Recall $x^{-m} = \frac{1}{x^m}$ ex. $x^{-3} = \frac{1}{x^3}$

Neg exponents → think Reciprocal

ex. $\frac{1}{x^5} = x^{-5}$

ex $\frac{1}{x^{-3}} = x^3$

ex. $f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -1x^{-1-1}$

$= -1x^{-2}$

$= \boxed{\frac{-1}{x^2}}$

ex. $f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{1/2-1}$

$= \frac{1}{2}x^{\frac{1}{2}-\frac{2}{2}}$

$= \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}}$

$= \boxed{\frac{1}{2\sqrt{x}}}$

Drop $\frac{d}{dx}$ once you take the derivative

ex. $\frac{d}{dx}\left[\frac{x^5+x^2}{x^3}\right]$

$\frac{d}{dx}\left[\frac{x^5}{x^3} + \frac{x^2}{x^3}\right]$

$= \frac{d}{dx}\left[x^2 + \frac{1}{x}\right] = \frac{d}{dx}\left[x^2 + x^{-1}\right]$

$= 2(x^{2-1}) + (-1)(x^{-1-1})$

$= 2(x) - x^{-2} = \boxed{2x - \frac{1}{x^2}}$

The constant mult. rule

If c is a constant and $f(x)$ is differentiable, then so is $cf(x)$ and

$$\frac{d}{dx}[c f(x)] = c \frac{d}{dx} f(x)$$

ex. $\frac{d}{dx}[4x^3] = 4\frac{d}{dx}[x^3]$

$= 4(3x^{3-1})$

$= 4 \cdot 3x^2$

$\boxed{= 12x^2}$

Recall $x^{m/n} = \sqrt[n]{x^m}$ or $(\sqrt[n]{x})^m$

ex. $\frac{d}{dx}\left[\frac{-2}{\sqrt[3]{x}}\right] = \frac{d}{dx}\left[-2 \cdot \frac{1}{\sqrt[3]{x}}\right]$

$= -2\frac{d}{dx}\left[\frac{1}{\sqrt[3]{x}}\right]$

$= -2\frac{d}{dx}\left[\frac{1}{x^{1/3}}\right]$

$= -2\frac{d}{dx}x^{-1/3}$

$= -2\left(-\frac{1}{3}\right)x^{-\frac{1}{3}-1}$ ← $\frac{2}{3}x^{-\frac{1}{3}-\frac{3}{3}}$ ← $\frac{2}{3}x^{-\frac{2}{3}}$ $= \frac{2}{\sqrt[3]{x^2}}$

1pm

The sum rule

If $f(x)$ & $g(x)$ are differentiable, then so is $s(x) = f(x) + g(x)$ and $s'(x) = f'(x) + g'(x)$

Skipped

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$ Start here

ex ① $\frac{d}{dx}[4x-2] = 4$ $\boxed{\frac{d}{dx}[mx+b] = m}$

③ Find $\frac{d}{dx}[4x^2 + x^{-1} + \pi]$

$= \frac{d}{dx} 4x^2 + \frac{d}{dx} x^{-1} + 0$

$= 4 \frac{d}{dx} x^2 + (-1)x^{-1-1}$

$= 4(2x^{2-1}) - x^{-2}$

$= \boxed{8x - x^{-2}}$

② ex. If $f(x) = 6x^5 + 4x^2 - 3x + 2$, find $f'(x)$

$f'(x) = 6(5x^4) + 4(2x) - 3x^0 + 0$

$\boxed{f'(x) = 30x^4 + 8x - 3}$

Imagine using the defn of the derivative (long route)

Skip if low on time

Application #59

A medical research team determines that t days after an epidemic begins, $N(t) = 10t^3 + 5t + \sqrt{t}$ people will be infected, for $0 \le t \le 20$. At what rate is the spread of infection increasing on the ninth day?

$N(t) = 10t^3 + 5t + \sqrt{t} = 10t^3 + 5t + t^{1/2}$

$N'(t) = 10(3t^{3-1}) + 5(1t^{1-1}) + \frac{1}{2}t^{1/2 - 2/2}$

$= 30t^2 + 5 + \frac{1}{2}t^{-1/2}$

$= 30t^2 + 5 + \frac{1}{2t^{1/2}}$

$N'(t) = 30t^2 + 5 + \frac{1}{2\sqrt{t}}$

$N'(t) = 30t^2 + 5 + \frac{1}{2\sqrt{t}}$

$t=9 \Rightarrow N'(9) = 30(9)^2 + 5 + \frac{1}{2\sqrt{9}}$

$= 30(81) + 5 + \frac{1}{6}$

$= \frac{6}{6} \cdot 2431 + 5 \cdot \frac{6}{6} + \frac{1}{6}$

$= \frac{14{,}580}{6} + \frac{30}{6} + \frac{1}{6}$

$= \frac{14580 + 31}{6}$

$= \frac{14{,}611}{6} \approx 2435.16$

$81 \times 30 = 2430$, $2430 \times 6 = 14580$, $14580 + 31 = 14{,}611$

$14{,}611.00 \div 6 = 2435.16$

The rate of spread on the 9th day is increasing at a rate of about 2435 people per day.

If time allows...
extra examples

ex) $y = \frac{-x^2}{16} + \frac{3.2}{x} - x^{5/2} + \frac{1}{3x^2} + \frac{x}{6} + 4$

rewrite

$y = -\frac{1}{16}x^2 + 3.2x^{-1} - x^{5/2} + \frac{1}{3}x^{-2} + \frac{1}{6}x + 4$

$y' = -\frac{1}{16}(2x^{2-1}) + 3.2(-1x^{-1-1}) - \frac{5}{2}x^{\frac{5}{2}-\frac{2}{2}} + \frac{1}{3}(-2x^{-2-1}) + \frac{1}{6}x^{1-1} + 0$

$= -\frac{1}{8}x - 3.2x^{-2} - \frac{5}{2}x^{3/2} - \frac{2}{3}x^{-3} + \frac{1}{6}x^0$

$\boxed{y' = -\frac{1}{8}x - \frac{3.2}{x^2} - \frac{5}{2}x^{3/2} - \frac{2}{3x^3} + \frac{1}{6}}$

11am

ex. Find the equation of the line that is tangent to the graph at the point

$y = x^5 - 3x^3 - 5x + 2$; $(1, -5)$

$\frac{dy}{dx} = 5x^4 - 9x^2 - 5$

Plug in $x = 1$

$\left.\frac{dy}{dx}\right|_{x=1} = 5(1)^4 - 9(1)^2 - 5$

$= \cancel{5} - 9 - \cancel{5}$

$= -9$ ← Slope

We have slope & point

$m = -9$ $(1, -5)$

Find equation

$y + 5 = -9(x - 1)$

$y = -9x + 9 - 5$

$\boxed{y = -9x + 4}$

Tricks:

talked about in 1pm class

ex. $f(x) = x(2x+3) = 2x^2 + 3x$

$f'(x) = ?$

$f'(x) = 4x^{2-1} + 3x^{1-1}$

$= 4x^1 + 3x^0$

$\boxed{f'(x) = 4x + 3}$

* we can lead with this in next section to illustrate product rule!