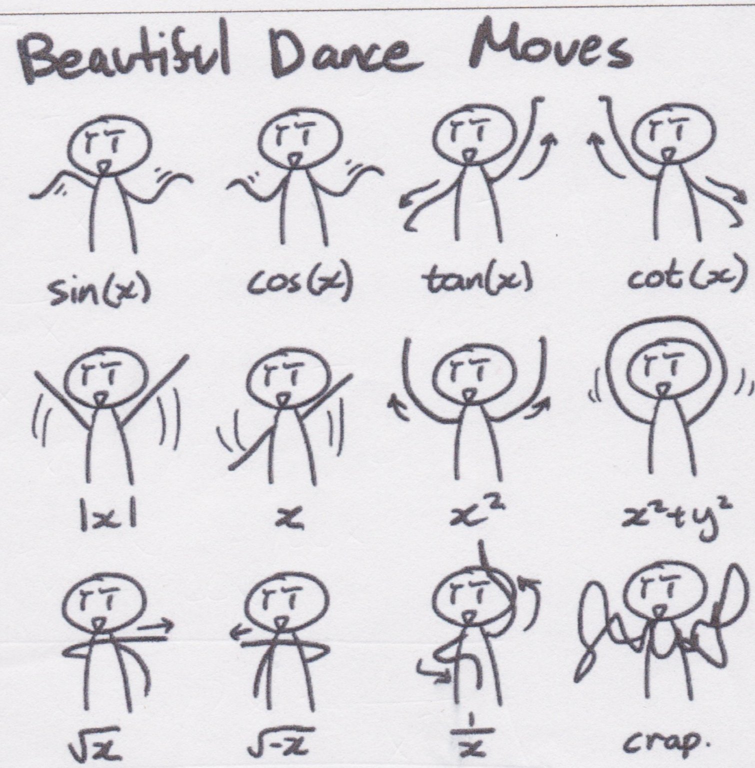
Math 105 - Bio-Calculus

- ✓ You have 50 minutes to take the exam.
- ✓ You may use a calculator (NO PHONES).
- ✓ You must SHOW ALL WORK and simplify all answers completely unless otherwise stated in order to receive full credit.
- ✓ Please indicate your answers by circling or boxing them.
- ✓ You may **NOT** use any notes, book, or neighbors during the exam.
- ✓ Try to leave answers exact (NOT as decimals) and use improper fractions when necessary.
- ✓ If you feel that you may be on the wrong track, put an x through work and try problem over on scratch paper. Many times, you are on the right track, but second-guess yourself.

I have read the above guidelines and agree to follow them. Also, the work contained on this exam is my own and I promise to adhere to academic honesty.

Name: Key

Signature: ______________________



Show as much work as possible. Answers with no work shown will receive no credit. Feel free to use sentences to explain answers as well.

1. Take the derivative and STOP. **DO NOT SIMPLIFY!**

5 a. $H(x)=\frac{7}{10}x^5-\frac{6}{x^3}+\sqrt{x}-\pi$ $=\frac{7}{10}x^5-6x^{-3}+\sqrt{x}-\pi$

$$\boxed{H'(x)=\frac{7}{10}\cdot 5x^4+18x^{-4}+\frac{1}{2\sqrt{x}}}$$

5 b. $g(t)=\left(-\frac{4}{7}t^7-3t^{-5}-\sqrt{3}\right)^8$

+1

$$\boxed{g'(t)=8\left(-\frac{4}{7}t^7-3t^{-5}-\sqrt{3}\right)^7\cdot\left(-4t^6+15t^{-6}-0\right)}$$

5 c. $f(x)=\sqrt[3]{2x^2+5x-6}$ $=(2x^2+5x-6)^{1/3}$

$\frac{1}{3}-\frac{3}{3}=\frac{-2}{3}$

-.5

$$\boxed{f'(x)=\frac{1}{3}(2x^2+5x-6)^{-2/3}\cdot(4x+5)}$$

Product Rule! $(f\cdot g)'=fg'+gf'$

5 d. $h(t)=7t^6\cdot(3t^2-5)^3$

$$\boxed{h'(t)=7t^6\cdot\left[3(3t^2-5)^2\cdot(6t)\right]+(3t^2-5)^3\cdot 42t^5}$$

No prod. rule +2.5/5

Quotient Rule $\left(\frac{hi}{lo}\right)'=\frac{lo\,Dhi-hi\,Dlo}{lolo}$

5 e. $f(x)=\frac{7x^3-6x^4+x-6}{3-5x}$

$$\boxed{f'(x)=\frac{(3-5x)(21x^2-24x^3+1)-(7x^3-6x^4x-6)(-5)}{(3-5x)^2}}$$

+1

Swap numerator terms +1. 3/5

/25

$\frac{2x^5}{y^2}+3$

2. Compute the derivative of the equation **using implicit differentiation**. Simplify!

dy/dx notation 5 a. $x^6 - 3y^2 = 20$

$$\frac{d}{dx}x^6 - \frac{d}{dx}3y^2 = \frac{d}{dx}20$$

$$6x^5 - 6y \cdot \frac{dy}{dx} = 0$$

$$-6y\frac{dy}{dx} = -6x^5$$

$$\frac{dy}{dx} = \frac{-\cancel{6}x^5}{-\cancel{6}y}$$

$$\boxed{\frac{dy}{dx} = \frac{x^5}{y}}$$

↓.5

y' notation 5 b. $5y^3 + 6 = x^3 + 3xy$ → Product rule! ~~x~~ NO

2.5/5

$$15y^2 \cdot y' + 0 = 3x^2 + [3x \cdot y' + y \cdot 3]$$

< Solve for y'

$$15y^2y' = 3x^2 + 3xy' + 3y$$

$$15y^2y' - 3xy' = 3x^2 + 3y$$

$$y'(15y^2 - 3x) = 3x^2 + 3y$$

$$y' = \frac{3x^2+3y}{15y^2-3x}$$

x 4.5/5

$$y' = \frac{3(x^2+y)}{3(5y^2-x)}$$

$$\boxed{y' = \frac{x^2+y}{5y^2-x}}$$

+3 used prod rule but mistake!

10 3. Compute the **second derivative** of the following function. SIMPLIFY COMPLETELY!

$g(x) = \frac{3}{4x-7}$

$$g'(x) = \frac{\cancel{(4x-7)(0)} - 3(4)}{(4x-7)^2}$$

$$g'(x) = \frac{-12}{(4x-7)^2}$$

$$g''(x) = \frac{\cancel{(4x-7)^2(0)} - (-12)(2(4x-7)\cdot 4)}{(4x-7)^4}$$

$$g''(x) = \frac{12 \cdot 8\cancel{(4x-7)}}{(4x-7)^{\cancel{4}\,3}}$$

$$\boxed{g''(x) = \frac{96}{(4x-7)^3}}$$

-1.5 not reduced

-96

+9/10

/20

4. For the function $f(x) = x^4 - 4x^3$

3 a. Find the **x and y intercepts** of the function if it exists. Answer as an ordered pair(s).

$\boxed{\text{x-int}} \Rightarrow f(x) = 0$

$0 = x^4 - 4x^3$

$0 = x^3(x-4)$

$x = 0 \quad x = 4$

$\boxed{(0,0) \quad (4,0) \text{ x-int}}$

$\boxed{\text{y int}} \Rightarrow x = 0$

$f(0) = 0^4 - 4(0)$

$= 0$

$\boxed{(0,0) \text{ y int}}$

7 b. Using calculus, find and state the **intervals of increase and decrease.** Answer in proper set-builder or interval notation.

Identify cps.

$f'(x) = 4x^3 - 12x^2$

$f'(x) = 4x^2(x-3)$

$0 = 4x^2(x-3)$

$4x^2 = 0 \qquad x - 3 = 0$

C.P.s $\rightarrow$ $x = 0 \qquad x = 3$

f': $- - - \;|\; - - - \;|\; + + + +$ (number line with -1, 0, 1, 3, 4)

$f'(-1) = +(-) = -$

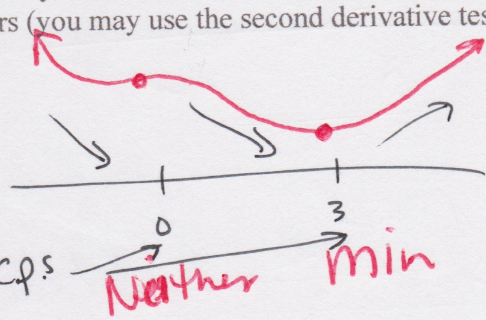
$f'(1) = +(-) = -$

$f'(4) = +(+) = +$

$\boxed{\begin{array}{l} f \text{ is increasing } (3, \infty) \\ f \text{ is decreasing } (-\infty, 0) \cup (0, 3) \end{array}}$ $\rightarrow$ $(-\infty, 3)$ +5.5/7

5 c. Identify **relative maxima and minima,** if they exist. Remember these are stated as ordered pairs (you may use the second derivative test for a max/min if you want to).

f' test



$X = 0$ neither a max or min

$X = 3$ min

$(3, f(3))$ min

$\boxed{(3, -27) \text{ minimum}}$

$f(3) = 3^4 - 4(3)^3$

$= 81 - 4(27)$ 4

$= 81 - 108$

$= -27$

/15

Recall, $f(x) = x^4 - 4x^3$

7 d. Using calculus, find the intervals where the function is **concave up and concave down**. Show work! Answer in set-builder or interval notation.

f'' test

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

$f''(x) = 12x(x-2)$

$0 = 12x(x-2)$

$12x = 0 \quad x - 2 = 0$

$\underbrace{x = 0 \quad x = 2}_{\text{possible I.P.'s}}$

f'': $++$ ∪ over $+++++$ | 0 | $- - - -$ ∩ over $- -$ | 2 | $++$ ∪ over $++++$ (test points -1, 1, 3)

$f''(-1) = (-)(-) = +$

$f''(1) = (+)(-) = -$

$f''(3) = (+)(+) = +$

+3

$\boxed{\begin{array}{l} f \text{ concave up } (-\infty, 0) \cup (2, \infty) \\ f \text{ concave down } (0, 2) \end{array}}$

5 e. State the **inflection point(s),** if any, as ordered pairs.

(2, 0) 4.5

Since concavity switches over both possible I.P.'s $x = 0, 2$ are both I.P.s

$\boxed{\begin{array}{l} (0,0) \\ (2,-16) \end{array} \text{ I.P.s}}$

$f(0) = 0$

$f(2) = 2^4 - 4(2)^3 = 16 - 4(8)$
$= 16 - 32$
$= -16$

6.5

18 f. Sketch a fairly accurate graph of *f(x)* based on the information in parts a-d. Label all intercepts/max/min/inflection points if they exist. **Be sure to scale and label axes!!**

.5 .5

other pts for accuracy:

$x = -2$

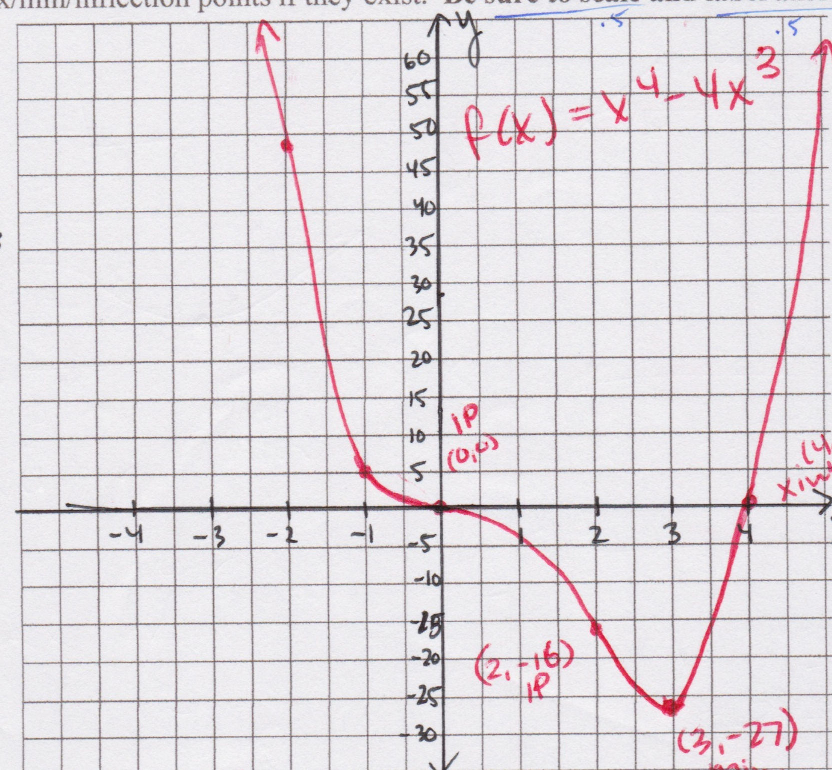
$f(-2) = (-2)^4 - 4(-2)^3$
$= 16 - 4(-8)$
$= 16 + 32$
$= 48$

$x = -1$

$f(-1) = (-1)^4 - 4(-1)^3$
$= 1 - 4(-1)$
$= 1 + 4$
$= 5$

$x = 1$

$f(1) = (1)^4 - 4(1)^3$
$= 1 - 4$
$= -3$



$f(5) = 5^4 - 4(5)^3$
$= 625 - 4(125) = 625 - 500 = 125$

+4 Good effort.

Summary

x	y	
0	0	x/y int/IP
4	0	x int
3	-27	min
2	-16	I.P.
-2	48	
-1	5	
1	-3	
5	125	

/20

5. A 5-year projection of the population trends suggests that t years from now, the population of a certain community will be $P(t) = -t^3 + 9t^2 + 48t + 50$ thousand.

a. At what time during the 5-year period will the population be growing most rapidly?

b. At what time is the rate of population growth changing most rapidly.

Extra Credit +2xtra

Find max of $P'(t)$

Find max of $P''(t)$

a) $P(t) = -t^3 + 9t^2 + 48t + 50$

+3

$P'(t) = -3t^2 + 18t + 48$ ← Find max

$P''(t) = -6t + 18$

$P''(t) = -6(t-3)$

$0 = -6(t-3)$

$-6 \neq 0 \quad t - 3 = 0$

$t = 3$

C.P. of P'

P'' + + + | − − − P'

1 3 4

$P''(1) = +$

$P''(4) = -$

$\boxed{t = 3 \text{ is max of } P'(t) \text{ aka fastest growth of } P(t)}$

b) $P''(t) = -6t + 18$

$P'''(t) = -6 \Rightarrow$

P'' is always decreasing.

$\boxed{\text{This means the max rate of pop growth occurs at time } t = 0.}$

6. 10 Find all critical points of the function. Show all work/explain if needed.

$f(x) = 9x^{1/3}$

+4 effort

Case 1:

$f'(x) = 0$

$f'(x) = 9 \cdot \frac{1}{3} x^{1/3 - 1}$

$f'(x) = 3x^{-2/3}$

$f'(x) = \frac{3}{x^{2/3}}$

$0 = \frac{3}{x^{2/3}}$

$0 \neq 3$

No C.P.'s Case 1

5/10

Case 2: $f'(c)$ DNE and $f(c)$ exists.

$f'(x) = \frac{3}{x^{2/3}}$

Denom $\neq 0$

$x^{2/3} \neq 0$

$x \neq 0$

Discontinuity of f'

$x = 0$

$f(0)$ exist?

$f(0) = 9(0)^{1/3}$

$= 9 \cdot 0$

$= 0$ ✓

$\boxed{\text{Since } f'(0) \text{ DNE \& } f(0) \text{ exists } x = 0 \text{ is a cp. by case 2}}$

7. Find the 1st through 3rd derivatives of the function $f(x) = 3x^4 - \frac{2}{x^4} + 5x^5 - 8x + \pi$. Simplify the 3rd derivative completely.

+7

$f(x) = 3x^4 - 2x^{-4} + 5x^5 - 8x + \pi$

$f'(x) = 12x^3 + 8x^{-5} + 25x^4 - 8 + 0$

$f''(x) = 36x^2 - 40x^{-6} + 100x^3$

$f'''(x) = 72x + 240x^{-7} + 300x^2$

$\boxed{f'''(x) = 300x^2 + 72x + \frac{240}{x^7}}$

/20

☺ Comments/Feedback: