Section 1.1 - Functions

~~HW#1 - 1.1 - 7, 10, 14, 19-22, 25-31 odd, 33, 35, 36, 39, 43, 50, 51, 54, 57, 62, 66, 75~~

~~Due Friday 26th~~

Functions

Motivation: In many real world situations things are related in certain ways. For instance, the amount of polution in a certain area may depend on the # of cars on the road, temperature in a room might depend on the number of people in the room. Rate that soup cools depends on room temp. These situations can be represented as functions.

Function Defn

A rule that assigns to each object in a set A exactly one object in a set B.

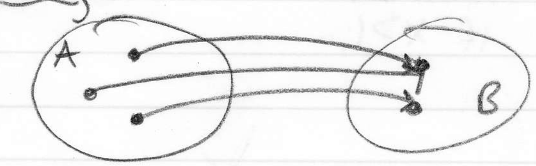
Set A = Domain (Real #s)

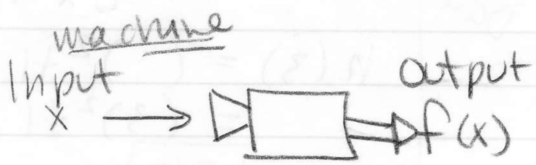
Set B = Range

*each x has only one y

Visually

map





Function Notation

$y = f(x)$

Plug in an x value to get a y value out

Function Evaluation

ex) Find $f(-3)$ if $f(x) = x^2 + 4x - 2$

$$f(-3) = (\ )^2 + 4(\ ) - 2$$
$$= (-3)^2 + 4(-3) - 2$$
$$= 9 - 12 - 2$$
$$= -3 - 2$$
$$= -5$$

Notation

$\boxed{f(-3) = -5} \longrightarrow$ Other representation: $x = -3$, $y = -5$, $(-3, -5)$

Evaluate

ex) $g(t) = (2t - 8)^{1/2}$ find $g(9)$, $g(4)$, $g(0)$

$$g(9) = \sqrt{2(9) - 8}$$
$$= \sqrt{18 - 8}$$
$$\boxed{= \sqrt{10}}$$

$$g(4) = \sqrt{2(4) - 8}$$
$$= \sqrt{0}$$
$$\boxed{= 0}$$

$$g(0) = \sqrt{2(0) - 8}$$
$$= \sqrt{-8}$$
$$\boxed{= \text{undefined (imaginary)}}$$

1pm Day 1

ex) $h(x) = \begin{cases} -2x + 4 & \text{if } x \le 1 \\ x^2 + 1 & \text{if } x > 1 \end{cases}$ find $h(3)$, $h(1)$

$$h(3) = (\ )^2 + 1$$
$$= (3)^2 + 1$$
$$\boxed{= 10}$$

$$h(1) = -2(\ ) + 4$$
$$= -2(1) + 4$$
$$\boxed{= 2}$$

Constant function $f(x) = 3$ find $f(-2)$

Add Constant Function?

Natural Domain

All x values for which the function is defined. Restrictions on domain

Linear $f(x) = mx + b$ Domain = All real numbers $\mathbb{R}$ or $(-\infty, \infty)$

11am Day 1

Polynomials $= \mathbb{R}$ ex) $f(x) = 5x^4 + 3x^2 - 2$ Dom $= \mathbb{R}$

Square Roots

Negative numbers do not have real square roots.

$\sqrt{\text{radicand}}$

Radicand must be positive or zero rad ≥ 0

Radicand ≥ 0

ex. $f(x) = \sqrt{-2x + 5}$

Domain Radicand ≥ 0

$$-2x + 5 \geq 0$$
$$-2x \geq -5$$
$$x \leq \frac{-5}{-2}$$
$$x \leq \frac{5}{2}$$

5/2

$\{x \mid x \leq \frac{5}{2}\}$ or $(-\infty, \frac{5}{2}]$

Rational Functions

$\frac{Q(x)}{R(x)}$ where $Q(x)$ & $R(x)$ are polynomials $\frac{\text{numerator}}{\text{Denom}}$

Denom $\neq 0$

Restriction on domain: Denom cannot equal zero

$R(x) \neq 0$

ex. $g(x) = \frac{3}{4x-5}$ Denom $\neq 0$

$$4x - 5 \neq 0$$
$$4x \neq 5$$
$$x \neq 5/4$$

5/4

$\{x \mid x \neq 5/4\}$ $(-\infty, \frac{5}{4}) \cup (\frac{5}{4}, \infty)$

Numerator

ex. $f(x) = \frac{\sqrt{\text{radicand}}}{3x \sqrt{2x+5}}$ Radicand ≥ 0

LEAVE OUT

Applications in Economics

Functions dealing with marketing

Demand - price $p = D(x)$ that is charged for each unit of commodity if x units are to be sold

Supply - price $p = S(x)$ at which producers are willing to supply x units to market.

Revenue - $R(x)$ is revenue from selling x units

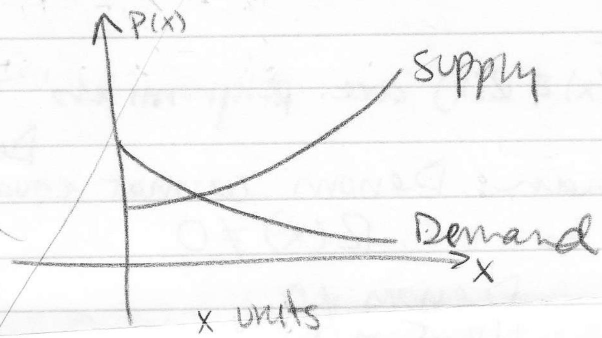
$$R(x) = (\text{\# of units sold}) \cdot (\text{price per item})$$
$$= x \cdot p(x)$$

Cost - $C(x)$ is the cost of producing x units

Profit - $P(x)$ is profit from selling x units of the commodity.

$$P(x) = \text{Revenue} - \text{Costs}$$
$$= R(x) - C(x)$$
$$= x \cdot p(x) - C(x)$$

(Higher the unit price, the fewer the # of units demanded & vice versa)



ex) Given $p = D(x)$ & $C(x)$ find

a) The revenue $R(x)$ & profit $P(x)$

b) All values of x for which production of the commodity is profitable

LEAVE OUT

ex. con't

$D(x) = -0.5x + 39 = p(x)$

$C(x) = 1.5x^2 + 9.2x + 67$

a) $R(x) = x \cdot p(x)$

$= x(-0.5x + 39)$

$\boxed{R(x) = -0.5x^2 + 39x}$

$P(x) = R(x) - C(x)$

$= -0.5x^2 + 39x - (1.5x^2 + 9.2x + 67)$

$= -0.5x^2 + 39x - 1.5x^2 - 9.2x - 67$

$\boxed{P(x) = -2x^2 + 29.8x - 67}$

b) $P(x) > 0$ (positive profit)

must find zeros

$-2x^2 + 29.8x - 67 > 0$

$-2x^2 + 29.8x - 67 = 0$

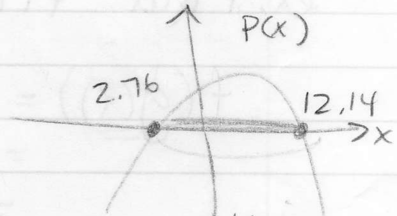
$a = -2 \quad b = 29.8 \quad c = -67$

$x = \dfrac{-b \pm \sqrt{b^2 - 2ac}}{2a}$

$= \dfrac{-29.8 \pm \sqrt{29.8^2 - 4(-2)(-67)}}{2(-2)}$

$= 2.76 \quad +$

$= 12.14 \quad -$

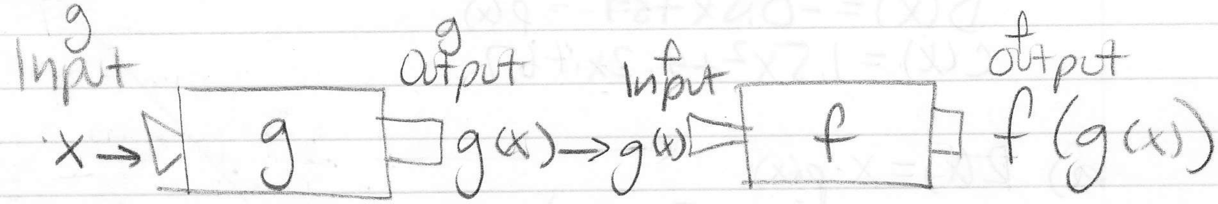


y value = 0

Profit > 0 when
$2.76 < x < 12.14$
$(2.76, 12.14)$

1pm Day 2

Function Composition



Notation

$(f \circ g)(x) = f(g(x))$ ← outer, inner

"f of g of x"

NOT times

$(g \circ f)(x) = g(f(x))$

In general $f(g(x)) \neq g(f(x))$

ex. Find $f(g(x))$ if $f(u) = u^2+4$, $g(x) = x-1$

$$
\begin{aligned}
f(g(x)) &= f(\quad) \\
&= f(x-1) \\
&= (\quad)^2 + 4 \\
&= (x-1)^2 + 4 \\
&= (x-1)(x-1) + 4 \\
&= x^2 - 2x + 1 + 4 \\
&\boxed{= x^2 - 2x + 5}
\end{aligned}
$$

composite function

ex. find $f(5x^2+4x)$ if $f(x) = 2x+3$

$f(5x^2+4x) = 2(5x^2+4x)+3$
$= \boxed{10x^2+8x+3}$

skip if needed

ex. If $f(x) = \sqrt{x+1}$, find $h(x)$ & $g(u)$ so that $f(x) = g(h(x))$ $h(x) = x+1$ $g(u) = \sqrt{u}$

ex. If $f(x) = \dfrac{1}{x^2+1}$, find $h(x)$ & $g(u)$ so that $f(x) = g(h(x))$

Option 1: $h(x) = x^2+1$
$g(u) = \dfrac{1}{u}$

Option 2: $h(x) = x^2$
$g(u) = \dfrac{1}{u+1}$

Option 3: $h(x) = x^2+1$
$g(u) = u^{-1}$

day 1

Difference Quotient (Expression)

$$\frac{f(x+h)-f(x)}{h}$$

* Important to define derivative
* Fundamental concept of calculus

11am Day 2 →
ex. $f(x)=5x$

ex. Find the difference quotient for $f(x)=3-x^2$

$$f(x)=\frac{2}{3x}$$

Pieces

$f(x+h)=3-(\quad)^2$
$=3-(x+h)^2$
$=3-(x+h)(x+h)$
$=3-(x^2+2xh+h^2)$
$=3-x^2-2xh-h^2$

$f(x)=3-x^2$

$$\frac{f(x+h)-f(x)}{h}=\frac{3-x^2-2xh-h^2-(3-x^2)}{h}$$

*Always goes away!

$$=\frac{\cancel{3}-\cancel{x^2}-2xh-h^2-\cancel{3}+\cancel{x^2}}{h}$$

$$=\frac{-2xh-h^2}{h}$$

$$=\frac{\cancel{h}(-2x-h)}{\cancel{h}}$$

can't cancel parts of a sum

$$\frac{8}{5}\overset{?}{=}\frac{5+3}{\cancel{5}}=3 \quad \text{NO!}$$

$$\boxed{=-2x-h}$$

Skip ~~Applications~~

~~#66~~ / page 13 #74 (similar to HW# 78)

74. Manufacturing cost
~~$C(q)=q^2+q+900$~~
~~$q(t)=25t$~~
a) Express cost as a function of time ($C(t)$)
$C(q(t))=625t^2+25t+900$
b) $C(3)=6600$ Sentence
c) ~~11000~~ $=625t^2$
~~$25t^2+t-404=0$~~
~~$x=-\frac{101}{25}$ or $x=4$~~
$t=4$
write out →

Another example of finding difference quotient.

1.1 8

Find the Difference Quotient given
$f(x) = \frac{3}{x+1}$

Scanned & emailed No class time

The Difference Quotient

$$\frac{f(x+h)-f(x)}{h} = \frac{\left[\frac{3}{(x+h)+1}\right] - \frac{3}{x+1}}{h}$$

$$= \frac{\frac{3}{x+h+1} - \frac{3}{x+1}}{h}$$

Now get a common denom & subtract

$$= \frac{\frac{3}{(x+h+1)}\left(\frac{x+1}{x+1}\right) - \frac{3}{(x+1)}\left(\frac{x+h+1}{x+h+1}\right)}{h}$$

parentheses important!

$$= \frac{\frac{3(x+1) - 3(x+h+1)}{(x+h+1)(x+1)}}{h}$$

← instead of dividing by $\frac{h}{1}$, mult. by $\frac{1}{h}$

$$= \frac{\cancel{3x}+\cancel{3} - \cancel{3x} - 3h - \cancel{3}}{(x+h+1)(x+1)} \cdot \frac{1}{h}$$

$$= \frac{-3\cancel{h}}{(x+h+1)(x+1)\cdot \cancel{h}}$$

$$= \boxed{\frac{-3}{(x+h+1)(x+1)}}$$

Don't worry about "wasting" paper. Write big & show steps!! :)

Too similar to the $f(x) = \frac{1}{x}$!

Find the difference quotient

$$f(x) = \frac{3}{x}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{3}{x+h} - \frac{3}{x}}{h}$$

$$= \frac{\frac{3}{(x+h)}\left(\frac{x}{x}\right) - \frac{3}{x}\left(\frac{x+h}{x+h}\right)}{h}$$

$$= \frac{\frac{3x}{x(x+h)} - \frac{3(x+h)}{x(x+h)}}{h}$$

$$= \frac{3x-(3x+3h)}{x(x+h)} \cdot \frac{1}{h}$$

$$= \frac{\cancel{3x}-\cancel{3x}+3h}{x(x+h)} \cdot \frac{1}{h}$$

$$= \frac{3\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}}$$

$$= \boxed{\frac{3}{x(x+h)}}$$

$f(x) = \sqrt{x}$ HW

removed from HW

$g(x) = -5x + 1$

$f(g(x))$

$= f(5x-1)$

$= \sqrt{-5x+1}$

$g(f(x)) = g(\sqrt{x})$

$= -5(\sqrt{x}) + 1$

$= -5\sqrt{x} + 1$

$$\left(\sqrt{-5x+1}\right)^2 = \left(-5\sqrt{x}+1\right)^2$$

$$-5x+1 = 25\sqrt{x}\sqrt{x} + 10\sqrt{x} + 1$$

$$-1$$

$$-5x + \cancel{1} = 25x + 10\sqrt{x}$$

$$\cancel{-1}$$

$$-5x = 25x - 10\sqrt{x}$$

$$-25x \qquad -25x$$

$$\frac{-30x}{-10} = \frac{-10\sqrt{x}}{-10}$$

$$(3x)^2 = (\sqrt{x})^2$$

$$9x^2 = x$$

$$9x^2 - x = 0$$

$$x(9x-1) = 0$$

$x = 0$ $\quad 9x = 1$ $\quad \cancel{x = \frac{1}{9}}$

$$\sqrt{-5\left(\tfrac{1}{9}\right)+1} \stackrel{?}{=} -5\sqrt{\tfrac{1}{9}}+1$$

$$\sqrt{\frac{-5}{9} + \frac{9}{9}} = -5\left(\frac{1}{3}\right) + 1$$

$$\sqrt{\frac{4}{9}} = -\frac{5}{3} + \frac{3}{3}$$

$$\frac{2}{3} = \frac{-2}{3} \quad \text{No}$$

$\cancel{0.4(8+0.2t^2)+1}$

$\cancel{0.4 \cdot 8 + 0.8t^2 + 1}$