

Factoring Patterns

$$a^2 - b^2 = (a + b)(a - b)$$

$$\text{Example: } 9y^2 - 25x^2 = (3y + 5x)(3y - 5x)$$

$$a^2 + b^2 = \text{Prime}$$

$$\text{Example: } 9y^2 + 25x^2 = \text{Prime}$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\begin{aligned}\text{Example: } 8x^3 + 125y^3 &= (2x)^3 + (5y)^3 \\ &= (2x + 5y)((2x)^2 - (2x)(5y) + (5y)^2) \\ &= (2x + 5y)(4x^2 - 10xy + 25y^2)\end{aligned}$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\begin{aligned}\text{Example: } 27r^3 - 64s^3 &= (3r)^3 - (4s)^3 \\ &= (3r - 4s)((3r)^2 + (3r)(4s) + (4s)^2) \\ &= (3r - 4s)(9r^2 + 12rs + 16s^2)\end{aligned}$$

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$\text{Example: } 4x^2 + 12xy + 9y^2 = (2x + 3y)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$\text{Example: } 16x^2 - 40xy + 25y^2 = (4x - 5y)^2$$

Factoring Hints

1) Always “take out” common factors

$$2x^3 - 8x^2 + 6x = 2x(x^2 - 4x + 3) \quad \text{--Note: this can be factored again}$$

2) Count the number of terms:

Two Terms: Use Formula if possible (think squares or cubes)

$$1) x^2 - y^2 = (x - y)(x + y)$$

$$\text{example: } 4x^2 - 9 = (2x - 3)(2x + 3)$$

$$2) x^2 + y^2 = \text{Prime}$$

$$3) x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$\text{example: } 8x^3 - 27y^3 = (2x - 3y)(4x^2 + 6xy + 9y^2)$$

$$4) x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$\text{example: } 64x^3 + 1 = (4x + 1)(16x^2 - 4x + 1)$$

squares

cubes

Three Terms: Trial and Error

$$\text{Example: } 2x^3 - 8x^2 + 6x$$

$$2x(x^2 - 4x + 3)$$

– step 1 above ("take out" common factors)

$$2x(\underline{x \quad})(\underline{x \quad})$$

– split up the x^2

$$2x(\underline{x - \quad})(\underline{x - \quad})$$

– decide on the signs

$$2x(\underline{x - 3})(\underline{x - 1})$$

– split up the 3 (the last term)

check with FOIL

Four Terms: Grouping

$$\text{Example: } 6x^2 + 3x - 8x - 4$$

$$(6x^2 + 3x) - (8x + 4) \quad \text{– a negative in middle means change 2nd sign}$$

$$3x(2x + 1) - 4(2x + 1) \quad \text{– factor each group separately}$$

$$(3x - 4)(2x + 1) \quad \text{– factor out the } (2x + 1) \text{ [check with FOIL]}$$

$$\text{Example: } 5x^2 + 2x + 10x + 4$$

$$(5x^2 + 2x) + (10x + 4) \quad \text{– divide into 2 groups}$$

$$x(5x + 2) + 2(5x + 2) \quad \text{– factor each group separately}$$

$$\underline{x(5x + 2)} + \underline{2(5x + 2)} \quad \text{– factor out the } (5x + 2)$$

$$(x + 2)(5x + 2) \quad \text{– check with FOIL}$$

3) Always check to be sure each factor is prime! $x^4 - 81 = (x^2 - 9)(x^2 + 9)$

$$\text{But } (x^2 - 9) = (x + 3)(x - 3), \text{ so } x^4 - 81 = (x + 3)(x - 3)(x^2 + 9)$$