Section 1.3 - Linear Functions

Linear Functions
Changes at a constant rate (slope)

Slope Intercept Form
\[ y = mx + b \]
\[ m = \text{slope} \]

Slope of a line through points \((x_1, y_1)\) and \((x_2, y_2)\)
\[ m = \frac{\text{rise}}{\text{run}} \]
\[ = \frac{\Delta y}{\Delta x} \]
\[ = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{y_1 - y_2}{x_1 - x_2} \]

- \( m > 0 \) (pos) uphill/increasing
- \( m < 0 \) (neg) downhill/decreasing

Horizontal/Vertical line

Horizontal
\[ y = b \]
Equation
\[ y = b \]
Slope = 0

Vertical
\[ x = a \]
Equation
\[ x = a \]
Slope is undefined
\[ \frac{\text{rise}}{0} = \text{un} \]
Point Slope form of the equation of the line
\[ y - y_1 = m(x - x_1) \]
where \( m \) is slope
\((x_1, y_1)\) is any point

Ex. Find the equation of the line thru
\((5, -2)\) with slope of \( \frac{1}{2} \).
\[
y - (-2) = \frac{1}{2} (x - 5)
y + 2 = \frac{1}{2}x - \frac{5}{2}
\]
\[
y = \frac{1}{2}x - \frac{5}{2} - \frac{2}{1} \left( \frac{2}{2} \right)
y = \frac{1}{2}x - \frac{5}{2} - \frac{4}{2}
y = \frac{1}{2}x - \frac{9}{4}
\]

Ex. Find the equation of the line thru
\((5, -1) \# (2, 3)\)
\[
m = \frac{3 - (-1)}{-2 - 5} = \frac{3 + 1}{-7} = -\frac{4}{7}
\]
\[
y - y_1 = m(x - x_1)
y + 1 = -\frac{4}{7} (x - 5)
y = -\frac{4}{7}x + \frac{20}{7} - 1 \left( \frac{7}{7} \right)
y = -\frac{4}{7}x + \frac{20}{7} - \frac{7}{7}
y = -\frac{4}{7}x + \frac{13}{7}
\]
Slope-Intercept Form
\[ y = mx + b \]
Point-Slope Form
\[ y - y_1 = m(x - x_1) \]
Standard Form
\[ Ax + By = C \] where \( A, B, \) and \( C \) are integers.
example: \( 5x - 3y = 6 \)

**Application**

At Verizon, I pay \$96 per month and if I go over my minute, I am charged 60 cents a minute for every minute I go over. Find a linear equation to model this scenario.

 Declare variables:

 Let \( y \) be my monthly bill and \( x \) be the number of minutes I go over my limit.

\[ y = 0.60x + 96 \]

\[ \$109.80 - 96 = 0.6x \]
\[ 13.8 = 0.6x \]
\[ \frac{13.8}{0.6} = x \]

Think about scale
Don't draw graphs
Too small

If I had a bill of \$109.80 how many minutes did I go over?

**Parallel & Perpendicular Lines**

\[ y_1 = mx_1 + b_1 \] Same
\[ y_2 = m_2x_2 + b_2 \] \( \parallel M_1 = M_2 \)

\[ y_2 = m_2x_2 + b_2 \] \( \perp \frac{a_1}{b_1} \]
\[ m_1 = \frac{a_1}{b_1} \]
\[ m_2 = -\frac{b_2}{a_2} \]}
\[ M_1 - M_2 = -1 \]
Write the equation of the line parallel to \( x + 3y = 5 \) thru \((-2, 3)\).

**Given:** a point

**Need Slope:** \( m = ? \)

\[
x + 3y = 5
\]

\[
3y = -x + 5 \quad \text{Same slope}
\]

\[
y = -\frac{1}{3}x + \frac{5}{3} \\ m_1 = -\frac{1}{3}
\]

\[
m_1 = -\frac{1}{3} (\frac{-2, 3}{x, y})
\]

\[
y - 3 = -\frac{1}{3} (x + 2)
\]

\[
y = -\frac{1}{3}x - \frac{2}{3} + \frac{3}{1} (\frac{3}{3})
\]

\[
y = -\frac{1}{3}x - \frac{2}{3} + \frac{9}{3} = -\frac{1}{3}x + \frac{7}{3}
\]

Equation of a line perpendicular to same line

\[
x + 3y = 5 \quad \text{thru} \ (\frac{5}{3}, y) \]

\[
m_1 = -\frac{1}{3} + m_2 = \frac{3}{3} = 1
\]

\[
y + 1 = 3(x - 5)
\]

\[
y = 3x - 15 - 1
\]

\[
y = 3x - 16
\]

\[
\text{You can check on graphing calc.}
\]