Solution

Using the given form, the difference quotient for f can be written as

$$\frac{f(x+h)-f(x)}{h} = \frac{[(x+h)^2 - 3(x+h)] - [x^2 - 3x]}{h} \quad \text{expand the numerator}$$

$$= \frac{[x^2 + 2xh + h^2 - 3x - 3h] - [x^2 - 3x]}{h} \quad \text{combine like terms in the numerator}$$

$$= \frac{2xh + h^2 - 3h}{h} \quad \text{divide the numerator and denominator by } h$$

$$= 2x + h - 3$$

EXERCISES ■ 1.1

In Exercises 1 through 14, compute the indicated values of the given function.

1. $f(x) = 3x + 5$; $f(0), f(-1), f(2)$
2. $f(x) = -7x + 1$; $f(0), f(1), f(-2)$
3. $f(x) = 3x^2 + 5x - 2$; $f(0), f(-2), f(1)$
4. $h(t) = (2t + 1)^3$; $h(-1), h(0), h(1)$
5. $g(x) = x + \frac{1}{x}$; $g(-1), g(1), g(2)$
6. $f(x) = \frac{x}{x^2 + 1}$; $f(2), f(0), f(-1)$
7. $h(t) = \sqrt{t^2 + 2t + 4}$; $h(2), h(0), h(-4)$
8. $g(u) = (u + 1)^{3/2}$; $g(0), g(-1), g(8)$
9. $f(t) = (2t - 1)^{-3/2}$; $f(1), f(5), f(13)$
10. $f(t) = \frac{1}{\sqrt{3 - 2t}}$; $f(1), f(-3), f(0)$
11. $f(x) = x - |x - 2|$; $f(1), f(2), f(3)$
12. $g(x) = 4 + |x|$; $g(-2), g(0), g(2)$
13. $h(x) = \begin{cases} -2x + 4 & \text{if } x \le 1 \\ x^2 + 1 & \text{if } x > 1 \end{cases}$; $h(3), h(1), h(0), h(-3)$
14. $f(t) = \begin{cases} 3 & \text{if } t < -5 \\ t + 1 & \text{if } -5 \le t \le 5 \\ \sqrt{t} & \text{if } t > 5 \end{cases}$; $f(-6), f(-5), f(16)$

In Exercises 15 through 18, determine whether or not the given function has the set of all real numbers as its domain.

15. $g(x) = \frac{x}{1 + x^2}$
16. $f(x) = \frac{x + 1}{x^2 - 1}$
17. $f(t) = \sqrt{1 - t}$
18. $h(t) = \sqrt{t^2 + 1}$

In Exercises 19 through 24, determine the domain of the given function.

19. $g(x) = \frac{x^2 + 5}{x + 2}$
20. $f(x) = x^3 - 3x^2 + 2x + 5$
21. $f(x) = \sqrt{2x + 6}$
22. $f(t) = \frac{t + 1}{t^2 - t - 2}$
23. $f(t) = \frac{t + 2}{\sqrt{9 - t^2}}$
24. $h(s) = \sqrt{s^2 - 4}$

In Exercises 25 through 32, find the composite function $f(g(x))$.

25. $f(u) = 3u^2 + 2u - 6$, $g(x) = x + 2$
26. $f(u) = u^2 + 4$, $g(x) = x - 1$
27. $f(u) = (u - 1)^3 + 2u^2$, $g(x) = x + 1$
28. $f(u) = (2u + 10)^2$, $g(x) = x - 5$
29. $f(u) = \frac{1}{u^2}$, $g(x) = x - 1$
30. $f(u) = \frac{1}{u}$, $g(x) = x^2 + x - 2$
31. $f(u) = \sqrt{u + 1}$, $g(x) = x^2 - 1$
32. $f(u) = u^2$, $g(x) = \frac{1}{x - 1}$

In Exercises 33 through 38, find the difference quotient, $\dfrac{f(x+h)-f(x)}{h}$.

33. $f(x) = 4 - 5x$

34. $f(x) = 2x + 3$

35. $f(x) = 4x - x^2$

36. $f(x) = x^2$

37. $f(x) = \dfrac{x}{x+1}$

38. $f(x) = \dfrac{1}{x}$

In Exercises 39 through 42, first obtain the composite functions f(g(x)) and g(f(x)), and then find all numbers x (if any) such that f(g(x)) = g(f(x)).

39. $f(x) = \sqrt{x}$, $g(x) = 1 - 3x$

40. $f(x) = x^2 + 1$, $g(x) = 1 - x$

41. $f(x) = \dfrac{2x+3}{x-1}$, $g(x) = \dfrac{x+3}{x-2}$

42. $f(x) = \dfrac{1}{x}$, $g(x) = \dfrac{4-x}{2+x}$

In Exercises 43 through 50, find the indicated composite function.

43. $f(x-2)$ where $f(x) = 2x^2 - 3x + 1$

44. $f(x+1)$ where $f(x) = x^2 + 5$

45. $f(x-1)$ where $f(x) = (x+1)^5 - 3x^2$

46. $f(x+3)$ where $f(x) = (2x-6)^2$

47. $f(x^2 + 3x - 1)$ where $f(x) = \sqrt{x}$

48. $f\left(\dfrac{1}{x}\right)$ where $f(x) = 3x + \dfrac{2}{x}$

49. $f(x+1)$ where $f(x) = \dfrac{x-1}{x}$

50. $f(x^2 - 2x + 9)$ where $f(x) = 2x - 20$

In Exercises 51 through 56, find functions h(x) and g(u) such that f(x) = g(h(x)).

51. $f(x) = (x-1)^2 + 2(x-1) + 3$

52. $f(x) = (x^5 - 3x^2 + 12)^3$

53. $f(x) = \dfrac{1}{x^2+1}$

54. $f(x) = \sqrt{3x-5}$

55. $f(x) = \sqrt[3]{2-x} + \dfrac{4}{2-x}$

56. $f(x) = \sqrt{x+4} - \dfrac{1}{(x+4)^3}$

BUSINESS AND ECONOMICS APPLIED PROBLEMS

57. PRODUCTION COST Suppose the total cost of manufacturing q units of a certain product is $C(q)$ thousand dollars, where

$$C(q) = 0.01q^2 + 0.9q + 2$$

a. Find the total cost and the average cost of producing 10 units.

b. Find the cost of producing the 10th unit.

58. PRODUCTION COST Answer the questions in Exercise 57 for the cost function

$$C(q) = q^3 - 30q^2 + 400q + 500$$

PROFITABILITY *In Exercises 59 through 62, the demand function p = D(x) and the total cost function C(x) for a particular commodity are given in terms of the level of production x. In each case, find:*

(a) The revenue R(x) and profit P(x).

(b) All values of x for which production of the commodity is profitable.

59. $D(x) = -0.02x + 29$
$C(x) = 1.43x^2 + 18.3x + 15.6$

60. $D(x) = -0.37x + 47$
$C(x) = 1.38x^2 + 15.15x + 115.5$

61. $D(x) = -0.5x + 39$
$C(x) = 1.5x^2 + 9.2x + 67$

62. $D(x) = -0.09x + 51$
$C(x) = 1.32x^2 + 11.7x + 101.4$

63. DISTRIBUTION COST Suppose that the number of worker-hours required to distribute new telephone books to $x\%$ of the households in a certain rural community is given by the function

$$W(x) = \frac{600x}{300 - x}$$

a. What is the domain of the function W?

b. For what values of x does $W(x)$ have a practical interpretation in this context?

c. How many worker-hours were required to distribute new telephone books to the first 50% of the households?

d. How many worker-hours were required to distribute new telephone books to the entire community?

e. What percentage of the households in the community had received new telephone books by the time 150 worker-hours had been expended?

64. DATA TRANSFER In the year 2000, Digicorp, a data management firm, began transferring files from antiquated databases and storing them on more modern systems. Measured in years after 2010, the function $R(t) = 30\sqrt{6 - t}$ represents the number of databases remaining to be transferred.

a. What is the domain of R?

b. How many databases were present when Digicorp began the transfer?

c. How many databases still needed to be transferred in 2007?

d. Approximately how many databases had been transferred as of 2011?

e. The data transfer was scheduled to be complete by 2015. Will the engineers accomplish this goal? Explain.

65. STOCK PRICES Apple Inc. (stock symbol AAPL) produces popular products such as the iPhone, iPad, and MacBook laptop computers. The company was not always as wildly successful, however. Taking into account stock splits, prices (in dollars) for AAPL shares can be represented by the following piecewise-defined function:

$$S(t) = \begin{cases} 14.7 + 0.6t & \text{if } t \le 4 \\ 14.2t^2 - 128t + 304 & \text{if } t > 4 \end{cases}$$

where t is the number of years after 2000.

a. Using this function, what was the share price of AAPL in 1990 (when $t = -10$)? In 2006?

b. In what year does the function predict that AAPL shares first reached the \$200 level?

c. What share value does the function predict for AAPL in the year 2012?

66. WORKER EFFICIENCY An efficiency study of the morning shift at a certain factory indicates that an average worker who arrives on the job at 8:00 A.M. will have assembled

$$f(x) = -x^3 + 6x^2 + 15x$$

television sets x hours later.

a. How many sets will such a worker have assembled by 10:00 A.M.? [*Hint:* At 10:00 A.M., $x = 2$.]

b. How many sets will such a worker assemble between 9:00 and 10:00 A.M.?

67. CONSUMER DEMAND An importer of Brazilian coffee estimates that local consumers will buy approximately $Q(p) = \dfrac{4{,}374}{p^2}$ kilograms of the coffee per week when the price is p dollars per kilogram. It is estimated that t weeks from now the price of this coffee will be

$$p(t) = 0.04t^2 + 0.2t + 12$$

dollars per kilogram.

a. Express the weekly demand (kilograms sold) for the coffee as a function of t.

b. How many kilograms of the coffee will consumers be buying from the importer 10 weeks from now?

c. When will the demand for the coffee be 30.375 kilograms?

68. PRODUCTION COST Arthur, the manager of a furniture factory, finds that the cost of producing q bookcases during the morning production run is $C(q) = q^2 + q + 500$ dollars. On a typical workday, $q(t) = 25t$ bookcases are produced during the first t hours of a production run for $0 \le t \le 5$.

a. Express the production cost C in terms of t.

b. How much will have been spent on production by the end of the 3rd hour? What is the average cost of production during the first 3 hours?

c. Arthur's budget allows no more than \$11,000 for production during the morning production run. When will this limit be reached?

LIFE AND SOCIAL SCIENCE APPLIED PROBLEMS

69. IMMUNIZATION Suppose that during a nationwide program to immunize the population against a new strain of influenza, public health officials found that the cost of inoculating $x\%$ of the population was approximately $C(x) = \dfrac{150x}{200 - x}$ million dollars.

a. What is the domain of the function C?

b. For what values of x does $C(x)$ have a practical interpretation in this context?

c. What was the cost of inoculating the first 50% of the population?

d. What was the cost of inoculating the second 50% of the population?

e. What percentage of the population had been inoculated by the time 37.5 million dollars had been spent?

70. TEMPERATURE CHANGE Suppose that t hours past midnight, the temperature in Miami was $C(t) = -\dfrac{1}{6}t^2 + 4t + 10$ degrees Celsius.

a. What was the temperature at 2:00 A.M.?
b. By how much did the temperature increase or decrease between 6:00 and 9:00 P.M.?

71. POPULATION GROWTH It is estimated that t years from now, the population of a certain suburban community will be $P(t) = 20 - \frac{6}{t+1}$ thousand people.
a. What will the population of the community be 9 years from now?
b. By how much will the population increase during the 9th year?
c. What happens to $P(t)$ as t gets larger and larger? Interpret your result.

72. EXPERIMENTAL PSYCHOLOGY To study the rate at which animals learn, Becky performed an experiment in which a rat was sent repeatedly through a laboratory maze. Suppose that the time required for the rat to traverse the maze on the nth trial was approximately

$$T(n) = 3 + \frac{12}{n}$$

minutes.
a. What is the domain of the function T?
b. For what values of n does $T(n)$ have meaning in the context of Becky's experiment?
c. How long did it take the rat to traverse the maze on the 3rd trial?
d. On which trial did the rat first traverse the maze in 4 minutes or less?
e. According to the function T, what will happen to the time required for the rat to traverse the maze as the number of trials increases? Will the rat ever be able to traverse the maze in less than 3 minutes?

73. BLOOD FLOW Biologists have found that the speed of blood in an artery is a function of the distance of the blood from the artery's central axis. According to **Poiseuille's law,*** the speed (in centimeters per second) of blood that is r centimeters from the central axis of an artery is given by the function $S(r) = C(R^2 - r^2)$, where C is a constant and R is the radius of the artery. Suppose that for a certain artery, $C = 1.76 \times 10^5$ and $R = 1.2 \times 10^{-2}$ centimeters.
a. Compute the speed of the blood at the central axis of this artery.
b. Compute the speed of the blood midway between the artery's wall and central axis.

74. POPULATION DENSITY Observations suggest that for herbivorous mammals, the number of animals N per square kilometer can be estimated by the formula $N = \frac{91.2}{m^{0.73}}$, where m is the average mass of the animal in kilograms.
a. Assuming that the average elk on a particular reserve has mass 300 kilograms, approximately how many elk would you expect to find per square kilometer in the reserve?
b. Using this formula, it is estimated that there is less than one animal of a certain species per square kilometer. How large can the average animal of this species be?
c. One species of large mammal has twice the average mass as a second species. If a particular reserve contains 100 animals of the larger species, how many animals of the smaller species would you expect to find there?

75. ISLAND ECOLOGY Observations show that on an island of area A square miles, the average number of animal species is approximately equal to $s(A) = 2.9\sqrt[3]{A}$.
a. On average, how many animal species would you expect to find on an island of area 8 square miles?
b. If s_1 is the average number of species on an island of area A and s_2 is the average number of species on an island of area $2A$, what is the relationship between s_1 and s_2?
c. How big must an island be to have an average of 100 animal species?

76. AIR POLLUTION An environmental study of a certain suburban community suggests that the average daily level of carbon monoxide in the air will be $c(p) = 0.4p + 1$ parts per million when the population is p thousand. It is estimated that t years from now the population of the community will be $p(t) = 8 + 0.2t^2$ thousand.
a. Express the level of carbon monoxide in the air as a function of time.
b. What will the carbon monoxide level be 2 years from now?
c. When will the carbon monoxide level reach 6.2 parts per million?

*Edward Batschelet, *Introduction to Mathematics for Life Scientists*, 3rd ed., New York: Springer-Verlag, 1979, pp. 101–103.

MISCELLANEOUS PROBLEMS

77. **POSITION OF A MOVING OBJECT** A ball has been dropped from the top of a building. Its height (in feet) after t seconds is given by the function $H(t) = -16t^2 + 256$.
   a. What is the height of the ball after 2 seconds?
   b. How far will the ball travel during the third second?
   c. How tall is the building?
   d. When will the ball hit the ground?

78. What is the domain of $f(x) = \dfrac{7x^2 - 4}{x^3 - 2x + 4}$?

79. What is the domain of $f(x) = \dfrac{4x^2 - 3}{2x^2 + x - 3}$?

80. For $f(x) = 2\sqrt{x - 1}$ and $g(x) = x^3 - 1.2$, find $g(f(4.8))$. Use two decimal places.

81. For $f(x) = 2\sqrt{x - 1}$ and $g(x) = x^3 - 1.2$, find $f(g(2.3))$. Use two decimal places.

SECTION 1.2 The Graph of a Function

Learning Objectives

1. Review the rectangular coordinate system.
2. Graph several functions.
3. Study intersections of graphs, the vertical line test, and intercepts.
4. Sketch and use graphs of quadratic functions in applications.

Graphs have visual impact. They also reveal information that may not be evident from verbal or algebraic descriptions. Two graphs depicting practical relationships are shown in Figure 1.3.

The graph in Figure 1.3a describes the variation in total industrial production in a certain country over a 4-year period of time. Notice that the highest point on the graph occurs near the end of the third year, indicating that production was greatest at that time. The graph in Figure 1.3b represents population growth when environmental factors impose an upper bound on the possible size of the population. It indicates that the *rate* of population growth increases at first and then decreases as the size of the population gets closer and closer to the upper bound.

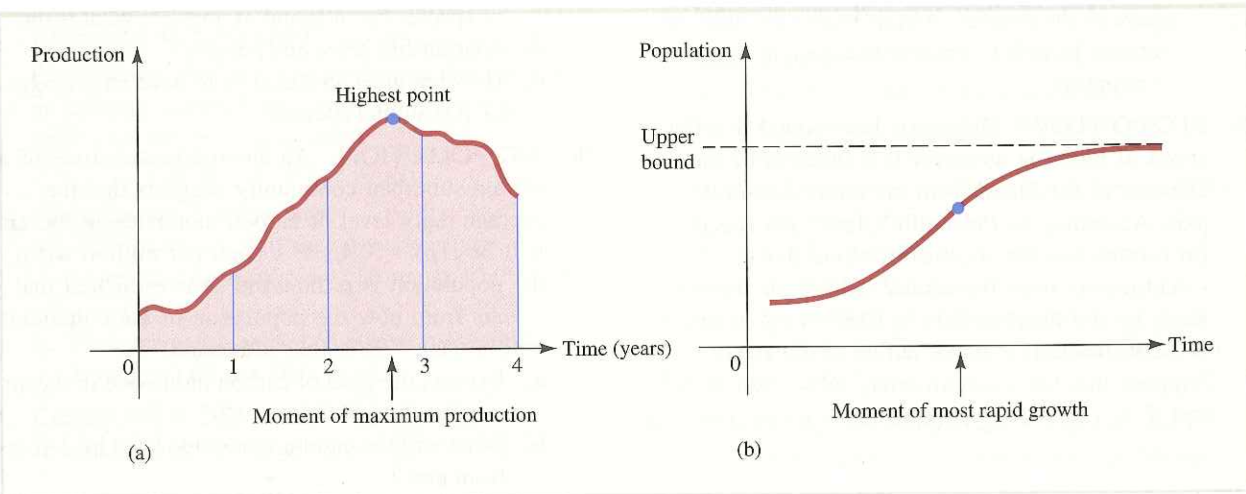

FIGURE 1.3 (a) A production function. (b) Bounded population growth.