Section 4.1 Exponential Functions

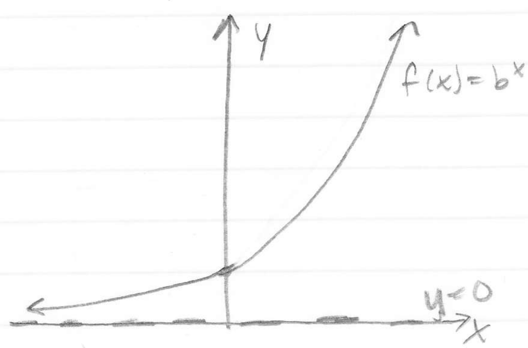
(✱)

$b^x \leftarrow$ exponent

$b \leftarrow$ base

Exponential Function

$f(x) = b^x \qquad b > 0\ ,\ b \neq 1$



$b > 1$

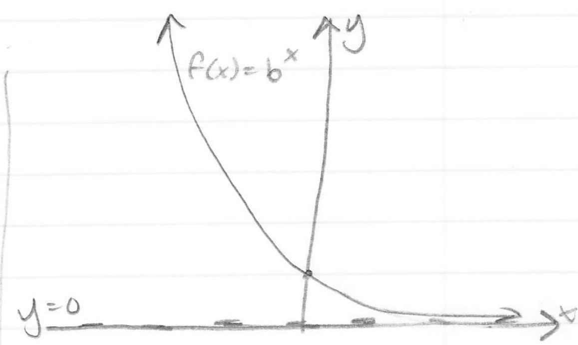
Dom $\mathbb{R}$

Range $(0, \infty)$

Always increasing

$\lim\limits_{x \to \infty} b^x = +\infty$

$\lim\limits_{x \to -\infty} b^x = 0$



$0 < b < 1$

Dom $\mathbb{R}$

Ran $(0, \infty)$

Always decreasing

$\lim\limits_{x \to \infty} b^x = 0$

$\lim\limits_{x \to -\infty} b^x = \infty$

Def'n of b^n for rational values of n

Integer powers

$$b^n = \underbrace{b \cdot b \cdot \cdots b}_{n \text{ factors}}$$

Fractional powers

$$b^{n/m} = \sqrt[m]{b^n} = \left(\sqrt[m]{b}\right)^n$$

Negative powers

$$b^{-n} = \frac{1}{b^n}$$

Zero power

$$b^0 = 1$$

ex. a) $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$= 4 \cdot 4 \cdot 2$
$= 16 \cdot 2$
$\boxed{= 32}$

b) $2^{-5} = \dfrac{1}{2^5}$
$= \boxed{\dfrac{1}{32}}$

c) $27^{2/3} = \left(\sqrt[3]{27}\right)^2$
$= \left(\sqrt[3]{3^3}\right)^2 =$
3^2
$\boxed{= 9}$

d) $\left(\dfrac{1}{9}\right)^{3/2}$
$= \left(\sqrt[2]{\dfrac{1}{9}}\right)^3 = \left(\dfrac{1}{3}\right)^3 = \boxed{\dfrac{1}{27}}$

(*) b^x is not to be confused with power function $f(x) = x^b$

exponential

b^x ← variable exponent

↑ constant base

Power

x^b ← constant exponent

var. base

Exponential Rules - $a, b > 0 \quad x, y \in \mathbb{R}$

- $b^x = b^y$ if and only if $x = y$

  ex. $2^3 = 2^x$ solve

  $\boxed{3 = x}$

- $b^x \cdot b^y = b^{x+y}$

  ex. $2^3 \cdot 2^2 = 2^{3+2} = 2^5 = \boxed{32}$

- $\dfrac{b^x}{b^y} = b^{x-y}$

  ex. $\dfrac{5^2}{5^6} = 5^{2-6} = 5^{-4} = \dfrac{1}{5^4} = \boxed{\dfrac{1}{625}}$

- $(b^x)^y = b^{xy}$

  ex. $(t^3)^6 = t^{18}$

  $(a \cdot b)^x = a^x b^x$

  ex. $(2\pi)^2 = 2^2 \pi^2$ $\boxed{= 4\pi^2}$

- $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

  ex. $\left(\dfrac{2}{9}\right)^2 = \dfrac{2^2}{9^2} = \boxed{\dfrac{4}{81}}$

- $\left(\dfrac{a}{b}\right)^{-x} = \left(\dfrac{b}{a}\right)^x$

  ex. $\left(\dfrac{4}{7}\right)^{-2} = \left(\dfrac{7}{4}\right)^2 = \boxed{\dfrac{49}{16}}$

ex. a) $8^{2/3}$

$= \left(\sqrt[3]{8}\right)^2$

$= \left(\sqrt[3]{2^3}\right)^2$

$= 2^2$

$\boxed{= 4}$

Prime factor!

Show work
NO T.I.!

b) $9^{3/2} + 16^{1/4}$

$= \left(\sqrt[2]{9}\right)^3 + \sqrt[4]{16}$

$= 3^3 + 2$

$= 27 + 2$

$\boxed{= 29}$

16
2 8
2 4
2 2

c) $(2^6)(2^{-3})$

$2^{6-3} = 2^3 = \boxed{8}$

or

$\frac{2^6}{2^3} = 2^{6-3} = \boxed{2^3}$

e is a # like π

d) $\left(\frac{\pi^2}{\sqrt{\pi}}\right)^{4/3} = \left(\frac{\pi^2}{\pi^{1/2}}\right)^{4/3} = \left(\pi^{2-\frac{1}{2}}\right)^{4/3}$

$= \left(\pi^{3/2}\right)^{4/3} = \pi^{(3/2)(4/3)}$

$= \pi^{4/2}$

$= \boxed{\pi^2}$

e) $\frac{(2^{2.5})(2^{1.5})}{2^2} = \frac{2^{2.5+1.5}}{2^2} = \frac{2^4}{2^2} = 2^{4-2} = \boxed{2^2}$

f) $\left(e^4 e^{2/3}\right)^{3/2}$

$= \left(e^{4(\frac{3}{2})} e^{(2/3)(3/2)}\right)$

$= \left(e^6 e^1\right) = e^{6+1} = \boxed{e^7}$

simplify (variables) Typically we leave with positive exponents

ex) a) $(x^{1/3})^{3/2}$
$= x^{\frac{1}{3}\cdot\frac{3}{2}}$
$= \boxed{x^{1/2}}$

b) $\left(\frac{x+y}{3x^3}\right)^0 = 1$

c) $(-2t^{-3})(3t^{2/3})$
$= -6t^{-3+\frac{2}{3}}$
$= -6t^{-7/3}$
$= \boxed{\frac{-6}{t^{7/3}}}$

$\frac{-3}{1}\left(\frac{3}{3}\right) + \frac{2}{3}$
$-\frac{9}{3} + \frac{2}{3}$
$-\frac{7}{3}$

d) $\left(\frac{x^8y^{-2}}{z^6}\right)^{1/4}$
$= \frac{(x^8y^{-2})^{1/4}}{(z^6)^{1/4}} = \frac{x^{8/4}\,y^{-2/4}}{z^{6/4}}$
$= \frac{x^2y^{-\frac{1}{2}}}{z^{3/2}} = \boxed{\frac{x^2}{y^{1/2}z^{3/2}}}$

Solve for x.

ex. a) $3^x = 9$
$3^x = 3^2$
$\boxed{x = 2}$

you can check it!

Get both sides to have same base... use $b^x = b^y$, $x = y$

b) $2^{3x+2} = 16$
$2^{3x+2} = 2^4$
$3x + 2 = 4$
$-2 \quad -2$
$3x = 2$
$\boxed{x = 2/3}$

Check it!

b) $(2.6)^{2x-1} = (2.6)^{1-2x}$

$2x - 1 = 1 - 2x$

$+2x \qquad +2x$

$4x - 1 = 1$

$+1 \quad +1$

$4x = 2$

$x = \frac{2}{4} = \boxed{\frac{1}{2}}$

c) $\left(\frac{1}{8}\right)^{x-1} = 2^{3-2x^2}$

$\left(\frac{1}{2^3}\right)^{x-1} = 2^{3-2x^2}$

$\left(2^{-3}\right)^{x-1} = 2^{3-2x^2}$

$2^{-3x+3} = 2^{3-2x^2}$

$-3x + 3 = 3 - 2x^2$

$+2x^2 \quad -3 \quad -3 \quad +2x^2$

$2x^2 - 3x = 0$

$x(2x - 3) = 0$

$\boxed{x = 0}$ or $2x - 3 = 0$

$2x = 3$

$\boxed{x = 3/2}$