Thus, $R(30) = 0 + 900 = 900$ and if c is any number other than 30, then

$$R(c) = -(c - 30)^2 + 900 < 900 \quad \text{since } -(c-30)^2 < 0$$

so the maximum revenue is \$90,000 when $x = 30$ (3,000 units).

EXERCISES ■ 1.2

In Exercises 1 through 6, plot the given points in a rectangular coordinate plane.

1. (4, 3) **2.** (−2, 7)
3. (5, −1) **4.** (−1, −8)
5. (0, −2) **6.** (3, 0)

In Exercises 7 through 10, find the distance between the given points.

7. (3, −1) and (7, 1) **8.** (4, 5) and (−2, −1)
9. (7, −3) and (5, 3) **10.** $\left(0, \frac{1}{2}\right)$ and $\left(-\frac{1}{5}, \frac{3}{8}\right)$

In Exercises 11 and 12, classify each function as a polynomial, a power function, or a rational function. If the function is not one of these types, classify it as "different."

11. a. $f(x) = x^{1.4}$
b. $f(x) = -2x^3 - 3x^2 + 8$
c. $f(x) = (3x - 5)(4 - x)^2$
d. $f(x) = \dfrac{3x^2 - x + 1}{4x + 7}$

12. a. $f(x) = -2 + 3x^2 + 5x^4$
b. $f(x) = \sqrt{x} + 3x$
c. $f(x) = \dfrac{(x - 3)(x + 7)}{-5x^3 - 2x^2 + 3}$
d. $f(x) = \left(\dfrac{2x + 9}{x^2 - 3}\right)^3$

In Exercises 13 through 28, sketch the graph of the given function. Include all x and y intercepts.

13. $f(x) = x$ **14.** $f(x) = x^2$
15. $f(x) = \sqrt{x}$ **16.** $f(x) = \sqrt{1 - x}$
17. $f(x) = 2x - 1$ **18.** $f(x) = 2 - 3x$
19. $f(x) = x(2x + 5)$ **20.** $f(x) = (x - 1)(x + 2)$
21. $f(x) = -x^2 - 2x + 15$
22. $f(x) = x^2 + 2x - 8$
23. $f(x) = x^3$
24. $f(x) = -x^3 + 1$

25. $f(x) = \begin{cases} x - 1 & \text{if } x \le 0 \\ x + 1 & \text{if } x > 0 \end{cases}$

26. $f(x) = \begin{cases} 2x - 1 & \text{if } x < 2 \\ 3 & \text{if } x \ge 2 \end{cases}$

27. $f(x) = \begin{cases} x^2 + x - 3 & \text{if } x < 1 \\ 1 - 2x & \text{if } x \ge 1 \end{cases}$

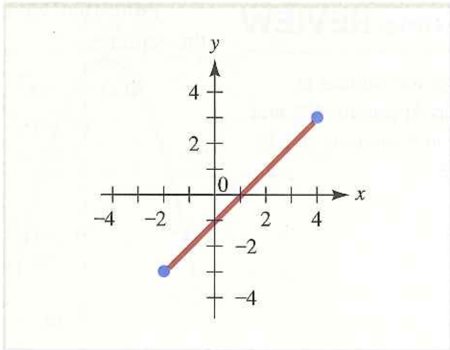
28. $f(x) = \begin{cases} 9 - x & \text{if } x \le 2 \\ x^2 + x - 2 & \text{if } x > 2 \end{cases}$

In Exercises 29 through 34, find the points of intersection (if any) of the given pair of curves and draw the graphs.

29. $y = 3x + 5$ and $y = -x + 3$
30. $y = 3x + 8$ and $y = 3x - 2$
31. $y = x^2$ and $y = 3x - 2$
32. $y = x^2 - x$ and $y = x - 1$
33. $3y - 2x = 5$ and $y + 3x = 9$
34. $2x - 3y = -8$ and $3x - 5y = -13$

In Exercises 35 through 38, the graph of a function $f(x)$ is given. In each case find:
(a) The y intercept.
(b) All x intercepts.
(c) The largest value of $f(x)$ and the value(s) of x for which it occurs.
(d) The smallest value of $f(x)$ and the value(s) of x for which it occurs.

35.



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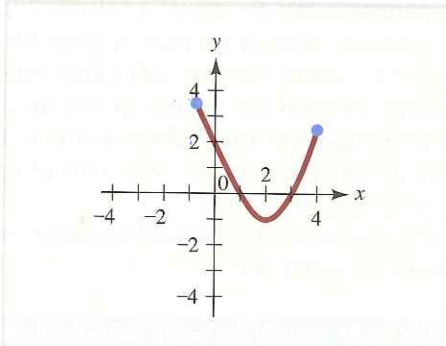

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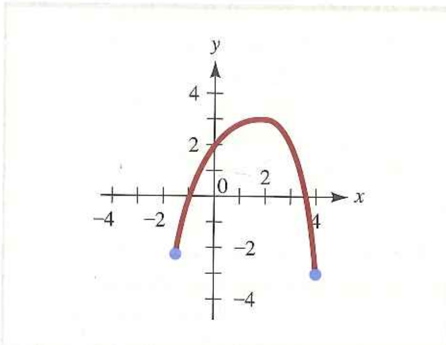

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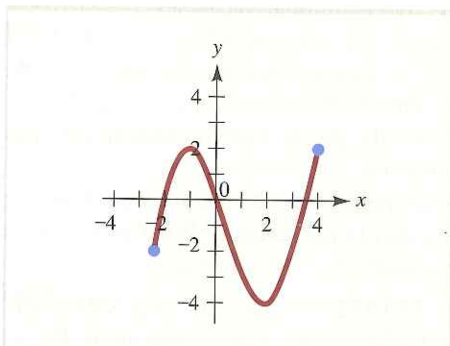


BUSINESS AND ECONOMICS APPLIED PROBLEMS

39. **MANUFACTURING COST** Vicki's company can produce digital recorders at a cost of \$40 apiece. It is estimated that if the recorders are sold for p dollars apiece, consumers will buy $120 - p$ of them a month. Express Vicki's monthly profit as a function of price, graph this function, and use the graph to estimate the optimal selling price.

40. **MANUFACTURING COST** A manufacturer can produce tires at a cost of \$20 each. It is estimated that if the tires are sold for p dollars apiece, consumers will buy $1{,}560 - 12p$ of them each month. Express the manufacturer's monthly profit as a function of price, graph this function, and use the graph to determine the optimal selling price. How many tires will be sold each month at the optimal price?

41. **RETAIL SALES** The owner of a toy store can obtain a popular board game at a cost of \$15 per set. She estimates that if each set sells for x dollars, then $5(27 - x)$ sets will be sold each week. Express the owner's weekly profit from the sales of this game as a function of price, graph this function, and estimate the optimal selling price. How many sets will be sold each week at the optimal price?

42. **RETAIL SALES** A bookstore can obtain an atlas from the publisher at a cost of \$10 per copy and estimates that if it sells the atlas for x dollars per copy, approximately $20(22 - x)$ copies will be sold each month. Express the bookstore's monthly profit from the sale of the atlas as a function of price, graph this function, and use the graph to estimate the optimal selling price.

43. **CONSUMER EXPENDITURE** Suppose $x = -200p + 12{,}000$ units of a particular commodity are sold each month when the market price is p dollars per unit. The total monthly consumer expenditure E is the total amount of money spent by consumers during each month.
    a. Express the total monthly consumer expenditure E as a function of the unit price p, and sketch the graph of $E(p)$.
    b. Discuss the economic significance of the p intercepts of the expenditure function $E(p)$.
    c. Use the graph in part (a) to determine the market price that generates the greatest total monthly consumer expenditure.

44. **CONSUMER EXPENDITURE** Suppose that x thousand units of a particular commodity are sold each month when the price is p dollars per unit, where

$$p(x) = 5(24 - x)$$

The total monthly consumer expenditure E is the total amount of money consumers spend during each month.
    a. Express total monthly expenditure E as a function of the unit price p, and sketch the graph of $E(p)$.
    b. Discuss the economic significance of the p intercepts of the expenditure function $E(p)$.

c. Use the graph in part (a) to determine the market price that generates the greatest total monthly consumer expenditure. How many units will be sold during each month at the optimal price?

45. **PROFIT** Suppose that when the price of a certain commodity is p dollars per unit, then x hundred units will be purchased by consumers, where $p = -0.05x + 38$. The cost of producing x hundred units is $C(x) = 0.02x^2 + 3x + 574.77$ hundred dollars.
   a. Express the profit P obtained from the sale of x hundred units as a function of x. Sketch the graph of the profit function.
   b. Find the average profit AP when the price is \$37 per unit.
   c. Use the profit curve found in part (a) to determine the level of production x that results in maximum profit. What unit price p corresponds to maximum profit?

46. **EQUIPMENT RENTAL** It costs \$90 to rent a piece of equipment plus \$21 for every day of use.
   a. Make a table showing the number of days the equipment is rented and the cost of renting for 2 days, 5 days, 7 days, and 10 days.
   b. Write an algebraic expression representing the cost y as a function of the number of days x.
   c. Graph the expression in part (b).

47. **MANUFACTURING OUTPUT** A company that manufactures lawnmowers has determined that a new employee can assemble N mowers per day after t days of training, where

$$N(t) = \frac{45t^2}{5t^2 + t + 8}$$

   a. Make a table showing the numbers of mowers assembled for training periods of lengths $t = 2$ days, 3 days, 5 days, 10 days, and 50 days.
   b. Based on the table in part (a), what do you think happens to $N(t)$ for very long training periods?
   c. Use your calculator to graph $N(t)$.

48. **PROFIT** Chuck owns several hot dog carts in a large downtown area. He finds that he can sell x hot dogs per day when the price of each hot dog is $p = 4.2 - 0.01x$ dollars. The cost of preparing x hot dogs per day is $C(x) = 0.002x^2 + 30$ dollars.
   a. Express Chuck's profit P as a function of sales x. Sketch the graph of the profit function.
   b. What price p should Chuck charge to maximize profit?

49. **CELL PHONE COST** A pay-as-you-go cell phone company offers a monthly plan for \$19 that includes 200 minutes of calls. After that, calls are an additional 4 cents per minute up to a maximum of 1,000 minutes. Let $C(m)$ be the cost in dollars of making m minutes of calls with a phone on this plan, for $0 \leq m \leq 1{,}000$.
   a. Write $C(m)$ as a piecewise-defined function.
   b. Sketch the graph of C.

LIFE AND SOCIAL SCIENCE APPLIED PROBLEMS

50. **BLOOD FLOW** Recall from Exercise 73, Section 1.1, that the speed of blood located r centimeters from the central axis of an artery is given by the function $S(r) = C(R^2 - r^2)$, where C is a constant and R is the radius of the artery. What is the domain of this function? Sketch the graph of $S(r)$.

51. **REAL ESTATE** Yuri manages 150 apartments in Irvine, California. All the apartments can be rented at a price of \$1,200 per month each, but for each \$100 increase in the monthly rent, there will be five additional vacancies.
   a. Express the total monthly revenue R obtained from renting apartments as a function of the monthly rental price p for each unit.
   b. Sketch the graph of the revenue function found in part (a).
   c. What monthly rental price p should Yuri charge to maximize total revenue? What is the maximum revenue?

52. **REAL ESTATE** Suppose it costs \$500 each month for Yuri's real estate company in Exercise 51 to maintain and advertise each unit that is unrented.
   a. Express the total monthly revenue R obtained from renting apartments as a function of the monthly rental price p for each unit.
   b. Sketch the graph of the revenue function found in part (a).
   c. What monthly rental price p should Yuri charge to maximize total revenue? What is the maximum revenue?

53. **AIR POLLUTION** Lead emissions are a major source of air pollution. Using data gathered by the U.S. Environmental Protection Agency in the 1990s, it can be shown that the formula

$$N(t) = -35t^2 + 299t + 3{,}347$$

estimates the total amount of lead emission N (in thousands of tons) occurring in the United States t years after the base year 1990.

a. Sketch the graph of the pollution function $N(t)$.

b. Approximately how much lead emission did the formula predict for the year 1995? (The actual amount was about 3,924 thousand tons.)

c. Based on this formula, when during the decade 1990–2000 would you expect the maximum lead emission to have occurred?

d. Can this formula be used to accurately predict the current level of lead emission? Explain.

54. **ARCHITECTURE** An arch over a road has a parabolic shape. It is 6 meters wide at the base and is just tall enough to allow a truck 5 meters high and 4 meters wide to pass.

a. Assuming that the arch has an equation of the form $y = ax^2 + b$, use the given information to find a and b. Explain why this assumption is reasonable.

b. Sketch the graph of the arch equation you found in part (a).

55. **ROAD SAFETY** When an automobile is being driven at v miles per hour, the average driver requires D feet of visibility to stop safely, where $D = 0.065v^2 + 0.148v$. Sketch the graph of $D(v)$.

56. **POSTAL RATES** Effective May 11, 2009, the cost of mailing a first-class letter was 44 cents for the first ounce and 17 cents for each additional ounce or fraction of an ounce. Let $P(w)$ be the postage required for mailing a letter weighing w ounces, for $0 \leq w \leq 3$.

a. Describe $P(w)$ as a piecewise-defined function.

b. Sketch the graph of P.

MISCELLANEOUS PROBLEMS

57. **MOTION OF A PROJECTILE** If an object is thrown vertically upward from the ground with an initial speed of 160 feet per second, its height (in feet) t seconds later is given by the function $H(t) = -16t^2 + 160t$.

a. Graph the function $H(t)$.

b. Use the graph in part (a) to determine when the object will hit the ground.

c. Use the graph in part (a) to estimate how high the object will rise.

58. **MOTION OF A PROJECTILE** A missile is projected vertically upward from an underground bunker in such a way that t seconds after launch, it is s feet above the ground, where

$$s(t) = -16t^2 + 800t - 15$$

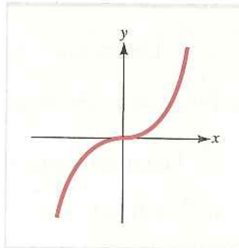
a. How deep is the bunker?

b. Sketch the graph of $s(t)$.

c. Use the graph in part (b) to determine when the missile is at its highest point. What is its maximum height?

In Exercises 59 through 62, use the vertical line test to determine whether or not the given curve is the graph of a function.

59.



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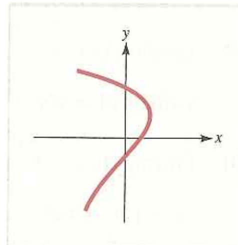

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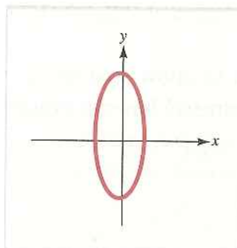

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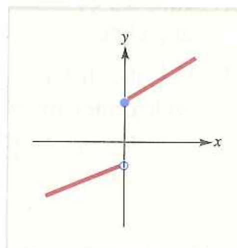


63. What viewing rectangle should be used to get an adequate graph for the quadratic function

$$f(x) = -9x^2 + 3{,}600x - 358{,}200?$$

64. What viewing rectangle should be used to get an adequate graph for the quadratic function

$$f(x) = 4x^2 - 2{,}400x + 355{,}000?$$

65. **a.** Graph the functions $y = x^2$ and $y = x^2 + 3$. How are the graphs related?

b. Without further computation, graph the function $y = x^2 - 5$.

c. Suppose $g(x) = f(x) + c$, where c is a constant. How are the graphs of f and g related? Explain.

66. **a.** Graph the functions $y = x^2$ and $y = -x^2$. How are the graphs related?

b. Suppose $g(x) = -f(x)$. How are the graphs of f and g related? Explain.

67. **a.** Graph the functions $y = x^2$ and $y = (x - 2)^2$. How are the graphs related?
b. Without further computation, graph the function $y = (x + 1)^2$.
c. Suppose $g(x) = f(x - c)$, where c is a constant. How are the graphs of f and g related? Explain.

68. Use your graphing utility to graph $y = x^4$, $y = x^4 - x$, $y = x^4 - 2x$, and $y = x^4 - 3x$ on the same coordinate axes, using [−2, 2]1 by [−2, 5]1. What effect does the added term involving x have on the shape of the graph? Repeat using $y = x^4$, $y = x^4 - x^3$, $y = x^4 - 2x^3$, and $y = x^4 - 3x^3$. Adjust the viewing rectangle appropriately.

69. Graph $f(x) = \dfrac{-9x^2 - 3x - 4}{4x^2 + x - 1}$. Determine the values of x for which the function is defined.

70. Graph $f(x) = \dfrac{8x^2 + 9x + 3}{x^2 + x - 1}$. Determine the values of x for which the function is defined.

71. Graph $g(x) = -3x^3 + 7x + 4$. Find the x intercepts.

72. Use the distance formula to show that the circle with center (a, b) and radius R has the equation

$$(x - a)^2 + (y - b)^2 = R^2$$

73. Use the result in Exercise 72 to solve the following problems.
a. Find an equation for the circle with center $(2, -3)$ and radius 4.
b. Find the center and radius of the circle with the equation

$$x^2 + y^2 - 4x + 6y = 11$$

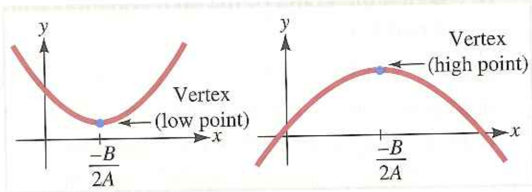
c. Describe the set of all points (x, y) such that

$$x^2 + y^2 + 4y = 2x - 10$$

74. Show that the vertex of the parabola $y = Ax^2 + Bx + C$ $(A \neq 0)$ occurs at the point where $x = \dfrac{-B}{2A}$. $\Big\{$*Hint:* First verify that

$$Ax^2 + Bx + C = A\left[\left(x + \frac{B}{2A}\right)^2 + \left(\frac{C}{A} - \frac{B^2}{4A^2}\right)\right].$$

Then note that the largest or smallest value of $f(x) = Ax^2 + Bx + C$ must occur where $x = \dfrac{-B}{2A}\Big\}$.



EXERCISE 74

SECTION 1.3 Lines and Linear Functions

Learning Objectives

1. Review properties of lines: slope, horizontal and vertical lines, and forms for the equation of a line.
2. Solve applied problems involving linear functions.
3. Study parallel and perpendicular lines.
4. Explore a least-squares linear approximation of data.

A **linear function** is one with the general form $f(x) = mx + b$, and the graph of such a function is a line. Linear functions play an important role in many practical applications, some of which we shall examine later in this section, but first we provide a brief review of the key properties of lines.

The Slope of a Line

A surveyor might say that a hill with a *rise* of 2 feet for every foot of *run* has a **slope** of

$$m = \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$$