3.2 Additional Example

ex. $f(x) = \dfrac{x^2}{x^2+3}$

* Find
Increase / Decrease
Max / min
Concavity
Inflection pts.
Graph it!

Dom $f = \mathbb{R}$

$$f'(x) = \frac{(x^2+3)(2x) - (x^2)(2x)}{(x^2+3)^2}$$

$$= \frac{\cancel{2x^3} + 6x - \cancel{2x^3}}{(x^2+3)^2}$$

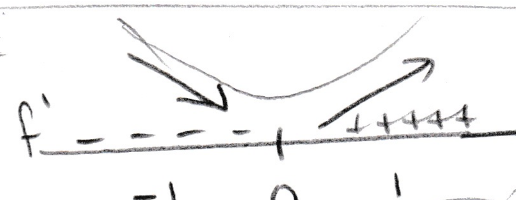
$$f'(x) = \frac{6x}{(x^2+3)^2}$$

C.P.'s $f'(x) = 0$ or ~~$f'(x) = DNE$~~

$$0 = \frac{6x}{(x^2+3)^2}$$

$$0 = 6x$$

$$0 = x$$



$f'(-1) = -$

$f'(1) = +$

Increasing $(0, \infty)$
Decreasing $(-\infty, 0)$
min @ $(0,0)$

I.P.'s & Concavity

Recall, $f'(x) = \dfrac{6x}{(x^2+3)^2}$

$$f''(x) = \frac{(x^2+3)^2(6) - 6x\left[2(x^2+3)'(2x)\right]}{(x^2+3)^4}$$

$$= \frac{\cancel{(x^2+3)}\left[(x^2+3)\cdot 6 - 6x \cdot 4x\right]}{\cancel{(x^2+3)^4}\,(x^2+3)^3}$$

$$= \frac{6x^2 + 18 - 24x^2}{(x^2+3)^3}$$

$$\to f''(x) = \frac{-18x^2 + 18}{(x^2+3)^3}$$

$$f''(x) = \frac{-18(x^2-1)}{(x^2+3)^3}$$

$$f''(x) = \frac{-18(x+1)(x-1)}{(x^2+3)^3} \longrightarrow$$

Recall,

$$f''(x) = \frac{-18(x+1)(x-1)}{(x^2+3)^3}$$

Possible I.P.'s

$f''(x) = 0$

OR

~~$f''(x) = DNE$~~

$$\frac{0}{-18} = \frac{-18(x+1)(x-1)}{-18}$$

$$0 = (x+1)(x-1)$$

$$x = \pm 1$$

f'': $- -$ $\cap$ on $(-\infty, -1)$; $+ +$ $\cup$ on $(-1, 1)$; $- -$ $\cap$ on $(1, \infty)$

$-2 \quad -1 \quad 0 \quad 1 \quad 2$

$$f''(-2) = -(-)(-)$$
$$= -$$

$$f''(0) = -(+)(-)$$
$$= +$$

$$f''(2) = -(+)(+)$$
$$= -$$

f is

Concave up $(-1, 1)$

concave down $(-\infty, -1) \cup (1, \infty)$

Inflection pts @ $x = \pm 1$

$$f(\pm 1) = \frac{(\pm 1)^2}{(\pm 1)^2 + 3}$$
$$= \frac{1}{1+3}$$
$$= \frac{1}{4}$$

$(1, \frac{1}{4})$

$(-1, \frac{1}{4})$

See graph $\longrightarrow$

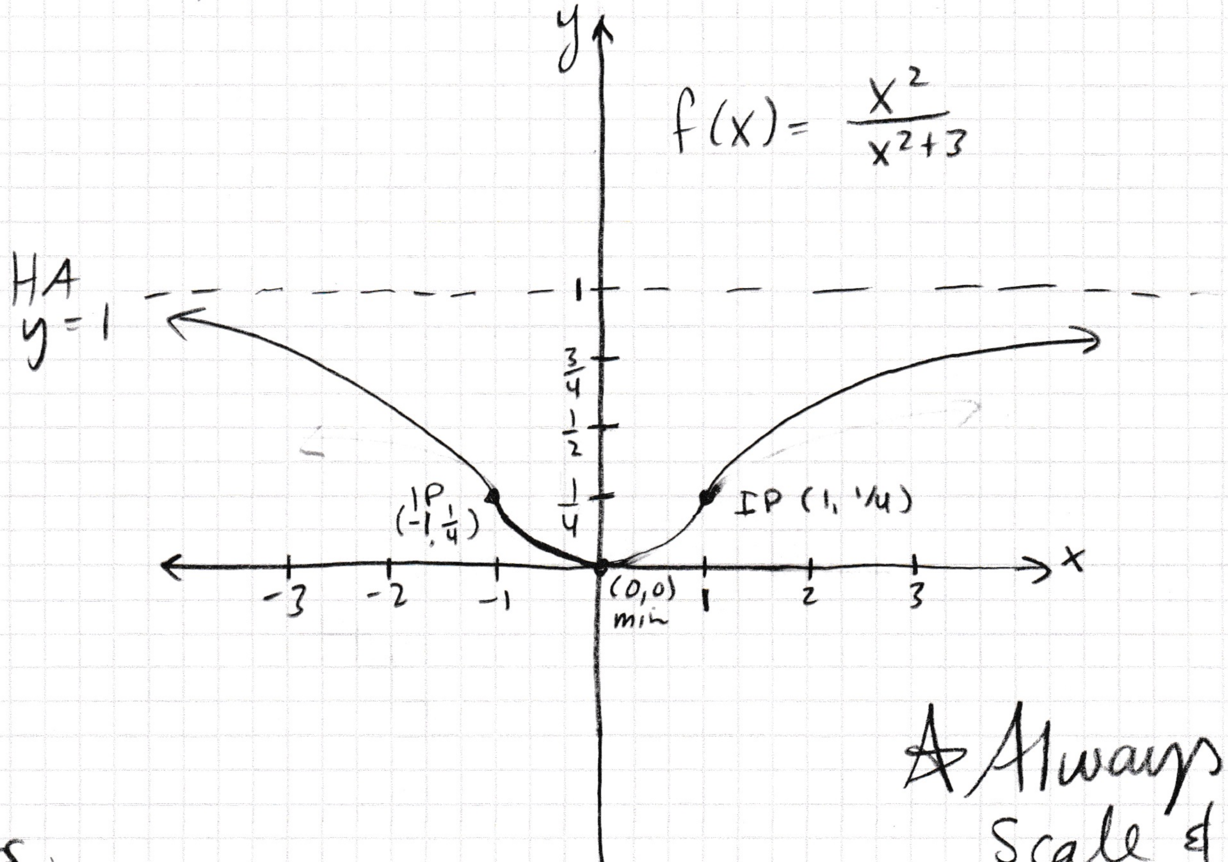
Summary of Info

Increasing $(0,\infty)$
Decreasing $(-\infty,0)$
Minimum @ $(0,0)$

Concave up $(-1,1)$
Concave down $(-\infty,-1)\cup(1,\infty)$
Inflection pts
$(-1,\frac{1}{4})\ (1,\frac{1}{4})$

Other useful info:

Dom $f(x)=\mathbb{R}$
HA @ $y=1$



more pts

x	y

Get more pts if needed for accuracy!!

★ Always Scale & label axes & use GRAPH paper for all graphs.