A few ways to calculate the derivative

① Using notation: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Using the same function $y = \frac{2}{3(5x^4+1)^2}$

Handout

Let $y(u) = \frac{2}{3u^2}$ and $u(x) = 5x^4+1$

Find $\frac{dy}{dx}$. First, let's find $\frac{dy}{du}$ & $\frac{du}{dx}$ separately.

• If $y(u) = \frac{2}{3u^2} = \frac{2}{3}u^{-2}$ then

$\frac{dy}{du} = \frac{2}{3}(-2u^{-3})$ Power Rule

$= \frac{-4}{3u^3}$

• If $u(x) = 5x^4+1$, then

$\frac{du}{dx} = 20x^3$

Now using the chain rule,

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

$$= \left(\frac{-4}{3u^3}\right)\left(20x^3\right)$$

Now plug $u(x) = 5x^4+1$ for u.

$$= \frac{-4 \cdot 20x^3}{3(5x^4+1)^3}$$

$$\boxed{= -\frac{80x^3}{3(5x^4+1)^3}}$$

② Using function composition notation

If $h(x) = f(g(x))$, then $h'(x) = f'(g(x)) \cdot g'(x)$

Where f is the outer function and g is the inner.

$f(u) = \frac{2}{3u^2}$ $\quad$ $g(x) = 5x^4+1$

$$h(x) = f(g(x)) = \frac{2}{3(5x^4+1)^2}$$

(2) con't $h(x) = f(g(x))$

$$= \frac{2}{3(5x^4+1)^2} = \frac{2}{3} \cdot \frac{1}{(5x^4+1)^2}$$

Need Quotient Rule

$$h'(x) = \frac{2}{3}\left[\frac{\cancel{(5x^4+1)^2(0)}^{\,0} - 1[40x^3(5x^4+1)]}{[(5x^4+1)^2]^2}\right]$$

$lo = (5x^4+1)^2$
$dlo = 2(5x^4+1) \cdot 20x^3$
$hi = 1$
$dhi = 0$

$$= \frac{2}{3}\left[\frac{-40x^3\cancel{(5x^4+1)}}{(5x^4+1)^{\cancel{4}\,3}}\right]$$

$$= \boxed{\frac{-80x^3}{3(5x^4+1)^3}}$$

Note: we get the same answer as part (1)!! :)

(3) Using the generalized power rule.

$$\frac{d}{dx}[h(x)]^n = n[h(x)]^{n-1} \cdot h'(x)$$

$y = \frac{2}{3(5x^4+1)^2}$ from last problem.

$= \frac{2}{3}(5x^4+1)^{-2}$ Power Rule

$$y' = \frac{2}{3}\left[\underbrace{-2(5x^4+1)^{-2-1}}_{n\,h(x)^{n-1}} \cdot \underbrace{20x^3}_{h'(x)}\right]$$

$$= \frac{2}{3}\left[-40x^3(5x^4+1)^{-3}\right]$$

$$= \boxed{\frac{-80x^3}{3(5x^4+1)^3}}$$

SAME as (1) & (2)

There are many more varieties of how to differentiate this function. I suggest you try to pick a method & stick with it.