Section 4.2 Logarithmic Functions

Logarithmic Function $x > 0$, $b > 0$, $(b \neq 1)$

$$y = \log_b x \text{ if and only if } b^y = x$$

→ the exponent we raise b to to get x out.

a) $\log_2 16 = 4$ since $2^4 = 16$

b) $\log_{10} 1000 = 3$ since $10^3 = 1000$

c) $\log_5 \frac{1}{125} = -3$ since $5^{-3} = \frac{1}{125}$

(logs & exponential are inverses)

$b^{\log_b x} = x$

& $\log_b b^x = x$

ex) Solve for x

a) $\log_2 x = -3$

$2^{-3} = x$

$\boxed{\frac{1}{8} = x}$

b) $\log_3 243 = x$

$3^x = 243$

$3^x = 3^5$ $\boxed{x = 5}$

c) $\log_x 27 = 3$

$(x^3)^{1/3} = (27)^{1/3}$

$x = 27^{1/3}$

$\boxed{x = 3}$

→ Standard Base $\log x = \log_{10} x$

Assume base 10 like # system (calc).

$243 = 3^5$

243 → 3, 81; 81 → 9, 9; 9 → 3, 3; 9 → 3, 3

Rules of logarithms

logs convert mult/div. → add/subt. Useful! It's easier to add/sub. than to mult/divide.

$b > 0$, $b \neq 1$, then

$\log_b 1 = 0$ $\log_b b = 1$

and if u and v are positive #'s, we have

→ $\log_b u = \log_b v$ if and only if $u = v$

→ $\log_b(uv) = \log_b u + \log_b v$

→ $\log_b \frac{u}{v} = \log_b u - \log_b v$ → $\log_b b^u = u$

→ $\log_b u^r = r \log_b u$

$5^2 \cdot 2^3$

25 . 8

Use logs to rewrite in terms of $\log_3 2$ and $\log_3 5$.

a) $\log_3(200)$

$= \log_3(2^3 5^2)$

$= \log_3 2^3 + \log_3 5^2$

$= \boxed{3\log_3 2 + 2\log_3 5}$

$200 = 2^3 5^2$

200

2 100

10 10

2 5 2 5

Use properties of logs to simplify (expand using prop's)

ex) a) $\log_5(z^6 y^4)$

$= \log_5 z^6 + \log_5 y^4$

$= \boxed{6\log_5 z + 4\log_5 y}$

b) $\log_6 \dfrac{x^5\sqrt{y}}{z^3}$

upstairs = +

down stairs = −

write as z^{-3} then bring down in front

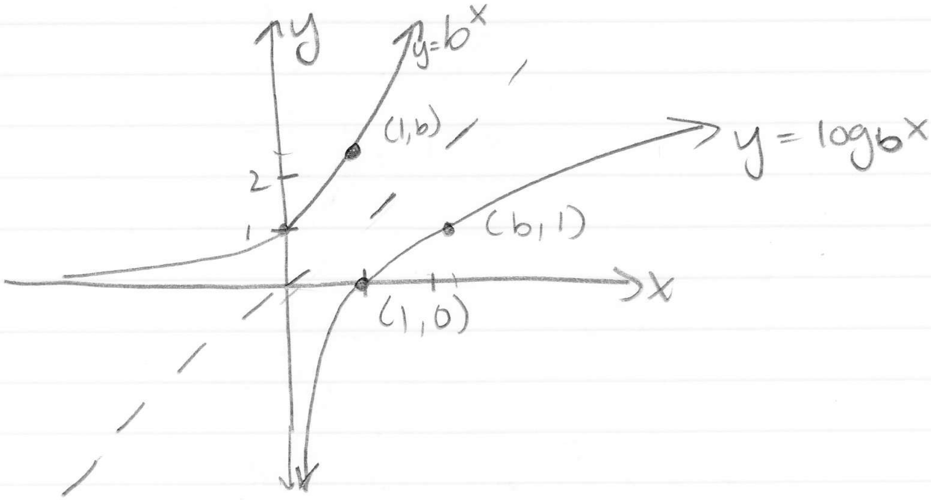
$= \log_6 x^5 y^{1/2} z^{-3}$

$= \log_6 x^5 + \log_6 y^{1/2} + \log_6 z^{-3}$

$= \boxed{5\log_6 x + \frac{1}{2}\log_6 y - 3\log_6 z}$

Graphs of logarithmic functions

Exponentials & logs are inverses
(inverses are symmetric w.r.t. the line $y = x$) Recall: To find inverse, switch x & y



→ Continuous for all $x > 0$
→ $x = 0$ is a V.A.
→ The x int (1,0), No y int
→ For $x > 0$, the graph is increasing when $b > 1$ and $0 < b < 1$ the graph is decreasing.

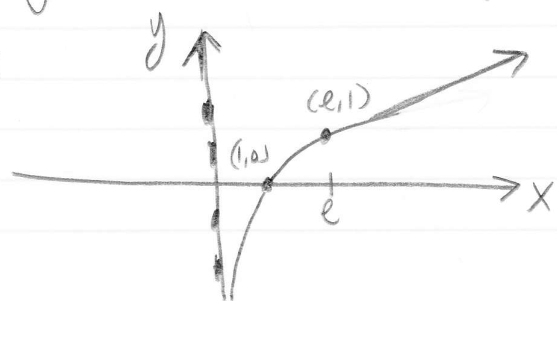
$\boxed{\text{Dom } (0, \infty) \quad \text{Range } (-\infty, \infty)}$

The Natural Logarithm

Most common (only has one base)
$\log_e x$ and is denoted $\ln x$ "el en of x"

for $x > 0$ $\quad y = \ln x$ if and only if $e^y = x$

Graph $y = \ln x$



e^x and $\ln x$ are inverses

That is

$$e^{\ln x} = x \quad x > 0$$

$$\ln(e^x) = x \quad \text{for all } x$$

(same rules as logs – ln is just a "special log)

ex. Break down using props of logs

a) $\ln \sqrt[3]{x^2 - x}$

$$= \ln (x^2 - x)^{1/3}$$

$$= \ln (x(x-1))^{1/3}$$

$$= \frac{1}{3}\left[\ln(x(x-1))\right]$$

$$= \boxed{\frac{1}{3}\left(\ln x + \ln(x-1)\right)}$$

ex. $\ln\left[\dfrac{\sqrt[3]{x+1}}{x^2\left(\sqrt{x^2-1}\right)}\right]$

$$= \ln (x+1)^{1/3} x^{-2} (x^2-1)^{-1/2}$$

$$= \boxed{\frac{1}{3}\ln(x+1) - 2\ln x - \frac{1}{2}\ln(x^2-1)}$$

Recall $b^x = b^y$ if & only if $x = y$

Also if $x = y$, then $\log_b x = \log_b y$ (Take log/ln of both sides)

and if $x = y$, then $\ln x = \ln y$

ex. Solve

$$e^{5x} = 4$$

$$\ln e^{5x} = \ln 4$$

$$5x = \ln 4 \quad \text{exact}$$

$$\boxed{x = \ln 4/5}$$

ex. $2^x = 5$

$$\ln 2^x = \ln 5$$

$$x \ln 2 = \ln 5$$

$$\boxed{x = \frac{\ln 5}{\ln 2}} \approx \text{calc}$$

you could take $\log_2$ of both sides but calc is base 10!

What about finding an approx. of $\log_2 5$?

To put in calc we have to use a formula

Change of Base Formula

$a, b, c > 0$ with $b \neq 1$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$$\text{or} = \frac{\log_e a}{\log_e b} = \boxed{\frac{\ln a}{\ln b}}$$

$$\log_2 5 = \frac{\log_{10} 5}{\log_{10} 2} \quad \text{or} \quad \frac{\ln 5}{\ln 2}$$

Try both on calc!

ex) Solve for x

a) $\log_5(3x+2) = 1$ Just convert to expenential!

$$5^1 = 3x+2$$
$$-2 \qquad -2$$
$$3 = 3x$$
$$\boxed{x = 1}$$

check it $3(1)+2$
$3+2 = 5 > 0$ ✓

b) $17 = 10 + 6e^{2x}$

$$-10 \quad -10$$
$$\frac{7}{6} = \frac{6e^{2x}}{6}$$
$$\frac{7}{6} = e^{2x}$$
$$\ln \frac{7}{6} = \ln e^{2x}$$
$$\left(\tfrac{1}{2}\right)\ln \frac{7}{6} = 2x\left(\tfrac{1}{2}\right)$$
$$\boxed{\tfrac{1}{2}\ln\left(\tfrac{7}{6}\right) = x}$$
$$\approx 0.077075$$

c) $\ln x = 2(\ln 3 - \ln 5)$

$\ln x = 2 \ln \frac{3}{5}$

$\ln x = \ln\left(\frac{3}{5}\right)^2$ $\qquad x = y$

$\cancel{e}^{\ln x} = \cancel{e}^{\ln\left(\frac{3}{5}\right)^2}$ $\qquad e^x = e^y$

$x = \left(\frac{3}{5}\right)^2$

$x = \frac{9}{25}$

d) $\frac{-3\ln x}{-3} = \frac{a}{-3}$

$\ln x = -\frac{a}{3}$

$\boxed{e^{-a/3} = x}$