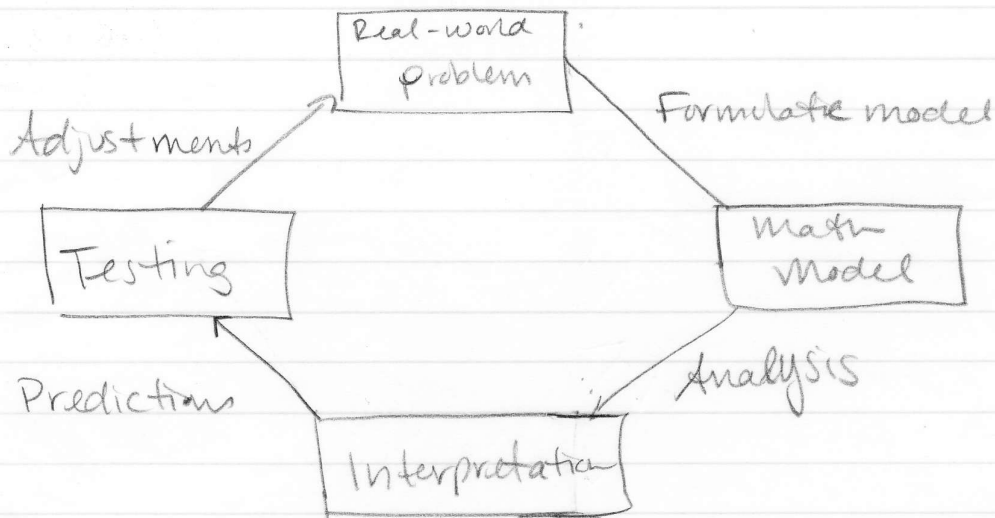


Section 1.4 Functional Models (Notes Handout)

Mathematical modeling

- Practical applications in business, econ, sciences
- Real world IS VERY complicated



Relate to cell phone example / my thesis.

Ghost moth caterpillars, nematodes, bush lupine system of three species.

Once model is refined, use it for land management, predictive purposes.

Steps to formulate model/Function

Step 1: Draw pictures (related)

Identify Pertinent information

* Declare variables.

Step 2: In this section, we are finding functions to model behavior

Ⓐ Define the function (typically in two variables) $A = l \cdot w$

Ⓑ State an equation relating two variables

Step 3: Solve equation Ⓑ for one variable $l = 2 + w$

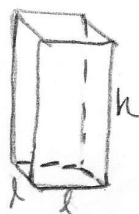
& substitute into Ⓐ so that the function only has one variable.

Step 4: Graph/Analyze Function if asked to do so.

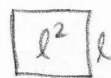
Section 1.4 - Lecture Notes/Examples

Set up and solve an equation(s) to solve the following problems. Be sure to declare all variables and answer all word problems in complete sentences.

1. A closed box with a square base has a surface area of 4,000 square centimeters. Express its volume as a function of the length of its base. Let l = length & h = height



Base (x2)



Sides (x4)



(A) Function

$$\begin{aligned} V &= l \cdot w \cdot h \\ &= l \cdot l \cdot h \\ V &= l^2 h \end{aligned}$$

(B) Equation

Surface Area (solve for h)

$$\begin{aligned} 4000 &= 2l^2 + 4lh \\ 4000 - 2l^2 &= 4lh \\ \frac{4000 - 2l^2}{4l} &= h \\ \frac{2(2000 - l^2)}{4l} &= h \end{aligned}$$

$$h = \frac{2000 - l^2}{2}$$

Now substitute back into function V .

$$V = l^2 h = l^2 \left(\frac{2000 - l^2}{2} \right)$$

$$V(l) = \frac{2000l - l^3}{2}$$

Function for volume in terms of length.

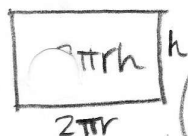
2. A closed cylindrical can has a surface area of 230π square inches. Express the volume of the can as a function of its radius. Let r = radius & h = height



Base x2



Side



(A) Volume of a cylinder

$$V = \pi r^2 h$$

(B) S.A. (solve for h)

$$\begin{aligned} 230\pi &= 2\pi r^2 + 2\pi r h \\ 230\pi - 2\pi r^2 &= 2\pi r h \\ \frac{230\pi - 2\pi r^2}{2\pi r} &= h \end{aligned}$$

$$h = \frac{2\pi(115 - r^2)}{2\pi r}$$

$$h = \frac{115 - r^2}{r}$$

Now substitute h into V .

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi r^2 \left(\frac{115 - r^2}{r} \right) \\ &= \pi (115 - r^2) \end{aligned}$$

$$V(r) = 115\pi - \pi r^2$$

Volume as a function of radius!

Proportionality Notes:

The quantity Q is said to vary:

directly with x if $Q = kx$ for some constant k
inversely with x if $Q = \frac{k}{x}$
jointly with x if $Q = kxy$

3. The spread of an epidemic. The rate at which an epidemic spreads through a community is jointly proportional to the number of people who have caught the disease and the number who have not. Express this rate as a function of the number of people who have caught the disease.

Let
 x = people with disease

y = people w/out disease

R = rate of spread

P = Total population

(A) $R = k \cdot x \cdot y$

(B) $P = x + y$

$P - x = y$

Substitute into R

$$R = kx(P-x)$$
$$R(x) = Pkx - kx^2$$

where P & k are constants.

4. A manufacturer can sell a certain product for \$110 per unit. Total cost consists of a fixed overhead of \$7,500 plus production costs of \$60 per unit.

- How many units must be sold for the manufacturer to break even. (Revenue = Cost)
- What is the manufacturer's profit or loss if 100 units are sold?
- How many units must be sold for the manufacturer to realize a profit of \$1250?

Let x = # of units and C = Cost, R = revenue
then $C(x) = 60x + 7500$, $R(x) = 110x$

a) Break even $\Rightarrow R(x) = C(x)$

Recall

b) Profit = Revenue - Cost

$$\begin{aligned} &= R(x) - C(x) \\ &= 110x - (60x + 7500) \\ &= 110x - 60x - 7500 \end{aligned}$$

$$P(x) = 50x - 7500$$

$x = 100$

$$\begin{aligned} P(100) &= 50(100) - 7500 \\ &= 5000 - 7500 \\ &= -2500 \end{aligned}$$

You would expect a loss of \$2500.

$$110x = 60x + 7500$$

$$50x = 7500$$

$x = 150$ The break even point is at 150 units.

c) $P(x) = 1250$ $x = ?$

$$\begin{aligned} 1250 &= 50x - 7500 \\ +7500 & \quad +7500 \end{aligned}$$

$$8750 = 50x$$

$$\frac{8750}{50} = x$$

$$175 = x$$

They would have to sell 175 units.