Perhaps, the process of factoring by removing the greatest common factor can be best stated as the reverse distributive property. In the distributive property, one is multiplying a certain factor to all of the terms. In factoring by GCF, one is dividing all of the terms by the GCF.

Consider this expression which utilizes the distributive property: \( 5x^2(4x^4 + 3) \).

Visually, this is the distributive process: \( 5x^2(4x^4 + 3) \).

To simplify using the distributive property, one multiplies \( 5x^2 \) times \( 4x^4 \), and then one multiplies \( 5x^2 \) times 3.

After simplifying using the distributive property, you get \( 20x^6 + 15x^2 \).

This section will now demonstrate how to factor by removing the GCF.

Let's now take your answer to the problem above: \( 20x^6 + 15x^2 \).

Using what was learned in the last lesson, the GCF of \( 20x^6 \) and \( 15x^2 \) is \( 5x^2 \).

Recall - this is because the greatest common factor of 20 and 15 is 5, and because the GCF of like variable quantities is always the lowest exponent.

Now, divide each term in the original expression by the GCF (\( 5x^2 \)). Divide \( 20x^6 \) by \( 5x^2 \), and divide \( 15x^2 \) by \( 5x^2 \).

Therefore, after dividing by the GCF, the expression is \( 4x^4 + 3 \).

To complete this reverse distributive process, write the GCF in front of a set of parentheses. Inside of the parentheses, place the expression that is left after dividing by the GCF.

\[
\begin{align*}
\text{GCF} & \quad \text{what's left after dividing} \\
5x^2 & \quad (4x^4 + 3)
\end{align*}
\]

So, after factoring by removing the GCF, the answer is \( 5x^2(4x^4 + 3) \). Note how this is the original question before distributing at the very top of the page.

Factor the greatest common factor: \( 8y^5 - 12y^3 + 4y \).

The GCF is of the three terms is \( 4y \), because the GCF of 8, 12, and 4 is 4, and the GCF of \( y^5 \), \( y^3 \), and \( y \) is \( y \). So, the GCF (\( 4y \)) will be placed in front of the parentheses, and all of the terms in the expression will be divided by \( 4y \).
\[8y^5 - 12y^3 + 4y\]
\[+ 4y\quad + 4y\quad + 4y\]
\[= 4y\left(2y^4 - 3y^2 + 1\right)\]

Therefore, the answer is \[4y(2y^4 - 3y^2 + 1)\].

Generating the last term in this expression is where many students make a mistake. In order to get "+1", one has to divide \(4y\) by \(4y\). Some students would think this is zero, and they would not write anything. However, it's important to see that \(4y + 4y = 1\).

Factor the greatest common factor: \(14z^8 + 24z^7 - 30z^3\).

First, the GCF of all three terms is \(2z^3\). Now, divide each of the terms by \(2z^3\).

\[14z^8 + 24z^7 - 30z^3\]
\[+2z^3\quad +2z^3\quad +2z^3\]
\[= 2z^3(7z^5 + 12z^4 - 15)\]

The answer is \[2z^3(7z^5 + 12z^4 - 15)\]

Factor the greatest common factor: \(16c^7 - 6c^3\).

The GCF is \(2c^3\). Now, you complete the problem below:

\[16c^7 - 6c^3\]
\[+\quad +\]
\[\downarrow\quad \downarrow\]

\[\frac{(\quad -)}{GCF}\quad \text{what's left after dividing}\]

For Questions 1-2, factor the greatest common factor.

1. \(25d^5 + 45d^4\)
2. \(9k^4 + 12k^3 - 6k\)
Notes on Factoring by GCF - Page II

Factor the greatest common factor: \(28a^3b^2 - 36a^2 - 17b^5\).

Note that the GCF of the coefficients (28, -36, and -17) is 1. Also, note that the terms do not all share any common variables.

Obviously, it makes little sense to write \(1(28a^3b^2 - 36a^2 - 17b^5)\).

When one is only factoring out the greatest common factor, and the GCF is 1, he/she should write that the expression is PRIME.

Homework on Factoring by Greatest Common Factor

Factor the greatest common factor out of the polynomial. If the GCF is 1, write PRIME.

1. \(8x^2 + 10x\)  
2. \(12y - 16\)  
3. \(-15d^5 + 45d^3\)

4. \(13a + 20b\)  
5. \(c^3 + c^2 - c\)  
6. \(6n^2 - 30n + 42\)

7. \(-7m^2 - 10m + 17\)  
8. \(18p^3 - 63p^2 - 9p\)  
9. \(18x^2 - 50y^2\)

10. \(100z^9 + 50z^6 - 75z^5\)  
11. \(36rs^2 - 108r^2s^3\)  
12. \(36k - 30\)

13. \(a^7b - a^{10}\)  
14. \(2c^5d^4 - 3c^4 + 4c^3\)  
15. \(3g^8 + 3g^7\)

16. \(18x^5 - 48x^4 + 56x^3 - 86x\)  
17. \(23y^{10} - 46y^7 + 68y^2 + 10y\)

---

1. \(2x(4x + 5)\)  
2. \(4(3y - 4)\)  
3. \(15d^3(-d^2 + 3)\) or \(-15d^3(d^2 - 3)\)

4. PRIME  
5. \(c(c^2 + c - 1)\)  
6. \(6(n^2 - 5n + 7)\)  
7. PRIME

8. \(9p(2p^2 - 7p - 1)\)  
9. \(2(9x^2 - 25y^2)\)  
10. \(25z^5(4z^4 + 2z - 3)\)

11. \(36rs^2(1 - 3rs)\)  
12. \(6(6k - 5)\)  
13. \(a^7(b - a^3)\)

14. \(c^3(2c^2d^4 - 3c + 4)\)  
15. \(3g^7(g + 1)\)  
16. \(2x(9x^4 - 24x^3 + 28x^2 - 43)\)

17. \(y(23y^9 - 46y^6 + 68y + 10)\)