Math 105 - Section 4.3 - Lecture Notes

1. The population (in millions) of a country is modeled by the function

   $P(t) = 100e^{-0.1t}$

   a. What was the initial population? $\Rightarrow t=0$

   $P(0) = 100e^0 = 100$

   $\boxed{\text{One hundred million was the initial population}}$

   b. At what percentage rate is the population changing with respect to time?

   $P(t) = 100e^{-0.1t}$ ← decay rate

   $\boxed{\text{Decreasing at 10\% per year.}}$

   c. What is the rate of change of population after 10 years? Is it increasing or decreasing? ← $P'(10)$

   $P'(t) = 100e^{-0.1t} \cdot (-0.1)$

   $P'(t) = -10e^{-0.1t}$

   $P'(10) = -10e^{-0.1(10)}$

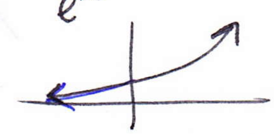
   $= -10e^{-1}$

   $= -10/e \approx -3.67879$

   $\boxed{\text{The population is decreasing at a rate of } 10/e \text{ million people per year.}}$

   d. What happens in the long run? (i.e. as t approaches infinity)

   e^x

   $\lim_{x \to -\infty} e^x = e^{-\infty} = 0$

   $\lim_{t \to \infty} 100e^{-0.1(t)}$

   $= 100 \lim_{t \to \infty} e^{-0.1(t)}$

   $= 100e^{-\infty}$

   $= 100(0)$

   $= 0$

   $\boxed{\text{In the long run, the population goes extinct.}}$

2. Suppose the percentage of alcohol in the blood t hours after consumption is given by $C(t) = 0.12te^{-t/2}$
   a. At what rate is the blood alcohol changing at time t?
   b. How much time passes before the blood alcohol level begins to decrease?
   c. At what rate is the blood alcohol level changing after 4 hours?

$C'(t) < 0$

negative rate of change

a) Rate $\rightarrow C'(t)$

$$C'(t) = 0.12t \cdot e^{-\frac{1}{2}t} \cdot \left(-\tfrac{1}{2}\right) + e^{-\frac{1}{2}t} \cdot 0.12$$

$$= e^{-\frac{1}{2}t}\left[0.12t \cdot \left(-\tfrac{1}{2}\right) + 0.12\right]$$

$$\boxed{C'(t) = e^{-\frac{1}{2}t}\left[-0.06t + 0.12\right]}$$

OR

$$C'(t) = 0.12e^{-\frac{1}{2}t}\left[-.5t + 1\right]$$

b) $C'(t) < 0$

$$\underbrace{0.12e^{-\frac{1}{2}t}}_{\text{Always positive}}\left[-.5t+1\right] < 0$$

$$-.5t + 1 < 0$$

$$1 < .5t$$

$$\frac{1}{.5} < \frac{.5t}{.5}$$

$$2 < t$$

$$t > 2$$

After 2 hours the blood alcohol percentage begins to decrease.

c) $C'(4) = e^{-\frac{1}{2}(4)}\left[-0.06(4) + 0.12\right]$

$= e^{-2}\left[-.12\right]$

$= -0.01624$

$\approx -1.624\%$

$\approx -2\%$

It is decreasing at a rate of 1.6% per hour or about 2%.