Section 3.4 Optimization

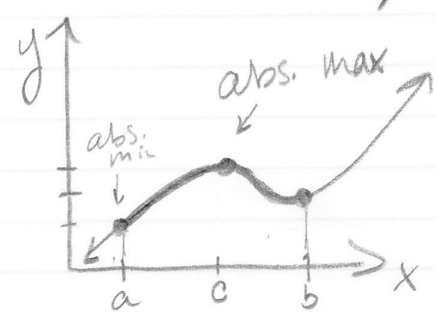
Absolute maxima/minima (extrema) of a function

Let f be a function on interval I that contains c, then f(c) is the absolute:

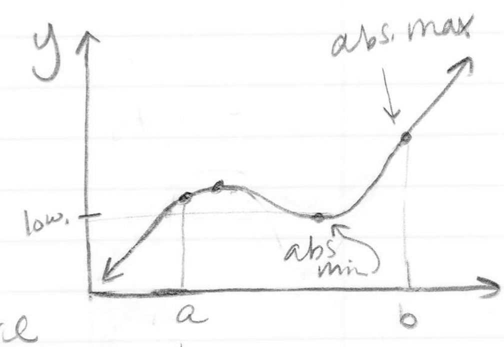
→ max of f on I if
$f(c) \geq f(x)$ for all $x \in I$ (largest y value)

→ min of f on I if
$f(c) \leq f(x)$ for all $x \in I$ (smallest y value)

Typically the absolute max/min coincide with relative max/min, but not always.



OR



we are only concerned w/ y values of cps & y values of endpoints

Extreme Value Property

skip?

Continuous function f(x) on the interval $a \leq x \leq b$ attains its absolute extrema on the interval either at an endpoint of the interval (a or b) or at a critical value c such that $a < c < b$.

Strategy to find absolute extrema on $a \le x \le b$

① Evaluate the function at its endpoints to find $f(a)$ & $f(b)$ y values.

② Find all critical points $(c, f(c))$ of f on the interval $a < x < b$. (You do not even need to classify them as max/min)

$(c, f(c))$

③ The largest & smallest function values (y values) are the absolute max and abs. min respectively on $a \le x \le b$.

ex. Find absolute extrema of

$$f(x) = 3t^5 - 5t^3 \qquad -2 \le t \le 0$$

① $f(-2) = 3(-2)^5 - 5(-2)^3$
$= 3(-32) - 5(-8)$
$= -96 + 40 \qquad (-2, -56)$
$= -56$

$f(0) = 0 \qquad (0,0)$

② $f'(x) = 15x^4 - 15x^2$
$= 15x^2(x^2 - 1)$
$f'(x) = 15x^2(x+1)(x-1)$
$0 = 15x^2(x+1)(x-1)$
$x = 0$ ✓ $\quad x = -1$ ✓ $\quad \cancel{x = 1}$

Are they in interval?

$f(-1) = 3(-1)^5 - 5(-1)^3$
$= 3(-1) - 5(-1)$
$= -3 + 5 \qquad (-1, 2)$
$= 2$

End Points

abs. min $(-2, -56)$

$(0,0)$

Critical Points

$(0,0)$

abs max $(-1, 2)$

③ There is an absolute max. @ $(-1, 2)$
There is an absolute min @ $(-2, -56)$

ex. Find absolute extrema.

$f(x) = (x^2-4)^5 \qquad -3 \le x \le 2$

(1) $f(-3) = ((-3)^2-4)^5 = (9-4)^5 = 5^5 = 3125$

$f(2) = (2^2-4)^5 = 0^5 = 0$

End Points
max (-3, 3125)
(2, 0)

(2) $f'(x) = 5(x^2-4)^4(2x)$

$f'(x) = 10x(x^2-4)^4$

$0 = 10x(x^2-4)^4$

$0 = x \qquad (x^2-4)^4 = 0$

$x^2-4 = 0$

$x^2 = 4$

$x = \pm\sqrt{4}$

$x = \pm 2$

C.P.
min (0, -1024)
(-2, 0)
(2, 0)

Find y values

$f(0) = (-4)^5 = -1024$

$f(-2) = [(-2)^2-4]^5 = 0^5 = 0$

$f(2) = 0$

There is an abs. max @ (-3, 3125)
There is an abs. min @ (0, -1024)

ex. Find abs. extrema

$f(x) = \frac{1}{x^2} = x^{-2} \qquad x > 0$ (1) (No equal sign means no endpoint)

(2)

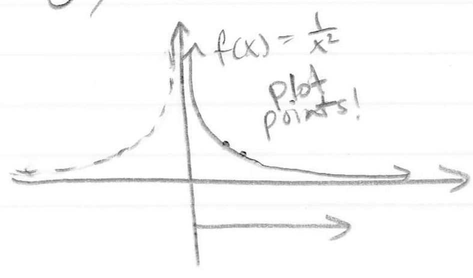
$f'(x) = -2x^{-3}$

$f'(x) = \frac{-2}{x^3}$ OR $f'(x)$ DNE & $f(x)$ exists

$0 = \frac{-2}{x^3}$ — $f'(x)$ DNE @ $x=0$

$0 \neq -2$ — $f(0)$ DNE

No cps



Function never equals zero!

Print Handout?

#34. An all-news radio station has made a survey of the local listening habits of local residents between 5pm & midnight. The survey indicates the percentage of local adult population that is tuned in to the station x hours after 5:00 pm is

$$f(x) = \frac{1}{8}(-2x^3+27x^2-108x+240)$$

a) At what time between 5pm & midnight are the most people listening?
What percentage of pop. is listening at this time?

b) At what time are the fewest people listening?
What percentage of pop. is listening at this time?

a/b) Find max of $f(x)$ on $0 \le x \le 7$

$f(x) = \frac{1}{8}(-2x^3+27x^2-108x+240)$

① $f(0) = \frac{1}{8}(0+0-0+240) = \frac{1}{8}(240) = 30$

$f(7) = \frac{1}{8}(-2(7)^3+27(7)^2-108(7)+240) = \frac{121}{8}$
$= 15.125$

End Points
max $(0,30)$
$(7, 15.125)$

CPs
~~$(6, 16.5)$~~
min $(3, 13.125)$

② $f'(x) = \frac{1}{8}(-6x^2+54x-108)$

$\frac{27}{27} \over 54$

$8 \quad 0 = \frac{1}{8}(-6x^2+54x-108) \quad 8$

~~$0 = \frac{1}{8}$~~

$0 = -6x^2+54x-108$

$0 = -6(x^2-9x+18)$

~~$0 = -6$~~ $\qquad 0 = x^2-9x+18$

$0 = (x-6)(x-3)$

$x = 6 \quad x = 3$ in $0 \le x \le 7$ yes

$f(6) = \frac{1}{8}(-2(6)^3+27(6)^2-108(6)+240) =$

$f(3) = \frac{1}{8}(-2(3)^3+27(3)^2-108(3)+240) = \frac{105}{8} \approx 13.125$

The max percent. of listeners is after 0 hrs and is 30%. The min percentage of listeners is after 3hrs and is at 13.125%.

#38. A poll indicates that x months after a particular candidate declares her candidacy, she will have the support of S(x) percent of the voters, where

$$S(x) = \frac{1}{29}(-x^3 + 6x^2 + 63x + 1080) \quad \text{for } 0 \le x \le 12$$

If the election is held in November, when should she announce her candidacy? Should she expect to win if needs at least 50% of the vote?

Find max percen. of support?

① $S(0) = \frac{1}{29}(1080) = \frac{1080}{29} \approx 37.24$

$S(12) = \frac{1}{29}(-(12)^3 + 6(12)^2 + 63(12) + 1080)$

$= \frac{972}{29} \approx 33.52$

Endpoint
(0, 37.24)
(12, 33.52)

max CP
(7, 50.76)

② $S'(x) = \frac{1}{29}(-3x^2 + 12x + 63)$

$0 = \frac{1}{29}(-3x^2 + 12x + 63)$

$0 = -3x^2 + 12x + 63$

$0 = -3(x^2 - 4x - 21)$

$0 = x^2 - 4x - 21$

$0 = (x-7)(x+3)$

$0 = x - 7$ $\quad \cancel{x = -3}$

$x = 7$

Max percentage of voters 7mo. after she announces cand.

$S(7) = \frac{1}{29}(-(7)^3 + 6(7)^2 + 63(7) + 1080)$

$= \frac{1472}{29} \approx 50.76$

She should announce in April so that she has max percent of support by Nov. She would have 50.76% support so yes, she should expect to win.