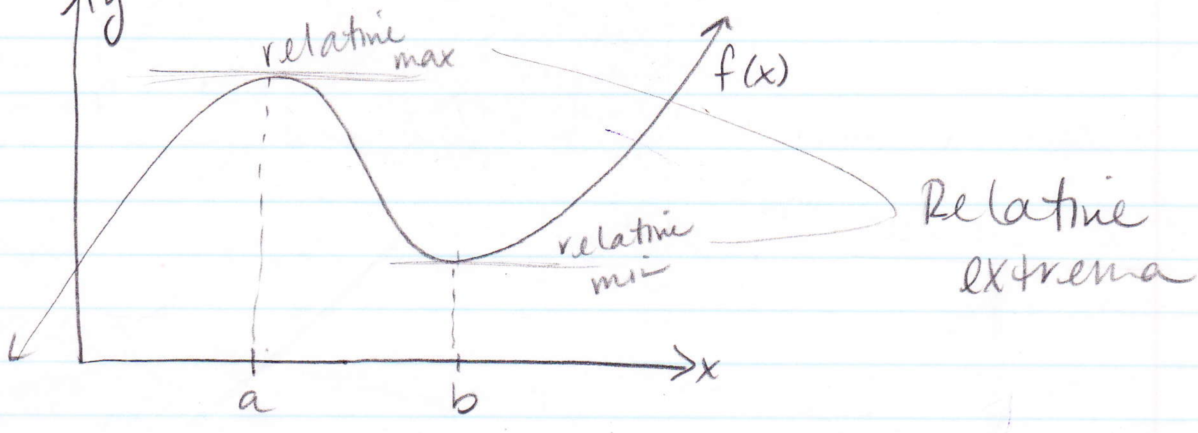
Section 3.1 Increasing / Decreasing Functions & Relative Extrema.



- How can we find $x=a$ & $x=b$ without being able to see the graph of $f(x)$?
- Where is $f(x)$ increasing / decreasing?

  $f \nearrow$ when $x<a$ and $x>b$

  $f \searrow$ when $a<x<b$

(a & b are special values)

11am

Critical Point

The point $(x_0, f(x_0))$ is a critical point of the function $f(x)$ if $f'(x_0)=0$ or $f'(x_0)$ DNE (but $f(x_0)$ does exist). ✱

Find the critical points: ① take deriv. set equal to zero & solve for x or find DNE

a) $f(x) = x^3 - x$

$f'(x) = 3x^2 - 1$

(Set $f'(x)=0$ to find cp.s)

$0 = 3x^2 - 1$

$-3x^2 = -1$

$x^2 = \frac{-1}{-3}$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$

b) $g(x) = \frac{1}{x}$ $= x^{-1}$ $\quad g'(x) = \frac{-1}{x^2}$

$\frac{-1}{\cancel{x^2}} = 0 \Rightarrow -1 = 0$

No places where $f'(x)=0$, but $f'(x)$ DNE @ $x_0 = 0$, but neither does $f(x)$

$\sqrt{x}$ $f(0)$ exists

$\frac{1}{2\sqrt{x}}$ ← DNE

1pm

c) $f(x) = 2x^4 - 4x^2 + 3$

$f'(x) = 8x^3 - 8x$

$0 = 8x^3 - 8x$

$0 = 8x(x^2-1)$

$0 = 8x(x+1)(x-1)$

$8x = 0$ $\quad \boxed{x=0}$ $\quad \boxed{x=\pm 1}$

Now to determine if CP's are max/min?

First derivative test for relative max/min of $f(x)$:

① Find critical points $x = x_0$ where $f'(x_0) = 0$ or $f'(x_0) = DNE$

② Mark these numbers on a number line/sign chart (This divides the # line into a number of open intervals)

③ Choose test #, c, in each separate interval and evaluate f' at the test points

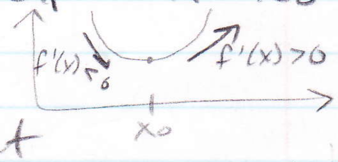
④ If $f'(c) > 0$, $f(x)$ is increasing (uphill) left to right on that interval

If $f'(c) < 0$, $f(x)$ is decreasing (downhill) left to right on the interval

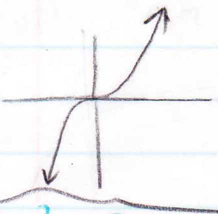
⑤ If $f'(x)$ goes from

$f'(x)>0$ $f'(x)<0$

- increasing to decreasing $\Rightarrow$ rel max at c.p. $x = x_0$
- decreasing to increasing $\Rightarrow$ rel min at c.p. $x = x_0$



- increasing to increasing or dec. to dec. then there isn't a max or min at c.p. $x = x_0$



rel. max/min are ordered pairs!

Great table in book pg 197.

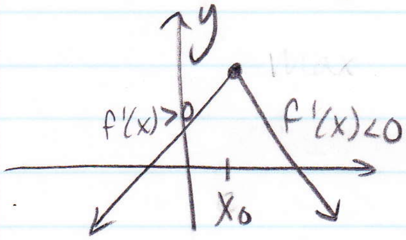
Find max/min of $f(x)$

$f(x) = 2x^3 + 3x^2 - 12x - 7$

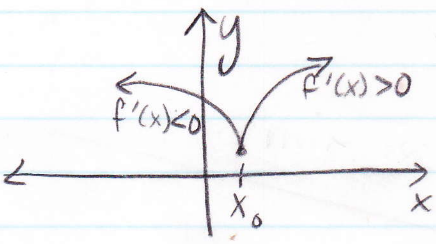
skip if No time

Examples where $f'(x)$ DNE

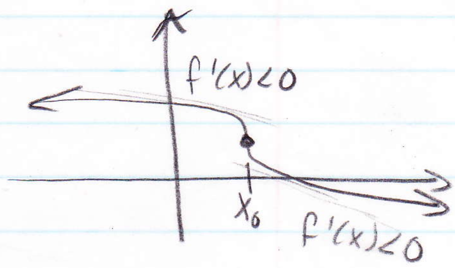
Max @ x_0



Min @ x_0

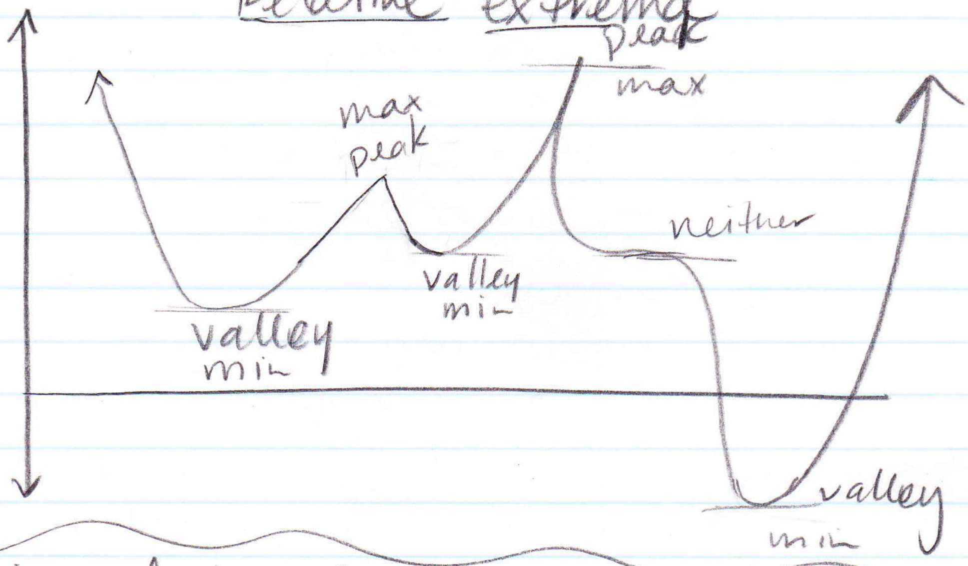


x_0 not rel. extreme



Relative Extrema

Skipped



① ex. Given $f(x) = 2x^3 + 3x^2 - 12x - 7$

① Find all c.p.'s (Think about domain

② Find all intervals where f(x) is incres/decreas.

③ Find all relative extrema, if they exist!

④ Use 1-3 to sketch the curve f(x)

① $f'(x) = 6x^2 + 6x - 12$ ← Domain $\mathbb{R}$ ⇒ There are no places where $f'(x)$ DNE

$0 = 6(x^2 + x - 2)$

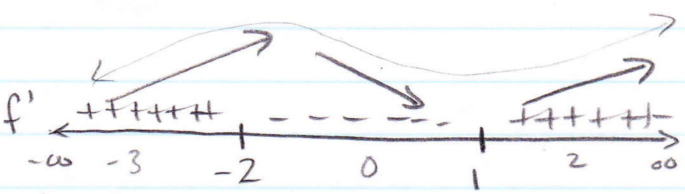
$0 = 6(x+2)(x-1)$

$\cancel{6=0}$ $\quad x+2=0 \quad x-1=0$

c.p.'s $\boxed{x=-2}$ $\quad \boxed{x=1}$

f(x) is a cubic similar to x^3

②



$f'(-3) = 6(-3+2)(-3-1)$
$= 6(-1)(-4)$
$= 24 > 0 \; +$

$f'(0) = -12 < 0 \; -$

$f'(2) = 6(2+2)(2-1)$
$= 6(4)(1)$
$= 24 > 0 \; +$

f(x) is increasing
$(-\infty, -2) \cup (1, \infty)$

f(x) is decreasing
$(-2, 1)$

Never include endpoints! [ ]

ex. cont

3 When $x=-2$ $y=?$

$f(-2)=2(-2)^3+3(-2)^2-12(-2)-7$

$=2(-8)+3(4)+24-7$

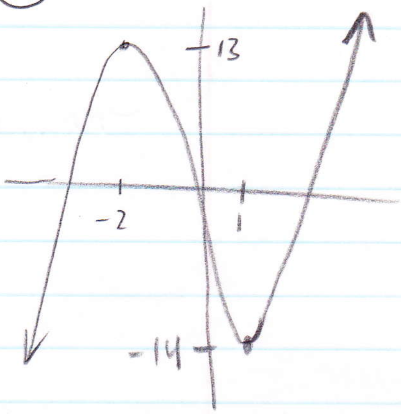
$=-16+12+17$

$=12+1$

$=13$

$\boxed{\text{Relative max at } (-2,13)}$

4 Rough sketch



even more accuracy in 3.2!

when $x=1$ $y=?$

$f(1)=2(1)^3+3(1)^2-12(1)-7$

$=2+3-12-7$

$=5-19$

$=-14$

$\boxed{\text{Relative min @ } (1,-14)}$

next page 2

I am here 11am

Same directions

ex. $H(u)=(16-u^2)^{1/2}$

3 $H'(u)=\frac{1}{\cancel{2}}(16-u^2)^{-1/2}(-\cancel{2}u)$

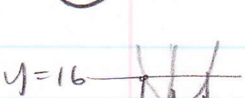
$=\dfrac{-u}{\sqrt{16-u^2}}$

$0=\dfrac{-u}{\sqrt{16-u^2}} \Rightarrow 0=-u$

1 $\boxed{u=0}$ c.p.

2 Think about domain of $f'(x)$ so we can draw # line/sign chart

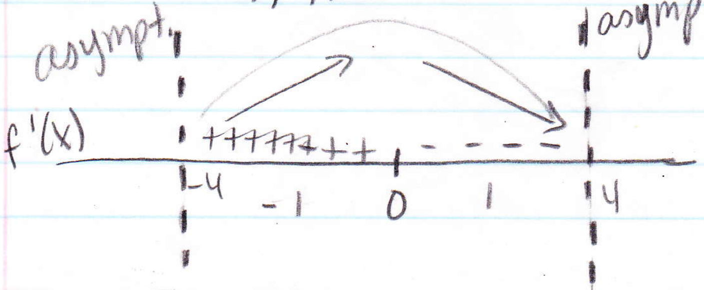
$16-u^2>0$



$16>u^2 \Rightarrow u^2=16 \Rightarrow u=\pm 4$ c.p.

$-4<u<4$ Domain

$(-4,4)$



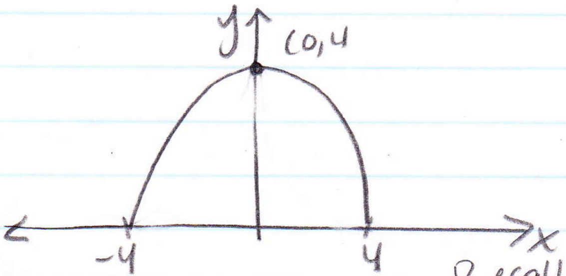
$H'(-1)=\dfrac{-(-1)}{\sqrt{16-(-1)^2}}=\dfrac{+}{+}=+$

$H'(1)=\dfrac{-1}{\sqrt{16-1^2}}=\dfrac{-}{+}=-$

② $\boxed{\begin{array}{l} H(u) \text{ is increasing } (-4,0) \\ H(u) \text{ is decreasing } (0,4) \end{array}}$

③ $u=0 \quad H(0)=\sqrt{16}=4$

$\boxed{\text{There is a relative max at } (0,4)}$



Recall domain

② ex. $f(x)=x^4+8x^3+18x^2-8$

$f'(x)=4x^3+24x^2+36x$ Domain $\mathbb{R}$

① $0=4x(x^2+6x+9)$

$0=4x(x+3)(x+3)$

$0=4x(x+3)^2$

$4x=0$ or $(x+3)^2=0$

$\boxed{x=0}$ $x+3=0$

CP's $\boxed{x=-3}$

② f' $- - - - \quad - - - \quad + + + + +$

$-\infty \quad -4 \quad -3 \quad -1 \quad 0 \quad 1 \quad \infty$

$f'(-1)=4(-1)(-1+3)^2$

$=-4(+)$

$=- \quad <0$

$f'(-4)=4(-4)(-4+3)^2$

$=-+$

$=- \quad <0$

$f'(1)=1(1+3)^2=+\cdot+=+>0$

$\boxed{\begin{array}{l} f(x) \text{ increasing } (0,\infty) \\ f(x) \text{ decreas. } (-\infty,-3)\cup(-3,0) \end{array}}$

③ $x=0 \quad f(0)=-8$

$\boxed{\text{Relative min at } (0,-8)}$

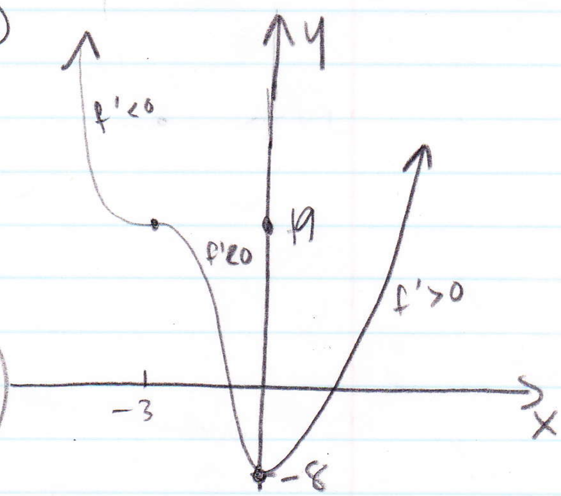
$x=-3$

$f(-3)=(-3)^4+8(-3)^3+18(-3)^2-8$

$=19$

flat spot at $(-3,19)$

④



ex. $f(x) = (x-1)^5$ → Domain $\mathbb{R}$

$f'(x) = 5(x-1)^4(1)$

① $0 = 5(x-1)^4$

$\cancel{0=5}$ $\quad (x-1)^4 = 0$

$x - 1 = \sqrt[4]{0}$

$x - 1 = 0$

c.p. $\boxed{x = 1}$

f' ++++ | +++++ on the number line at 0, 1, 2

② Increasing $\mathbb{R}$

③ No relative extrema

ex. 4

ex. If given $f'(x)$ below, find all cp's & classify as max/min/neither

$$f'(x) = \frac{(x+1)^2(4-3x)^3}{(x^2+1)^2}$$

← Dom $= \mathbb{R}$

① $0 = (x+1)^2(4-3x)^3$

$x+1 = 0 \quad 4-3x = 0$

c.p.'s $\boxed{x = -1}$ $\quad -3x = -4$

$\boxed{x = \frac{4}{3}}$

② f' +++++ | ++++++++ | - - - - on the number line at −2, −1, 0, 4/3, 2

$f'(-2) = (4-3(-2))^3$
$= (4+6)^3 = + > 0$

$f'(0) = (4-3(0)^3) = + > 0$

$f'(2) = (4-3(2))^3 = (4-6)^3 = - < 0$

$x = 2$ is a relative max

$x = -1$ is neither a max or min

Cool application #59. Advertising type up!