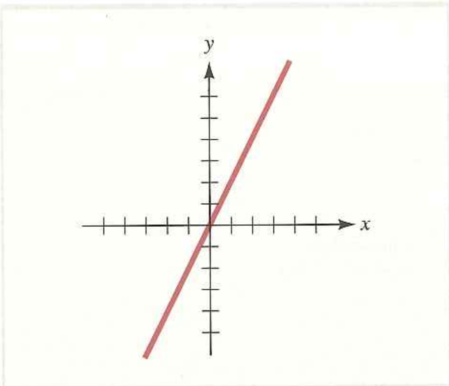
EXERCISES ■ 1.3

In Exercises 1 through 8, find the slope (if defined) of the line that passes through the given pair of points.

1. $(2, -3)$ and $(0, 4)$
2. $(-1, 2)$ and $(2, 5)$
3. $(2, 0)$ and $(0, 2)$
4. $(5, -1)$ and $(-2, -1)$
5. $(2, 6)$ and $(2, -4)$
6. $\left(\frac{2}{3}, -\frac{1}{5}\right)$ and $\left(-\frac{1}{7}, \frac{1}{8}\right)$
7. $\left(\frac{1}{7}, 5\right)$ and $\left(-\frac{1}{11}, 5\right)$
8. $(-1.1, 3.5)$ and $(-1.1, -9)$

In Exercises 9 through 12, find the slope and intercepts of the line shown. Then find an equation for the line.

9.



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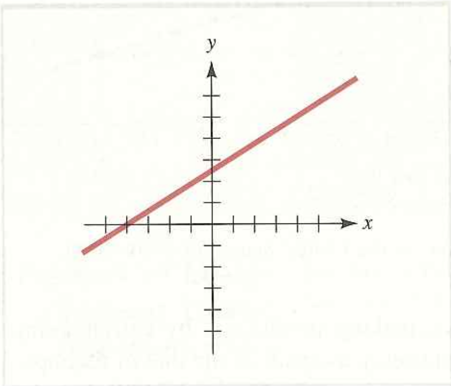

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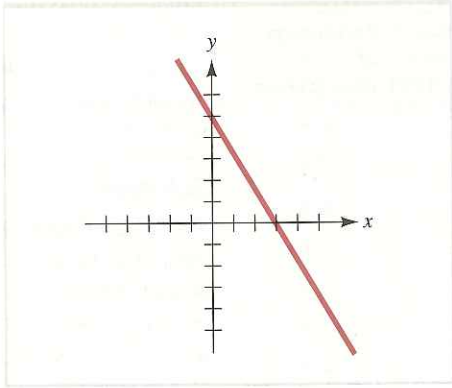
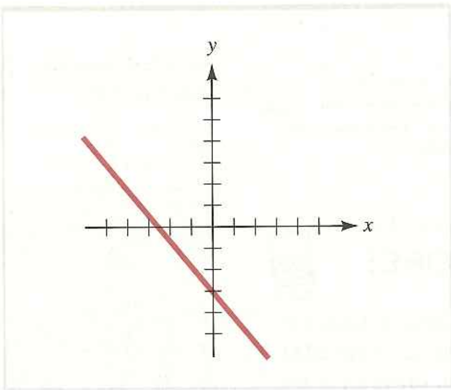


12.



In Exercises 13 through 20, find the slope and intercepts of the line whose equation is given and sketch the graph of the line.

13. $x = 3$
14. $y = 5$
15. $y = 3x$
16. $y = 3x - 6$
17. $3x + 2y = 6$
18. $5y - 3x = 4$
19. $\frac{x}{2} + \frac{y}{5} = 1$
20. $\frac{x + 3}{-5} + \frac{y - 1}{2} = 1$

In Exercises 21 through 36, write an equation for the line with the given properties.

21. Through $(2, 0)$ with slope 1
22. Through $(-1, 2)$ with slope $\frac{2}{3}$
23. Through $(5, -2)$ with slope $-\frac{1}{2}$
24. Through $(0, 0)$ with slope 5
25. Through $(2, 5)$ and parallel to the x axis

26. Through (2, 5) and parallel to the y axis

27. Through (1, 0) and (0, 1)

28. Through (2, 5) and (1, −2)

29. Through $\left(-\frac{1}{5}, 1\right)$ and $\left(\frac{2}{3}, \frac{1}{4}\right)$

30. Through (−2, 3) and (0, 5)

31. Through (1, 5) and (3, 5)

32. Through (1, 5) and (1, −4)

33. Through (4, 1) and parallel to the line $2x + y = 3$

34. Through (−2, 3) and parallel to the line $x + 3y = 5$

35. Through (3, 5) and perpendicular to the line $x + y = 4$

36. Through $\left(-\frac{1}{2}, 1\right)$ and perpendicular to the line $2x + 5y = 3$

BUSINESS AND ECONOMICS APPLIED PROBLEMS

37. MANUFACTURING COST A manufacturer's total cost consists of a fixed overhead of \$5,000 plus production costs of \$60 per unit.

- **a.** Express the total cost as a function of the number of units produced, and sketch its graph.
- **b.** Find the average cost function $AC(x)$. What is the average cost of producing 20 units?

38. MANUFACTURING COST A manufacturer estimates that it costs \$75 to produce each unit of a particular commodity. The fixed overhead is \$4,500.

- **a.** Express the total cost of production as a function of the number of units produced, and sketch the graph.
- **b.** Find the average cost function $AC(x)$. What is the average cost of producing 50 units?

39. CREDIT CARD DEBT A credit card company estimates that the average cardholder owed \$7,853 in the year 2005 and \$9,127 in 2010. Suppose average cardholder debt D grows at a constant rate.

- **a.** Express D as a linear function of time t, where t is the number of years after 2005. Draw the graph.
- **b.** Use the function in part (a) to predict the average cardholder debt in the year 2015.
- **c.** Approximately when will the average cardholder debt be double the amount in the year 2005?

40. CAR RENTAL A car rental agency charges \$75 per day plus 70 cents per mile.

- **a.** Express the cost of renting a car from this agency for 1 day as a function of the number of miles driven, and draw the graph.
- **b.** How much does it cost to rent a car for a 1-day trip of 50 miles?
- **c.** The agency also offers a rental for a flat fee of \$125 per day. How many miles must you drive on a 1-day trip for this to be the better deal?

41. LINEAR DEPRECIATION Dr. Adams owns \$1,500 worth of medical books which, for tax purposes, are assumed to depreciate linearly to zero over a 10-year period. That is, the value of the books decreases at a constant rate so that it is equal to zero at the end of 10 years. Express the value of the doctor's books as a function of time, and draw the graph.

42. LINEAR DEPRECIATION A manufacturer buys \$20,000 worth of machinery that depreciates linearly so that its trade-in value after 10 years will be \$1,000.

- **a.** Express the value of the machinery as a function of its age, and draw the graph.
- **b.** Compute the value of the machinery after 4 years.
- **c.** When does the machinery become worthless?
- **d.** The manufacturer might not wait this long to dispose of the machinery. Discuss the issues the manufacturer may consider in deciding when to sell.

43. ACCOUNTING For tax purposes, the book value of certain assets is determined by depreciating the original value of the asset linearly over a fixed period of time. Suppose an asset originally worth V dollars is linearly depreciated over a period of N years, at the end of which it has a scrap value of S dollars.

- **a.** Express the book value B of the asset t years into the N-year depreciation period as a linear function of t. [*Hint:* Note that $B = V$ when $t = 0$ and $B = S$ when $t = N$.]
- **b.** Suppose a \$50,000 piece of office equipment is depreciated linearly over a 5-year period, with a scrap value of \$18,000. What is the book value of the equipment after 3 years?

44. PRINTING COST A publisher estimates that the cost of producing between 1,000 and 10,000 copies of a certain textbook is \$50 per copy;

between 10,001 and 20,000, the cost is $40 per copy; and between 20,001 and 50,000, the cost is $35 per copy.

a. Find a function $F(N)$ that gives the total cost of producing N copies of the text for $1{,}000 \le N \le 50{,}000$.

b. Sketch the graph of the function $F(N)$ you found in part (a).

45. STOCK PRICES A certain stock had an initial public offering (IPO) price of $10 per share and is traded 24 hours a day. Sketch the graph of the share price over a 2-year period for each of the following cases:

a. The price increases steadily to $50 a share over the first 18 months and then decreases steadily to $25 per share over the next 6 months.

b. The price takes just 2 months to rise at a constant rate to $15 a share and then slowly drops to $8 over the next 9 months before steadily rising to $20.

c. The price steadily rises to $60 a share during the first year, at which time an accounting scandal is uncovered. The price gaps down to $25 a share and then steadily decreases to $5 over the next 3 months before rising at a constant rate to close at $12 at the end of the 2-year period.

46. EQUIPMENT RENTAL A rental company rents a piece of equipment for a $60.00 flat fee plus an hourly fee of $5.00 per hour.

a. Make a chart showing the number of hours the equipment is rented and the cost for renting the equipment for 2 hours, 5 hours, 10 hours, and t hours of time.

b. Write an algebraic expression representing the cost y as a function of the number of hours t. Assume t can be measured to any decimal portion of an hour. (In other words, assume t is any nonnegative real number.)

c. Graph the expression from part (b).

d. Use the graph to approximate, to two decimal places, the number of hours the equipment was rented if the bill is $216.25 (before taxes).

47. APPRECIATION OF ASSETS Aria owns a rare book that doubles in value every 10 years. In 1900, the book was worth $100.

a. How much was the book worth in 1930? In 2000? How much should Aria expect her book to be worth in the year 2020?

b. Is the value of the book a linear function of its age? Answer the question by interpreting an appropriate graph.

48. UNEMPLOYMENT RATE In the solution to Example 1.3.8, we noted that the line that best fits the unemployment data in the example in the sense of least-squares approximation has the equation $y = -0.389x + 7.338$. The data in the example stop at the year 2000. The accompanying table gives the unemployment data for the years 2001 through 2009.

Year	Number of Years from 2001	Percentage of Unemployed
2001	0	4.7
2002	1	5.8
2003	2	6.0
2004	3	5.5
2005	4	5.1
2006	5	4.6
2007	6	4.6
2008	7	5.8
2009	8	9.3

TABLE FOR EXERCISE 48 Percentage of Civilian Unemployment 2001–2009

a. Plot the data in the table in a coordinate plane. Then apply the calculator instructions in the Explore! box on page 39 to show that the line that best fits these data in the sense of least-squares approximation has the equation $y = 0.245x + 4.731$.

b. Interpret the slope of the line in part (a) in terms of the rate of unemployment.

c. Does the least-squares line do a good job of predicting these new data? Explain.

LIFE AND SOCIAL SCIENCE APPLIED PROBLEMS

49. COURSE REGISTRATION Students at a state college may preregister for their fall classes by mail during the summer. Those who do not preregister must register in person in September. The registrar can process 35 students per hour during the September registration period. Suppose that after 4 hours in September, a total of 360 students (including those who preregistered) have been registered.

a. Express the number of students registered as a function of time and draw the graph.
b. How many students were registered after 3 hours?
c. How many students preregistered during the summer?

50. **ENTOMOLOGY** It has been observed that the number of chirps made by a cricket each minute depends on the temperature. Crickets won't chirp if the temperature is 38°F or less, and observations yield the following data:

Number of chirps (C)	0	5	10	20	60
Temperature T (°F)	38	39	40	42	50

a. Express T as a linear function of C.
b. How many chirps would you expect to hear if the temperature is 75°F? If you hear 37 chirps in a 30-second period of time, what is the approximate temperature?

51. **GROWTH OF A CHILD** The average height H in centimeters of a child of age A years can be estimated by the linear function $H = 6.5A + 50$. Use this formula to answer these questions.
a. What is the average height of a 7-year-old child?
b. How old must a child be to have an average height of 150 cm?
c. What is the average height of a newborn baby? Does this answer seem reasonable?
d. What is the average height of a 20-year-old? Does this answer seem reasonable?

52. **MEMBERSHIP FEES** Membership in a swimming club costs \$250 for the 12-week summer season. If a member joins after the start of the season, the fee is prorated; that is, it is reduced linearly.
a. Express the membership fee as a function of the number of weeks that have elapsed by the time the membership is purchased and draw the graph.
b. Compute the cost of a membership that is purchased 5 weeks after the start of the season.

53. **WATER CONSUMPTION** Since the beginning of the month, a local reservoir has been losing water at a constant rate. On the 12th of the month the reservoir held 200 million gallons of water, and on the 21st it held only 164 million gallons.
a. Express the amount of water in the reservoir as a function of time, and draw the graph.
b. How much water was in the reservoir on the 8th of the month?

54. **CAR POOLING** To encourage motorists to form car pools, the transit authority in a certain metropolitan area has been offering a special reduced rate at toll bridges for vehicles containing four or more persons. When the program began 30 days ago, 157 vehicles qualified for the reduced rate during the morning rush hour. Since then, the number of vehicles qualifying has been increasing at a constant rate, and today 247 vehicles qualified.
a. Express the number of vehicles qualifying each morning for the reduced rate as a function of time, and draw the graph.
b. If the trend continues, how many vehicles will qualify during the morning rush hour 14 days from now?

55. **TEMPERATURE CONVERSION**
a. Temperature measured in degrees Fahrenheit is a linear function of temperature measured in degrees Celsius. Use the fact that 0° Celsius is equal to 32° Fahrenheit and 100° Celsius is equal to 212° Fahrenheit to write an equation for this linear function.
b. Use the function you obtained in part (a) to convert 15° Celsius to Fahrenheit.
c. Convert 68° Fahrenheit to Celsius.
d. What temperature is the same in both the Celsius and Fahrenheit scales?

56. **NUTRITION** Each ounce of Food I contains 3 g of carbohydrate and 2 g of protein, and each ounce of Food II contains 5 g of carbohydrate and 3 g of protein. Suppose x ounces of Food I are mixed with y ounces of Food II. The foods are combined to produce a blend that contains exactly 73 g of carbohydrate and 46 g of protein.
a. Explain why there are $3x + 5y$ g of carbohydrate in the blend and why we must have $3x + 5y = 73$. Find a similar equation for protein. Sketch the graphs of both equations.
b. Where do the two graphs in part (a) intersect? Interpret the significance of this point of intersection.

57. **COLLEGE ADMISSIONS** The average scores of incoming students at an eastern liberal arts college in the SAT mathematics examination have been declining at a constant rate in recent years. In 2005, the average SAT score was 575, while in 2010 it was 545.
a. Express the average SAT score as a function of time.

b. If the trend continues, what will the average SAT score of incoming students be in 2015?

c. If the trend continues, when will the average SAT score be 527?

58. ALCOHOL ABUSE CONTROL Ethyl alcohol is metabolized by the human body at a constant rate (independent of concentration). Suppose the rate is 10 milliliters per hour.

a. How much time is required to eliminate the effects of a liter of beer containing 3% ethyl alcohol?

b. Express the time T required to metabolize the effects of drinking ethyl alcohol as a function of the amount A of ethyl alcohol consumed.

c. Discuss how the function in part (b) can be used to determine a reasonable "cutoff" value for the amount of ethyl alcohol A that each individual may be served at a party.

59. AIR POLLUTION In certain parts of the world, the number of deaths N per week have been observed to be linearly related to the average concentration x of sulfur dioxide in the air. Suppose there are 97 deaths when $x = 100$ mg/m^3 and 110 deaths when $x = 500$ mg/m^3.

a. What is the functional relationship between N and x?

b. Use the function in part (a) to find the number of deaths per week when $x = 300$ mg/m^3. What concentration of sulfur dioxide corresponds to 100 deaths per week?

c. Research data on how air pollution affects the death rate in a population.* Summarize your results in a one-paragraph essay.

MISCELLANEOUS PROBLEMS

60. ASTRONOMY The following table gives the length of year L (in earth years) of each planet in the solar system, along with the mean (average) distance D of the planet from the sun, in astronomical units (1 astronomical unit is the mean distance of the earth from the sun).

Planet	Mean Distance from Sun, D	Length of Year, L
Mercury	0.388	0.241
Venus	0.722	0.615
Earth	1.000	1.000
Mars	1.523	1.881
Jupiter	5.203	11.862
Saturn	9.545	29.457
Uranus	19.189	84.013
Neptune	30.079	164.783

a. Plot the point (D, L) for each planet on a coordinate plane. Do these quantities appear to be linearly related?

b. For each planet, compute the ratio $\frac{L^2}{D^3}$.

Interpret what you find by expressing L as a function of D.

c. What you have discovered in part (b) is one of Kepler's laws, named for the German astronomer Johannes Kepler (1571–1630). Read an article about Kepler, and describe his place in the history of science.

61. AN ANCIENT FABLE In Aesop's fable about the race between the tortoise and the hare, the tortoise trudges along at a slow, constant rate from start to finish. The hare starts out running steadily at a much more rapid pace, but halfway to the finish line, stops to take a nap. Finally, the hare wakes up, sees the tortoise near the finish line, desperately resumes his old rapid pace, and is nosed out by a hair. On the same coordinate plane, graph the respective distances of the tortoise and the hare from the starting line, regarded as functions of time.

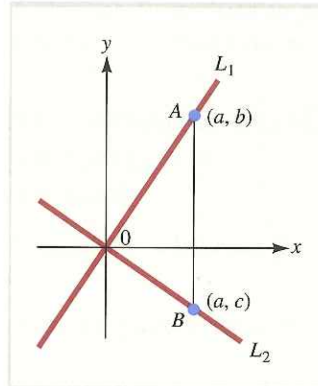
62. Graph $y = \frac{54}{270}x - \frac{63}{19}$ and $y = \frac{139}{695}x - \frac{346}{14}$ on the same set of coordinate axes using $[-10, 10]1$ by $[-10, 10]1$ for a starting range. Adjust the range settings until both lines are displayed. Are the two lines parallel?

63. Graph $y = \frac{25}{7}x + \frac{13}{2}$ and $y = \frac{144}{45}x + \frac{630}{229}$ on the same set of coordinate axes using $[-10, 10]1$ by $[-10, 10]1$. Are the two lines parallel?

*You may find the following articles helpful: D. W. Dockery, J. Schwartz, and J. D. Spengler, "Air Pollution and Daily Mortality: Associations with Particulates and Acid Aerosols," *Environ. Res.*, Vol. 59, 1992, pp. 362–373; Y. S. Kim, "Air Pollution, Climate, Socioeconomics Status and Total Mortality in the United States," *Sci. Total Environ.*, Vol. 42, 1985, pp. 245–256.

64. PARALLEL LINES Show that two nonvertical lines are parallel if and only if they have the same slope.

65. PERPENDICULAR LINES Show that if a nonvertical line L_1 with slope m_1 is perpendicular to a line L_2 with slope m_2, then $m_2 = \dfrac{-1}{m_1}$.
[*Hint:* Find expressions for the slopes of the lines L_1 and L_2 in the accompanying figure. Then apply the Pythagorean theorem along with the distance formula from Section 1.2 to the right triangle OAB to obtain the required relationship between m_1 and m_2.]



EXERCISE 65

SECTION 1.4 Functional Models

Learning Objectives

1. Study general modeling procedure.
2. Explore a variety of applied models.
3. Investigate market equilibrium and break-even analysis in economics.

Practical problems in business, economics, and the physical and life sciences are often too complicated to be precisely described by simple formulas, and one of our basic goals is to develop mathematical methods for dealing with such problems. Toward this end, we shall use a procedure called **mathematical modeling,** which may be described in terms of four stages displayed schematically in Figure 1.30:

Stage 1 (Formulation): Given a real-world situation (for example, the U.S. trade deficit, the AIDS epidemic, global weather patterns), we make enough simplifying assumptions to allow a mathematical formulation. This may require gathering and analyzing data and using knowledge from a variety of different areas to identify key variables and establish equations relating those variables. This formulation is called a **mathematical model.**

Stage 2 (Analysis of the Model): We use mathematical methods to analyze or "solve" the mathematical model. Calculus will be the primary tool of analysis in this text, but in practice, a variety of tools, such as algebra, statistics, numerical analysis, and computer methods may be brought to bear on a particular model.

Stage 3 (Interpretation): After the mathematical model has been analyzed, any conclusions that may be drawn from the analysis are applied to the original real-world problem, both to gauge the accuracy of the model and to make predictions. For instance, analysis of a model of a particular business may predict that profit will be maximized by producing 200 units of a certain commodity.

Stage 4 (Testing and Adjustment): In this final stage, the model is tested by gathering new data to check the accuracy of any predictions inferred from the analysis. If the predictions are not confirmed by the new evidence, the assumptions of the model are adjusted and the modeling process is repeated. Referring to the business example described in stage 3, it may be found that profit begins to wane at a production level significantly less than 200 units, which would indicate that the model requires modification.