More examples of curve sketching for 3.1

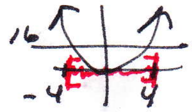
Ex. $g(t) = \sqrt{16 - t^2}$

Rewrite: $g(t) = (16 - t^2)^{1/2}$

$\longrightarrow$ Domain of g

$16 - t^2 \geq 0$

$16 \geq t^2$



Dom $= \{t \mid -4 \leq t \leq 4\}$
$= [-4, 4]$

Find critical points

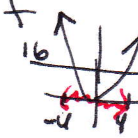
① Find $f'(x)$

② Set $f'(x) = 0$ and solve for x

$$g'(t) = \frac{1}{\cancel{2}}(16 - t^2)^{-1/2}(-\cancel{2}t)$$

$$g'(t) = \frac{-t}{\sqrt{16 - t^2}}$$

$\longrightarrow$ Domain of $g'(t)$



$16 - t^2 > 0$ ← Not equal to!

$16 > t^2$

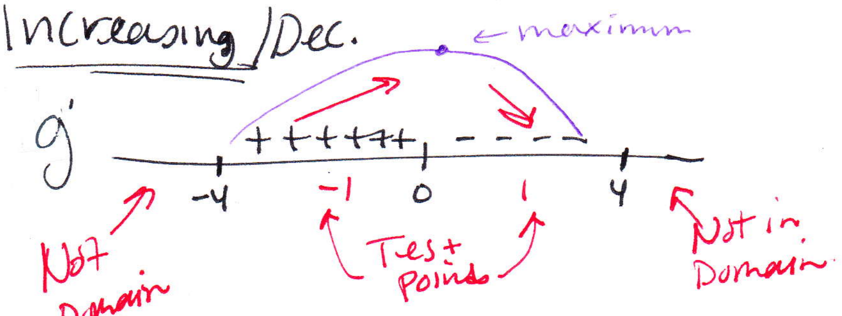
Dom $= \{t \mid -4 < t < 4\}$

$$0 = \frac{-t}{\sqrt{16 - t^2}}$$ ← Denom ≠ 0

$0 = -t$

$t = 0$ ← critical value

Increasing/Dec.



$$g'(-1) = \frac{-(-1)}{\sqrt{16 - (-1)^2}} = \frac{1}{\sqrt{15}} = +$$

$$g'(1) = \frac{-(1)}{\sqrt{16 - (1)^2}} = \frac{-1}{\sqrt{15}} = -$$

$\boxed{g(t) \text{ is increasing } (-4, 0) \text{ and decreasing } (0, 4).}$

Since g is increasing, then decreasing, g has a relative maximum at $t = 0$

ex. con't

There is a relative maximum at $t=0$

$(0, ?)$ Plug zero into $g(t)$

$$g(t) = \sqrt{16-t^2}$$
$$g(0) = \sqrt{16-0}$$
$$= \sqrt{16}$$
$$= 4$$

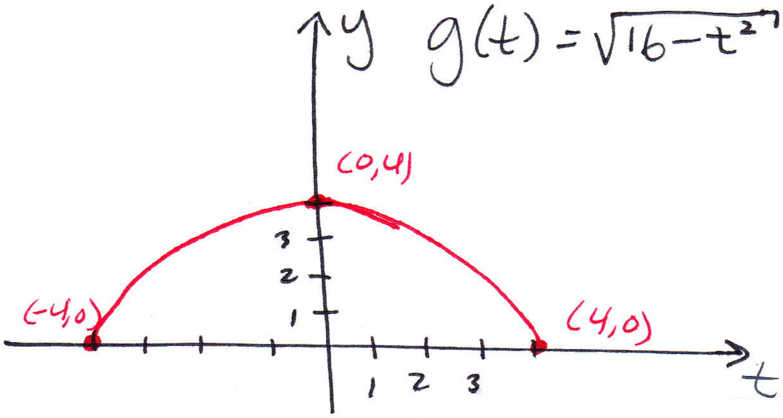
$\boxed{(0,4) \text{ is a relative maximum.}}$

To sketch, find ordered pairs for endpoints and plot along with max. & sketch a curve through the points.

Endpoints -4 & 4

$t=-4$ $\quad g(-4) = \sqrt{16-(-4)^2}$
$$= 0$$
$$(-4, 0)$$

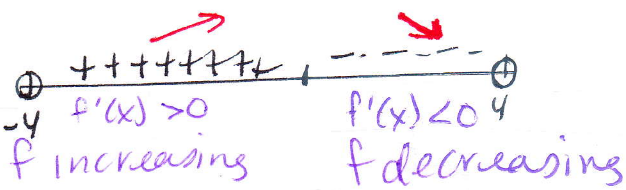
$t=4$ $\quad g(4) = \sqrt{0}$
$$= 0$$
$$(4, 0)$$



Recall Domain of $g(t)$ from beginning!

Dom $= [-4, 4]$

f' Sign Chart



Another Example.

ex. $f(x) = \dfrac{1}{9-x^2}$

Rewrite: $f(x) = (9-x^2)^{-1}$

Domain of $f(x)$

Denom $\neq 0$

$9 - x^2 \neq 0$

$9 \neq x^2$

$x \neq \pm\sqrt{9}$

$\text{Dom} = \{x \mid x \neq \pm 3\}$

Vertical Asymptotes @ $x = 3$ & $x = -3$

Find c.p.s

$f'(x) = -1(9-x^2)^{-2} \cdot (-2x)$

$f'(x) = \dfrac{2x}{(9-x^2)^2}$

Dom $f'(x)$

$(9-x^2)^2 \neq 0$

$9 - x^2 \neq 0$

$x^2 \neq 9$

$x \neq \pm 3$

asymptotes

$0 = \dfrac{2x}{(9-x^2)^2}$

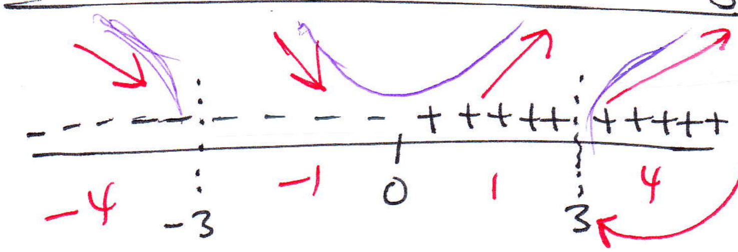
$0 = 2x$

$0 = x$

$x = 0 \leftarrow$ c.p.

Increasing / Decreasing

f'



$f'(-4) = \dfrac{2(-4)}{\text{Positive Always}} = -$

$f'(-1) = 2(-1) = -$

$f'(1) = 2(1) = +$

$f'(4) = 2(4) = +$

$f(x)$ is Increasing $(0,3) \cup (3,\infty)$ and decreasing $(-\infty,-3) \cup (-3,0)$

Since f(x) goes from decreasing to increasing at $x=0$, there is a relative minimum at $x=0$.

$x=0 \quad y=?$

$$f(0) = \frac{1}{9-0} = \frac{1}{9}$$

$\boxed{(0, \tfrac{1}{9}) \text{ is the relative minimum.}}$

To sketch, graph vertical asymptotes and plot minimum then sketch a curve!