## Summary of Factoring Techniques

### Factoring the Greatest Common Factor from a Polynomial

**Example:** $24a^2b^2 - 4a^2b^2 - 16a^2b^4$

1. Find the G.C.F of all terms. 
   \[ \text{G.C.F} = 4a^2b^2 \]
2. Factor the G.C.F from each term of the polynomial.

### Factoring by Grouping

**Example #1**

1. Both terms have a factor of $(x + y)$.
2. Factor out $(x + y)$ from each term.

**Example #2**

1. Group with parentheses the 1st two terms and the last two terms.
2. Factor out the G.C.F from each group. 
   Notice: Both terms have a factor of $(b + 3)$.
3. Factor out $(b + 3)$ from each term.

### Factoring a Trinomial of the Form $x^2 + bx + c$ (Leading coefficient is 1)

**Example #1**

1. What 2 numbers: MULTIPLY to $= 20$ and ADD to $= 12$ ???
   
   Factors of 20: $egin{cases} 1 \cdot 20 \rightarrow 1 + 20 = 21 \\ 2 \cdot 10 \rightarrow 2 + 10 = 12 \leftarrow 2 \text{ and } 10!!!! \\ 4 \cdot 5 \rightarrow 4 + 5 = 9 \end{cases}$
2. List the factors of 20 and check the sums.
3. Factor.

**Example #2**

1. What 2 numbers: MULTIPLY to $= 56$ and ADD to $= -15$ ???
   
   Factors of 56: $egin{cases} (-1)(-56) \rightarrow \\ (-2)(-28) \rightarrow \\ (-4)(-14) \rightarrow \\ (-7)(-8) \rightarrow (-7) + (-8) = -15 \leftarrow -7 \text{ and } -8!!!! \end{cases}$
2. List the factors of 56 and check the sums.
3. Factor.

**Example #3**

1. Steps 1, 2, 3 from above.
   $7 \cdot (-5) = -35$, \text{ and } $7 + (-5) = 2 \leftarrow$ The numbers are 7 and $-5$.
2. Factor.

\[ = (x + 7)(x - 5) \]
Factoring a Trinomial of the Form \( ax^2 + bx + c \), \( a \neq 1 \) (Leading coefficient is **not** 1)

**Method #1**: The “AC” method

Example: \( 2x^2 + 7x - 4 \)

1. Multiply 2 and \(-4 = -8\).
2. What 2 numbers: MULTIPLY to \(-8\) and ADD to \(7\)???
   \((8)(-1) = -8, \text{ and } 8 + (-1) = 7\)
   The numbers are 8 and \(-1\).
3. Split up the 7x term \(\rightarrow -1x + 8x\)
4. Group the 1\(^{st}\) 2 terms and the last 2 terms.
5. Factor out the GCF from each group.
6. Factor out the common factor of \((2x - 1)\).

\[
\begin{align*}
2x^2 - 1x + 8x - 4 \\
= 2x^2 - 1x + 8x - 4 \\
= x(2x - 1) + 4(2x - 1) \\
= (2x - 1)(x + 4)
\end{align*}
\]

**Method #2**: Factoring by “Trial and Error”

Example: \( 2x^2 + 7x - 4 \)

1. Constant term is negative
2. List factors of 2 and \(-4\).

\[
\begin{align*}
1, 2 & \quad 1, -4 \\
4, -1 & \quad 2, -2 \\
-2, 2 & \quad \text{Create TRIAL factors and FOIL out to pick the CORRECT factorization.}
\end{align*}
\]

\[
\begin{align*}
(x + 1)(2x - 4) &= 2x^2 - 4x + 2x - 4 \neq \text{original polynomial} \\
(x - 1)(2x + 4) &= 2x^2 + 4x - 2x - 4 \neq \text{original polynomial} \\
(x + 4)(2x - 1) &= 2x^2 - 1x + 8x - 4 = 2x^2 + 7x - 4 \\
\text{and so on} & \quad \text{is the correct factorization.}
\end{align*}
\]

**Special Factoring**

**Difference of 2 squares**: \( A^2 - B^2 = (A - B)(A + B) \)

Example: \( x^2 - 49 \)

1. Let \( A = x, B = 7 \)
2. Factor using the rule of difference of squares.

\[
\begin{align*}
x^2 - 49 \\
= x^2 - 7^2 \\
= (x - 7)(x + 7)
\end{align*}
\]

**Perfect Square trinomial**: \( A^2 + 2AB + B^2 = (A + B)(A + B) = (A + B)^2 \)

\[
A^2 - 2AB + B^2 = (A - B)(A - B) = (A - B)^2
\]

Example: \( x^2 - 10x + 25 \)

1. Let \( A = x, B = 5 \)
2. Factor using the rule of perfect sq. trinomial.

\[
\begin{align*}
x^2 - 10x + 25 \\
= x^2 - 2 \cdot 5 \cdot x + 5^2 \\
= (x + 5)(x + 5) = (x + 5)^2
\end{align*}
\]