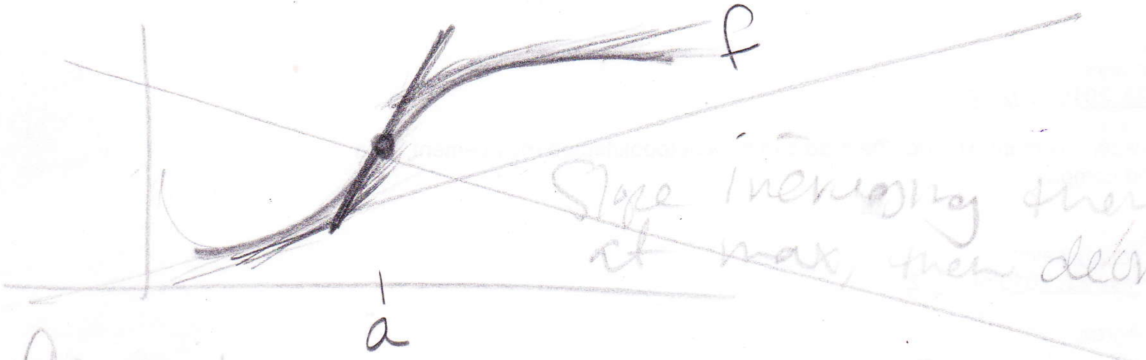
3.2 concavity and points of inflection

In 3.1 we used sign of f' to tell ~~whether~~ where original function f ~~graph~~ is increasing or decreasing and find rel. extrema.

Well, the second derivative tells some important info about f as well.

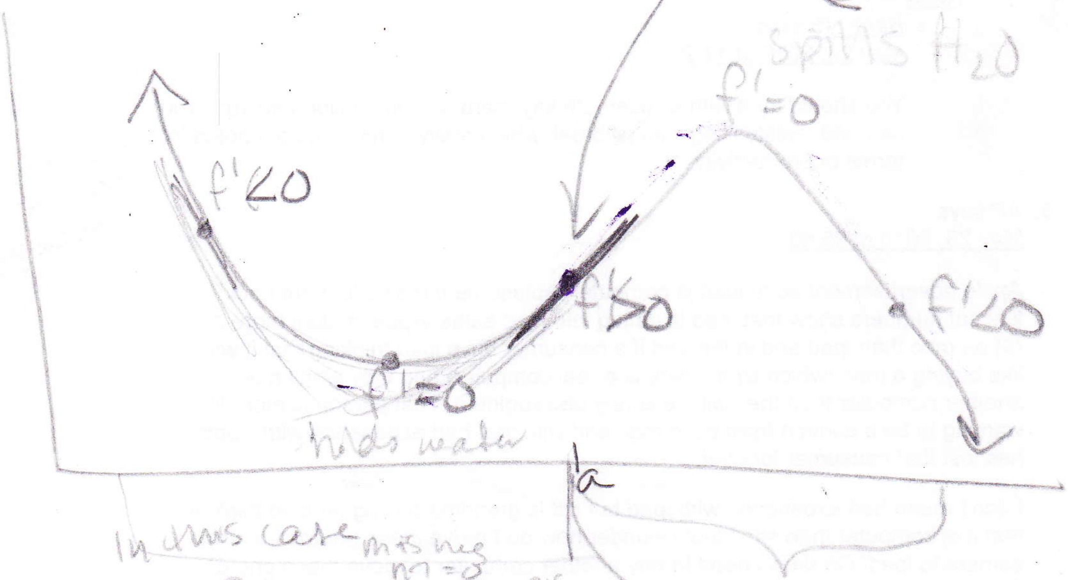


Concavity

f is concave up if

pt of inflection (f goes from concave up → down or vice versa)

Spills H_2O



In this case m is neg
m = 0
m pos

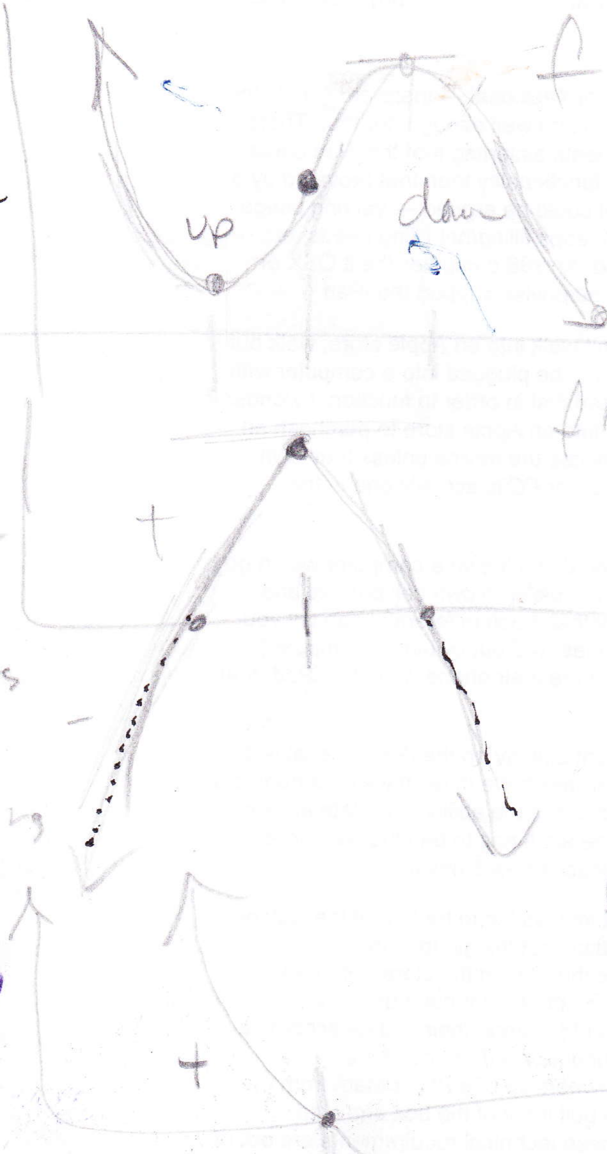
Slope is increasing

Concave up

Slope is decreasing
pos → 0 → neg
Concave Down

Slope is at its max

Slope neg = below x axis on derivative graph

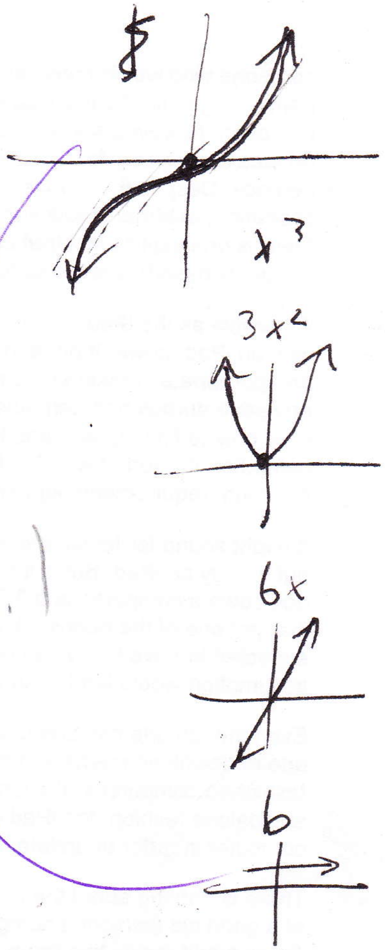


$f' = 0$ critical pt

$f' < 0$ f decreasing

$f' > 0$ f increasing

aka!! :)



3.1

$f'' = 0$ is the point of inflection if f'' goes from + to − or − to + (∪ → ∩ or ∩ → ∪)

3.2

$f'' > 0$ f concave up

$f'' < 0$ f concave down

$f'' = 0$ pt(s) of inflection

If concavity doesn't change, then it is not an inflection pt.

$f'' < 0$ $f'' > 0$

f'' DNE

To determine inflection pts and intervals of concavity

① Find $f'' = 0$ (where possible inflection pts occur) or f'' DNE

② Draw # line & pick test pts.

~~If~~ $f'' > 0 \Rightarrow f$ is concave up

~~If~~ $f'' < 0 \Rightarrow f$ is concave down

on $a < x < b$

ex. Find where ~~f~~ is concave up or concave down. Find inflection pts, concavity

$f(x) = x^4 - 6x^3 + 7x - 5$

$f'(x) = 4x^3 - 18x^2 + 7$

$f''(x) = 12x^2 - 36x$

$0 = 12x^2 - 36x$

$0 = 12x(x-3)$

$12x = 0 \quad x - 3 = 0$

$\boxed{x=0} \quad \boxed{x=3}$

$f''(-1) = 48 > 0$

$f''(1) = -24 < 0$

$f''(4) = 48 > 0$

x coord. of inflection points.

$x = 0 \quad f(0) = -5$

$x = 3 \quad f(3) = -65$

$\boxed{(0,-5) \quad (3,-65)}$

$3^4 - 6(3)^3 \ldots$

Concave up $(-\infty, -1) \cup (4, \infty)$

Concave down $(0,3)$

Find inflec. points!

ex. $f(x) = 3x^5 - 5x^4 - 1$

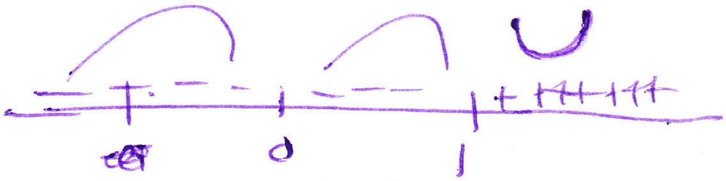
$f'(x) = 15x^4 - 20x^3$

$f''(x) = 60x^3 - 60x^2$

$0 = 60x^2(x-1)$

$60x^2 = 0 \quad x - 1 = 0$

$\boxed{x=0} \quad \boxed{x=+1}$



0, is not inflect. pts. $(1,-3)$ inflect pt

Imp Second Deriv test for max/min

If $f'(c)=0$ & $x=c$ critical

$f''(c)>0$, f has relative min at $x=c$

$f''(c)<0$, f " " max @ $x=c$

If $f''=0$ or f'' DNE inconclusive

a) c

max

b) min

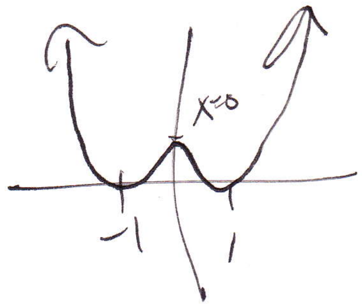
c) Inconclusive

$f''(c)=0$

& $f'(c)=0$

3.2 (5)

Use second deriv. test to find maxima | minima



$f(x) = x^4 - 2x^2 + 3$

$f'(x) = 4x^3 - 4x$

$\frac{0}{0^2} = \frac{4x(x^2-1)}{4}$

$0 = x(x^2-1)$

$0 = x \qquad x^2 = 1$

$x = \pm\sqrt{1} = \pm 1$

No # line needed or plug right into f''!

$f'(x) = 0$

$x = 0, 1, -1$

$f''(x) = 12x - 4$

$f''(0) = -4 < 0$ concave down $\Rightarrow$ x=0 (max)

ordered pairs!

$f''(1) = 12(1) - 4$

$= 12 - 4 > 0$ concave up $\Rightarrow$ x=1 min

$f''(-1) = 12(-1) - 4$

$= -12 - 4 < 0$ down $\Rightarrow$ x=-1 min

If we would've got $f''(c) = 0$ we can't say max/min

ex. If time

Increasing/decreasing
Concavity
Relative extrema
Inflection pts
Sketch

$f(x) = (x-2)^3$

$f'(x) = 3(x-2)^2 \cdot 1$

$f''(x) = 3 \cdot 2(x-2)^1$

$= 6x - 12$

f' ++++ | ++++ (0, 2, 3)

f increasing $(-\infty, 2) \cup (2, \infty)$

decreasing never

Inc/dec/max/min

$f' = 0$

$0 = 3(x-2)^2$

$\frac{0}{3} = \frac{3(x-2)^2}{3}$

$0 = (x-2)^2$

$0 = x - 2$

$x = 2$

test pts

$x=0$ $f'(0) = 3(-2)^2 = 12 > 0$

$x=3$ $f'(3) = 3(1)^2 = 3 > 0$

No max/mins

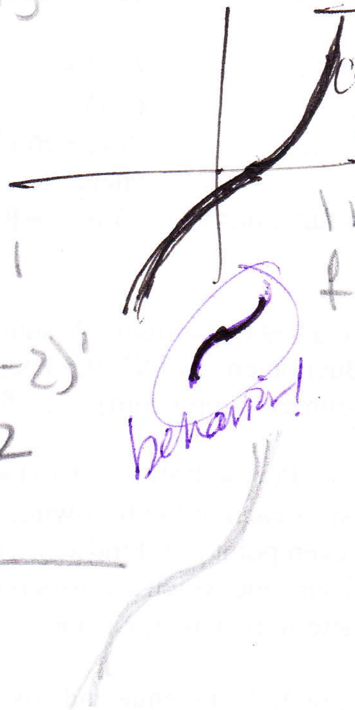
No max/min 3.2

f' ++++ | ++++ (0, 2, 3)

f'' ∩ | ∪ (0, 2, 3)

concave down | 2 | concave up

inflection pt (2,0)



f'' Concavity inf. pts

$f'' = 6x - 12 = 0$

$6x = 12$

$x = 2$ $f(2) = 0$

$\boxed{(2,0) \text{ inflection pt}}$

∩ | ∪ (1, 2, 3)

$f''(1) = -6 < 0$

$f''(3) = 6 > 0$