Rationalize $\lim_{x \to 25} \frac{\sqrt{x} + 5}{x - 25}$ → $\sqrt{5}+1$ can't tell ~~where~~ value top is approaching!

$\frac{\sqrt{x+1}+3}{x-9}$ H/W

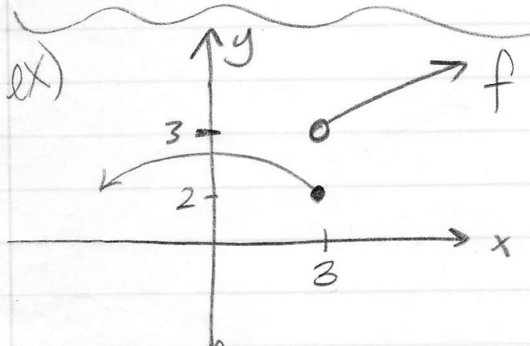
Section 1.6 – One sided limits & continuity

One sided limits

If f(x) approaches L as x tends toward c from the left, we write $\lim_{x \to c^-} f(x) = L$.

Likewise, if f(x) approaches M as x tends towards c from the right, then
$\lim_{x \to c^+} f(x) = M$

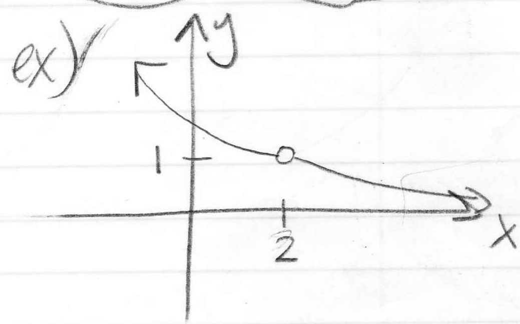
ex)



from left $\lim_{x \to 3^-} f(x) = 2$

from right $\lim_{x \to 3^+} f(x) = 3$

ex)



$\lim_{x \to 2^-} f(x) = 1 = \lim_{x \to 2^+} f(x)$

$\lim_{x \to 2} f(x) = 1$

Note: $\lim_{x \to 3} f(x) = \text{DNE}$

Vertical asymptotes

insted $\lim_{x \to 0^-} \frac{1}{x} = -\infty$ & $\lim_{x \to 0^+} \frac{1}{x} = +\infty$ Plug in close value

$x = 5$

ex. Find $\lim_{x \to 5^-} \frac{x+1}{x-5}$ & $\lim_{x \to 5^+} \frac{x+1}{x-5}$

If you can plug it in, then do it!!

Can't factor to simplify so think about numbers really close to the left/right respectively.

$\lim_{x \to 5^-} \frac{4.99+1}{4.99-5} = \frac{+}{-} = \boxed{-\infty}$

$\lim_{x \to 5^+} \frac{5.001+1}{5.001-5} = \frac{+}{+} = \boxed{+\infty}$ (0.001)

$\lim_{x \to 5} f(x) = \text{DNE}$

$\frac{\#}{\pm\infty} = 0$ concept

$\frac{\#}{0} = \pm\infty$

Existence of a limit

The two sided limit $\lim_{x \to c} f(x)$ exists

if and only if the two one sided limits
$\lim_{x \to c^-} f(x)$ & $\lim_{x \to c^+} f(x)$ both exist & are equal

That is

$$\lim_{x \to c} f(x) = \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$$

---

ex. Determine whether $\lim_{x \to 1} f(x)$ exists where

$$f(x) = \begin{cases} 3x+4 & \text{if } x \geq 1 \\ 4x^2-x+2 & \text{if } x < 1 \end{cases}$$

$\lim_{x \to 1^-} 4x^2-x+2$

$= 4(-1)^2-(-1)+2$

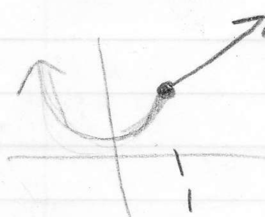
$= 4+1+2 = \boxed{7}$

$\lim_{x \to 1^+} 3x+4$

$= 3(1)+4$

$= 3+4$

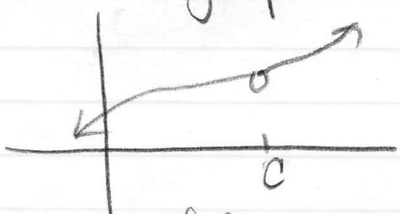
$\boxed{= 7}$



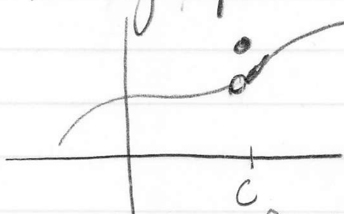
$\lim_{x \to 2} f(x) = 7$ since the left & right hand limits exist are are equal.

Examples of
Discontinuity (But limit still exists)

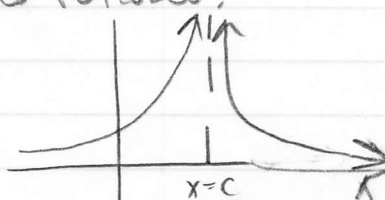
Holes & gaps in the graph occur as follows:



$f(c) = DNE$



$\lim_{x \to c} f(x) = $ EXISTS $\neq f(c)$

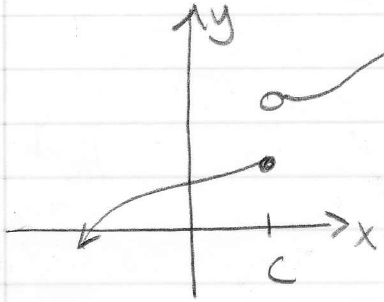


$\lim_{x \to c^+} f(x) = +\infty$

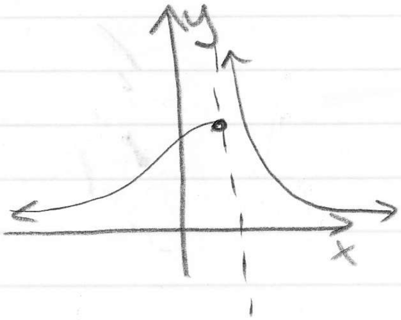
$\lim_{x \to c^-} f(x) =$

f(x) has

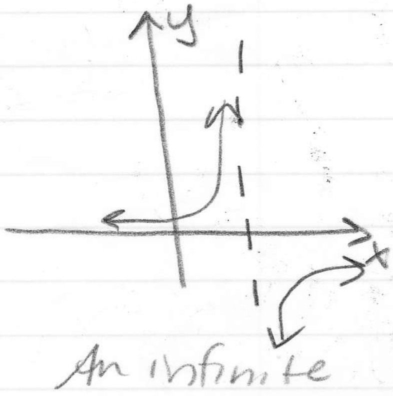
Discontinuity (limit doesn't exist)



A finite gap.



An infinite gap.



An infinite gap.

In all cases the left & right-hand limits are not equal.

1pm

Continuity – In short can you draw graph w/out picking up pencil.

Defn

A function f is continuous at c if all three conditions are statisfied

a) f(c) is defined.

b) $\lim_{x \to c} f(x)$ exists

c) $\lim_{x \to c} f(x) = f(c)$

examples of cont functions

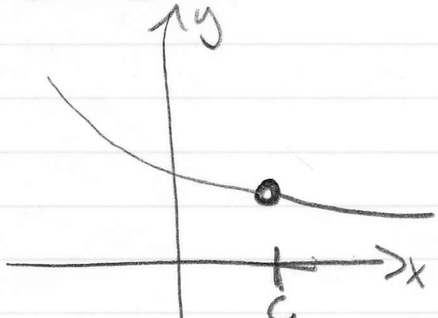
x^n, $mx+b$, $\sin x$, e^x, $\ln x$

polynomials.

Discontinuous

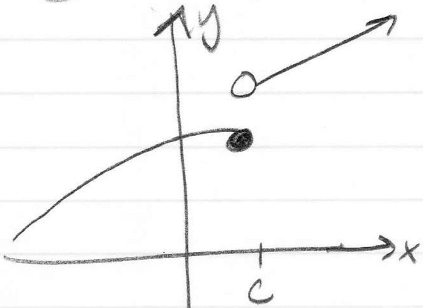
piecewise functions, rational funt.

① & ③ Fail



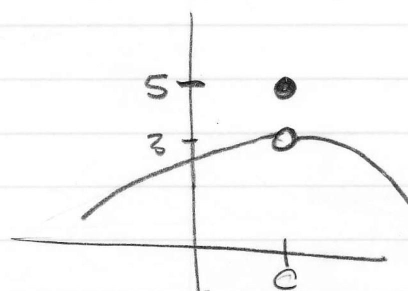
f(c) not defined

② & ③ Fail



f(c) defined but $\lim_{x \to c} f(x) = DNE$

③ Fails only



① $f(c)=5$

② $\lim_{x \to c} f(x) = 3$

Determine if f is continuous at
a) $x=1$
b) $x=3$

$f(x) = \dfrac{1}{x-3}$ Use definition of continuity to prove/disprove

a) $x=1$ $c=1$

(1) $f(1) = \dfrac{1}{1-3} = -\dfrac{1}{2}$ ✓

(2) $\lim_{x\to 1} \dfrac{1}{x-3} = \dfrac{1}{1-3} = -\dfrac{1}{2}$ ✓

(3) $\lim_{x\to 1} \dfrac{1}{x-3} = f(1) = -\dfrac{1}{2}$ ✓

Since all three conditions are met, the function is continuous at $x=1$.

b) $x=3$ DC

(1) $f(3) = \dfrac{1}{3-3} = \dfrac{1}{0} =$ UND

Since $f(3)$ is undefined, the function is not continuous at $x=3$.
← talk about diff between hole & asymptote →

11am

ex) Find all places where $f(x)$ is discontinuous
Look for places where you divide by zero.

Skip if low on time

a) $f(x) = 3x^2+6x-9$ None Continuous for $\mathbb{R}$ x,

b) $f(x) = \dfrac{x^2-1}{x+1}$ $< f(x) = \frac{1}{2x+3}$ $x \neq -1$

c) $f(x) = \dfrac{x}{x^2-x} = \dfrac{x}{x(x-1)}$ $x \neq 0$, $x \neq 1$

Con't at $\mathbb{R}$ except $x=0$, $x=1$.

d) $f(x) = \frac{1}{x^2+1}$ None

e) $f(x) = \begin{cases} 2x+3 & \text{if } x \leq 1 \\ 6x-1 & \text{if } x > 1 \end{cases}$

None

$$\frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$$

$x+2$

4

ex. Continuous at $x = 2$?

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 5 & \text{if } x = 2 \end{cases}$$

① $f(2) = 5$ ✓

② $\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{x^2-4}{x-2}$

$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$

$= \lim_{x \to 2} x+2$

$= 4$ ✓

③ $\lim_{x \to 2} f(x) = 4 \neq f(2) = 5$

The function is not continuous at $x=2$

Extension:

What if $f(x) = 4$ when $x = 2$?

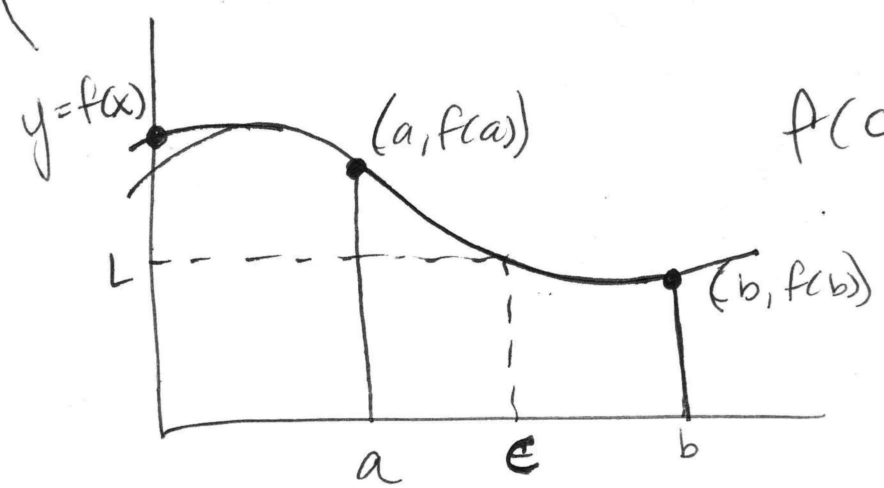
then it would prove continuous!

Intermediate Value property

If f is continuous on $a \le x \le b$ and L is a number between $f(a)$ & $f(b)$, then $f(c) = L$ for some # c between a & b

("A con't function attains all values between any two of its values)

Ex. ~~Abelen~~ my daughter weighed 9 lbs 8 oz when born, at some point in time she weighed exactly 15 lbs. during her 5 yrs of life. (weight is a con't funct of time)



$f(c) = L$ for some $c \in [a, b]$

Already covered