Summary of Factoring Techniques

Factoring the Greatest Common Factor from a Polynomial

Example:	$24a^3b^2 - 4a^2b^2 - 16a^2b^4$	1. Find the G.C.F of all terms. G.C.F = $4a^2b^2$
	$= 4a^2b^2\left(6a - 1 - 4b^2\right)$	2. Factor the GCF from each term of the polynomial.

Factoring by Grouping

Example #1	$3(x+y) + a(x+y)$	1. Both terms have a factor of $(x+y)$.
	$= (x+y)(3+a)$	2. Factor out $(x+y)$ from each term.

Example #2	$a^2b + 3a^2 + 2b + 6$	
	$= \underline{a^2b + 3a^2} + \underline{2b + 6}$	1. Group with parentheses the 1st two terms and the last two terms.
	$= a^2(b+3) + 2(b+3)$	2. Factor out the GCF from each group. Notice: Both terms have a factor of $(b+3)$.
	$= (b+3)(a^2+2)$	3. Factor out $(b+3)$ from each term.

Factoring a Trinomial of the Form $x^2 + bx + c$ (Leading coefficient is 1)

Example #1 $\quad x^2 + 12x + 20$

1. What 2 numbers: MULTIPLY to = 20 and ADD to = 12 ???

$$\text{Factors of 20: } \begin{cases} 1 \cdot 20 \to 1 + 20 = 21 \\ 2 \cdot 10 \to 2 + 10 = 12 \\ 4 \cdot 5 \to 4 + 5 = 9 \end{cases}$$

← 2 and 10!!!! (beside $2 \cdot 10 \to 2 + 10 = 12$)

2. List the factors of 20 and check the sums.
3. Factor.

$= (x+2)(x+10)$

Example #2 $\quad x^2 - 15x + 56$

1. What 2 numbers: MULTIPLY to = 56 and ADD to = –15 ???

$$\text{Factors of 56: } \begin{cases} (-1)(-56) \to \\ (-2)(-28) \to \\ (-4)(-14) \to \\ (-7)(-8) \to (-7) + (-8) = -15 \end{cases}$$

← –7 and –8!!!! (beside $(-7)(-8) \to (-7) + (-8) = -15$)

2. List the factors of 56 and check the sums.
3. Factor.

$= (x-7)(x-8)$

Example #3 $\quad x^2 + 2x - 35$ $\qquad$ Steps 1, 2, 3 from above.

$7 \cdot (-5) = -35, \; \textit{and} \; 7 + (-5) = 2$ $\qquad$ ←The numbers are 7 and –5.

$= (x+7)(x-5)$

Factoring a Trinomial of the Form $ax^2 + bx + c$, $a \neq 1$ (Leading coefficient is **not** 1)

Method #1: The "AC" method

Example: $2x^2 + 7x - 4$

1. Multiply 2 and –4 = –8.
2. What 2 numbers: MULTIPLY to = –8 and ADD to = 7 ???

$(8)(-1) = -8, \quad and\ 8 + (-1) = 7$

The numbers are 8 and –1.

$= 2x^2 - 1x + 8x - 4$

$= \overline{2x^2 - 1x} + \overline{8x - 4}$

$= x(2x - 1) + 4(2x - 1)$

$= (2x - 1)(x + 4)$

3. Split up the 7x term → $-1x + 8x$
4. Group the 1st 2 terms and the last 2 terms.
5. Factor out the GCF from each group.
6. Factor out the common factor of $(2x - 1)$.

Method #2: Factoring by "Trial and Error"

Example: $2x^2 + 7x - 4$

1. Constant term is negative
2. List factors of 2 and –4.

1, 2

1, – 4
4, –1
2, –2
–2, 2

3. Create TRIAL factors and FOIL out to pick the CORRECT factorization.

$(1x + 1)(2x - 4) = 2x2 - 4x + 2x - 4 \neq$ *original polynomial*

$(1x - 1)(2x + 4) = 2x^2 + 4x - 2x - 4 \neq$ *original polynomial*

$(x + 4)(2x - 1) = 2x^2 - 1x + 8x - 4 = 2x^2 + 7x - 4$

and so on…

$= (x + 4)(2x - 1)$ is the correct factorization.

Special Factoring

Difference of 2 squares: $A^2 - B^2 = (A - B)(A + B)$

Example: $x^2 - 49$

$= x^2 - 7^2$

$= (x - 7)(x + 7)$

1. Let $A = x$, $B = 7$
2. Factor using the rule of difference of squares.

Perfect Square trinomial: $A^2 + 2AB + B^2 = (A + B)(A + B) = (A + B)^2$

$A^2 - 2AB + B^2 = (A - B)(A - B) = (A - B)^2$

Example: $x^2 - 10x + 25$

$= x^2 - 2 \cdot 5 \cdot x + 5^2$

$= (x + 5)(x + 5) = (x + 5)^2$

1. Let $A = x$, $B = 5$
2. Factor using the rule of perfect sq. trinomial.