

The localized dynamics of a Ca^{2+} channel (10 minute-talk)

Bori Mazzag
Graduate Seminar
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This work was done in collaboration with:

Christopher Tignanelli

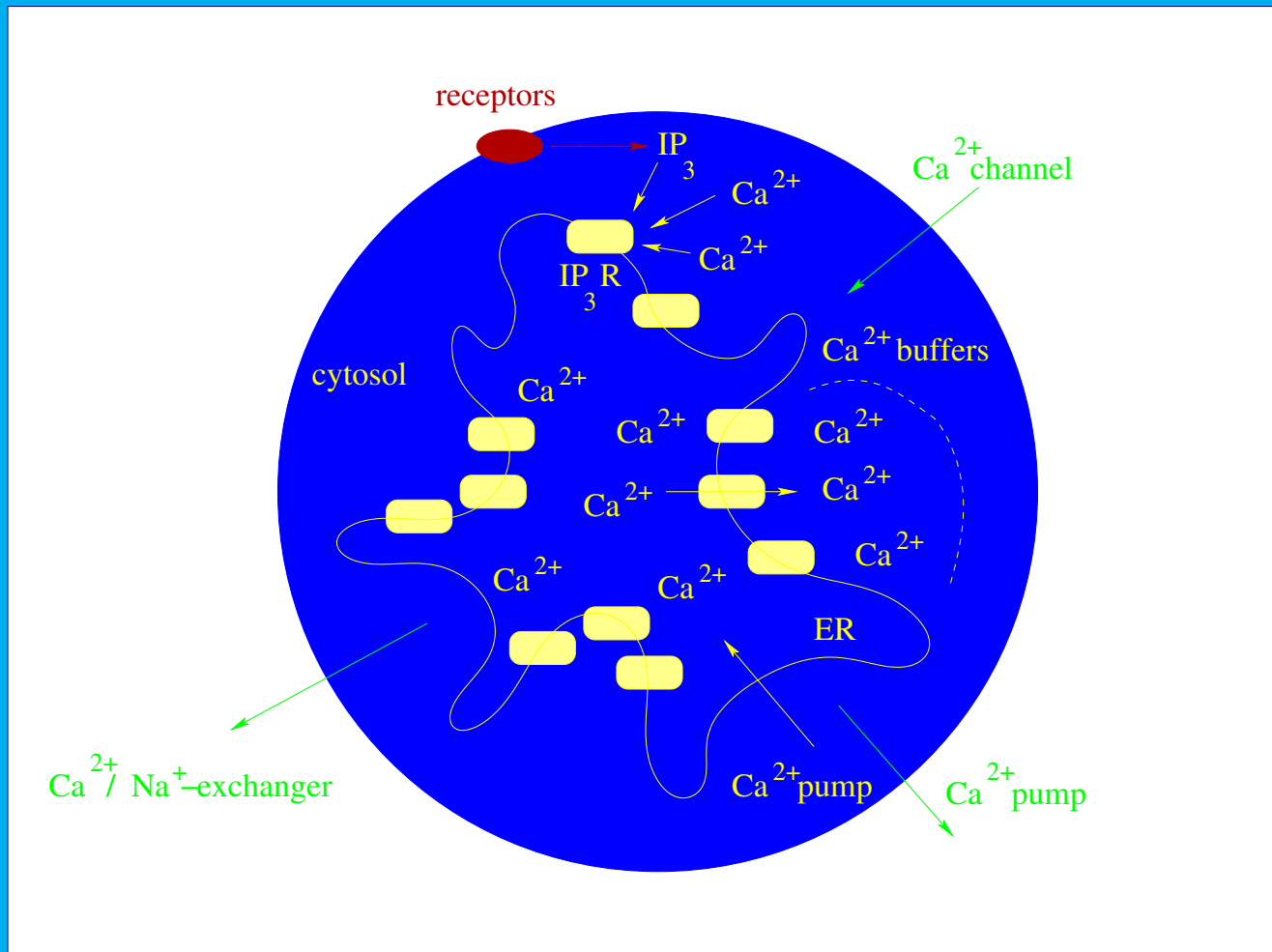
&

Gregory D. Smith

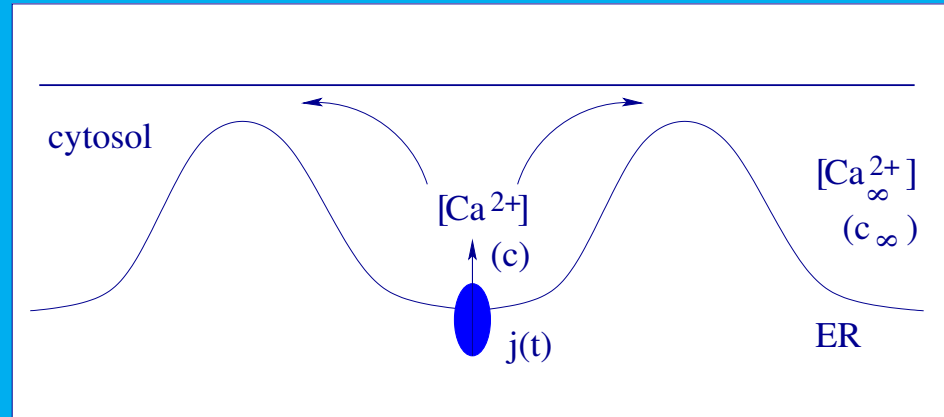
Applied Science Department

College of William and Mary

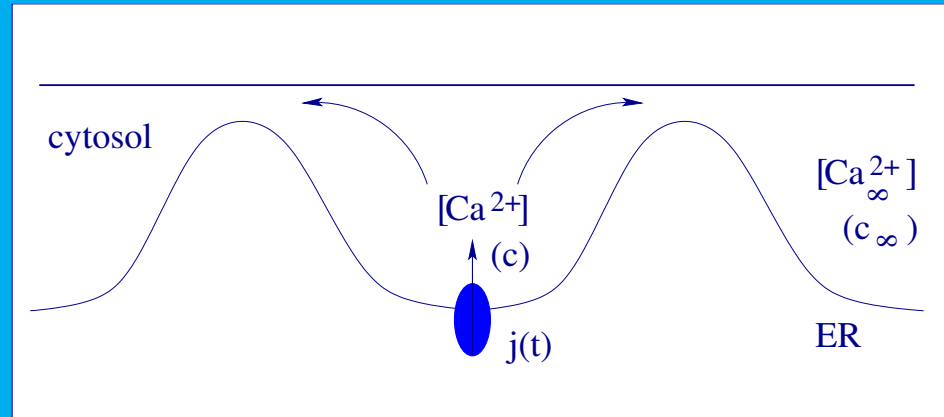
Ca^{2+} signalling in cells



Simple model – Ca^{2+} dynamics

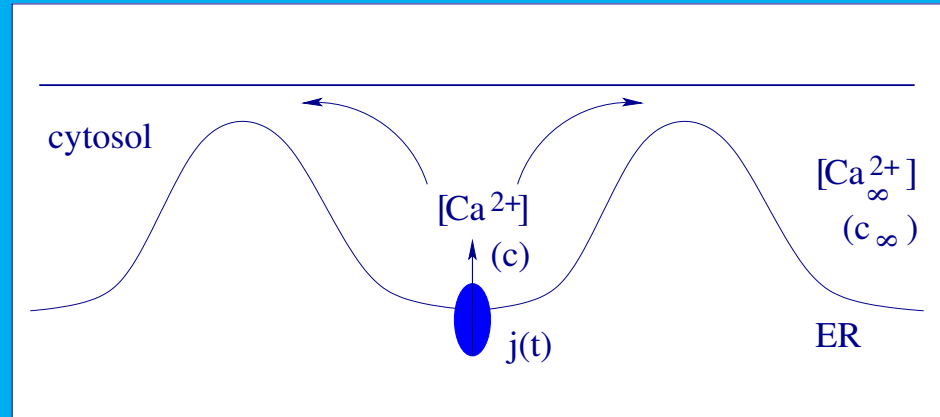


Simple model – Ca^{2+} dynamics



$$\frac{dc}{dt} = j - \frac{c - c_\infty}{\tau}$$

Simple model – Ca^{2+} dynamics

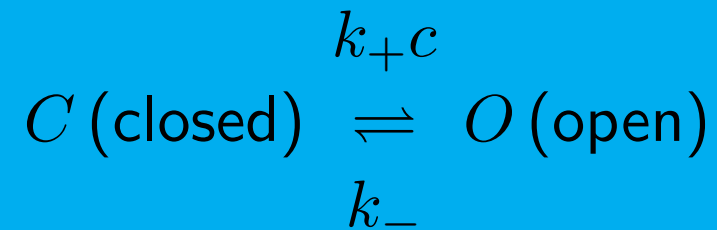


$$\frac{dc}{dt} = j - \frac{c - c_\infty}{\tau} \quad j(t) = \begin{cases} 0 & \text{when } S(t) = C \text{ (closed)} \\ j_0 & \text{when } S(t) = O \text{ (open)} \end{cases}$$

$$\text{with } j_0 = \frac{c_{ss} - c_\infty}{\tau}$$

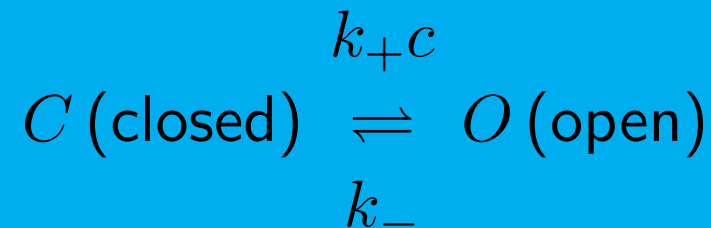
Channel description

Stochastically gating Ca^{2+} -activated channel:



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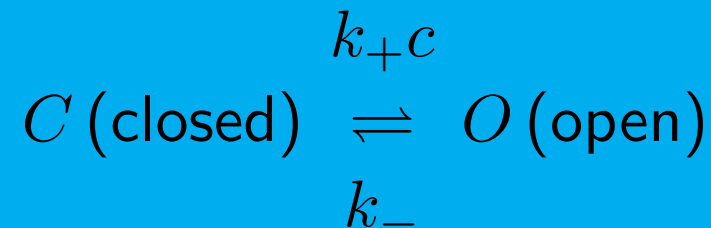


Transition probability matrix:

$$W = \begin{bmatrix} \Pr\{C, t + \Delta t | C, t\} & \Pr\{O, t + \Delta t | C, t\} \\ \Pr\{C, t + \Delta t | O, t\} & \Pr\{O, t + \Delta t | O, t\} \end{bmatrix}$$

Channel description

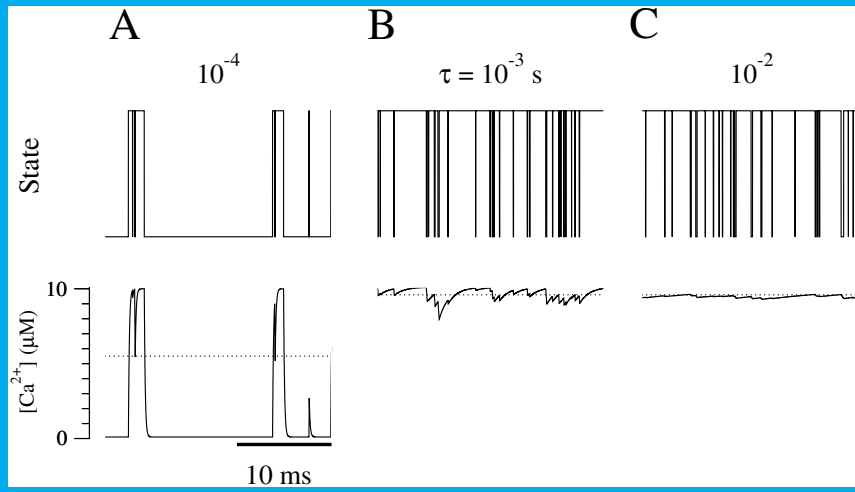
Stochastically gating Ca^{2+} -activated channel:



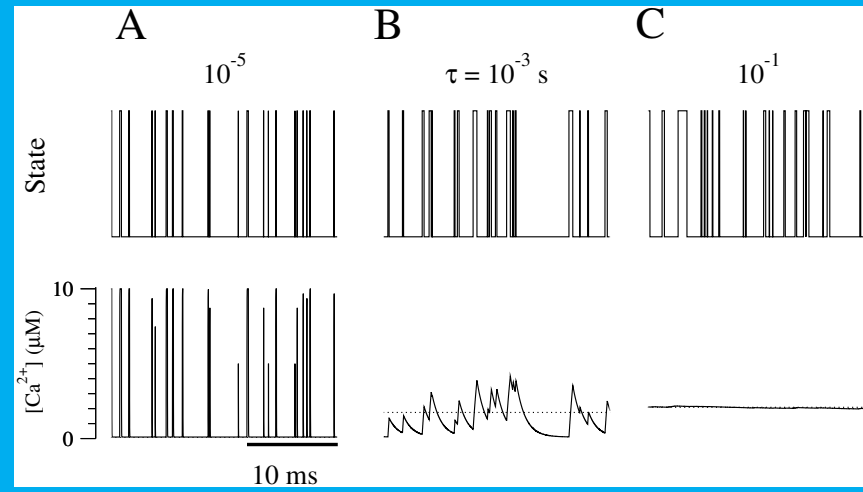
Transition probability matrix:

$$\begin{aligned} W &= \begin{bmatrix} \Pr\{C, t + \Delta t | C, t\} & \Pr\{O, t + \Delta t | C, t\} \\ \Pr\{C, t + \Delta t | O, t\} & \Pr\{O, t + \Delta t | O, t\} \end{bmatrix} \\ &= \begin{bmatrix} 1 - k_+c\Delta t & k_+c\Delta t \\ k_-\Delta t & 1 - k_-\Delta t \end{bmatrix} \end{aligned}$$

Monte Carlo simulation results

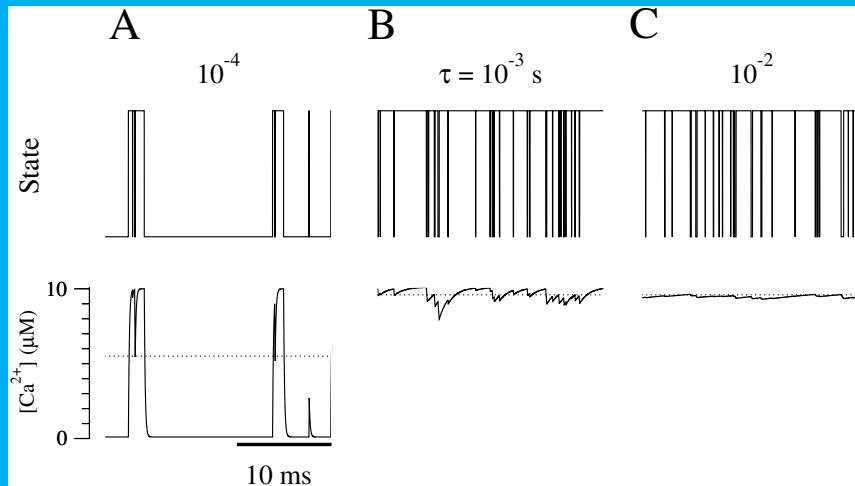


Ca^{2+} -activated channel

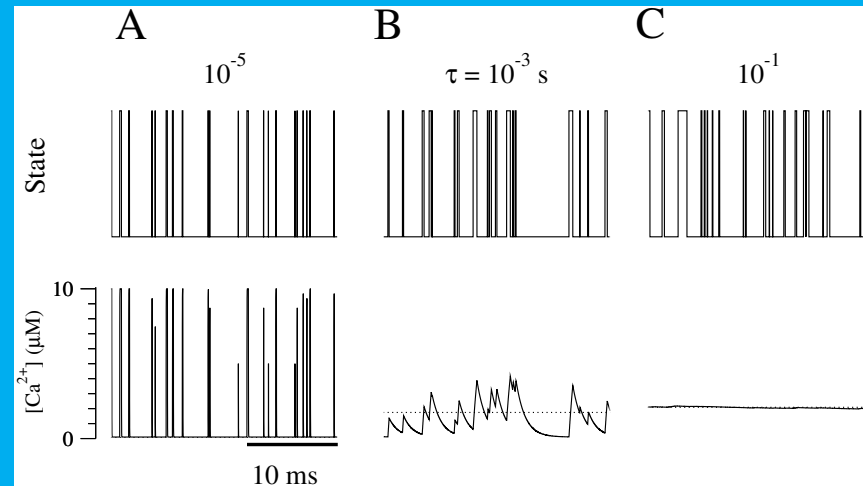


Ca^{2+} -inactivated channel

Monte Carlo simulation results



Ca^{2+} -activated channel

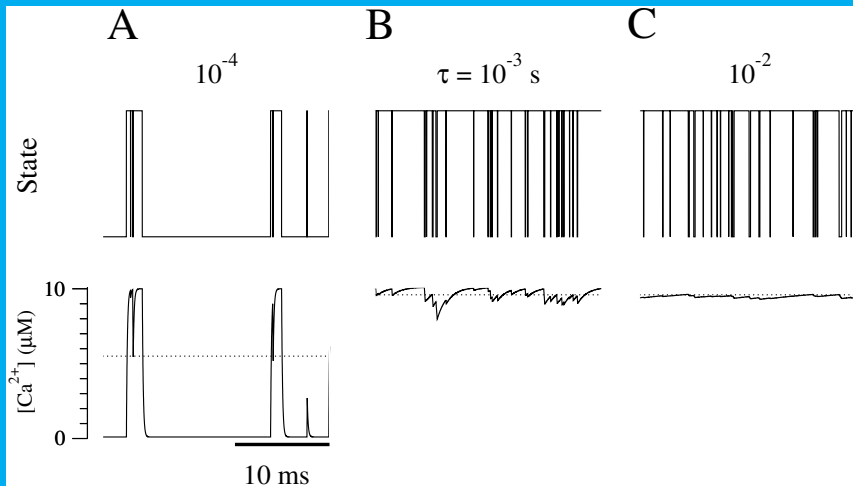


Ca^{2+} -inactivated channel

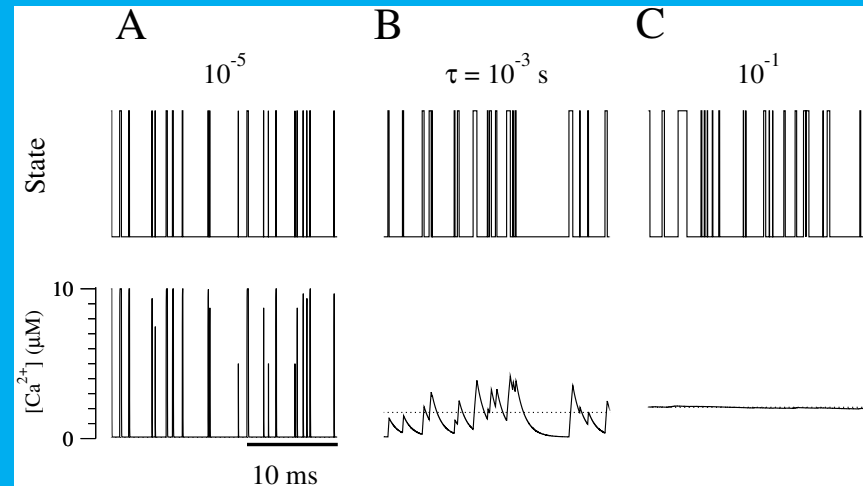
Fast domain (small τ):

$$p_{open} = \frac{k_+ c_\infty}{k_+ c_\infty + k_-} = \frac{c_\infty}{c_\infty + K}$$

Monte Carlo simulation results



Ca^{2+} -activated channel



Ca^{2+} -inactivated channel

Fast domain (small τ):

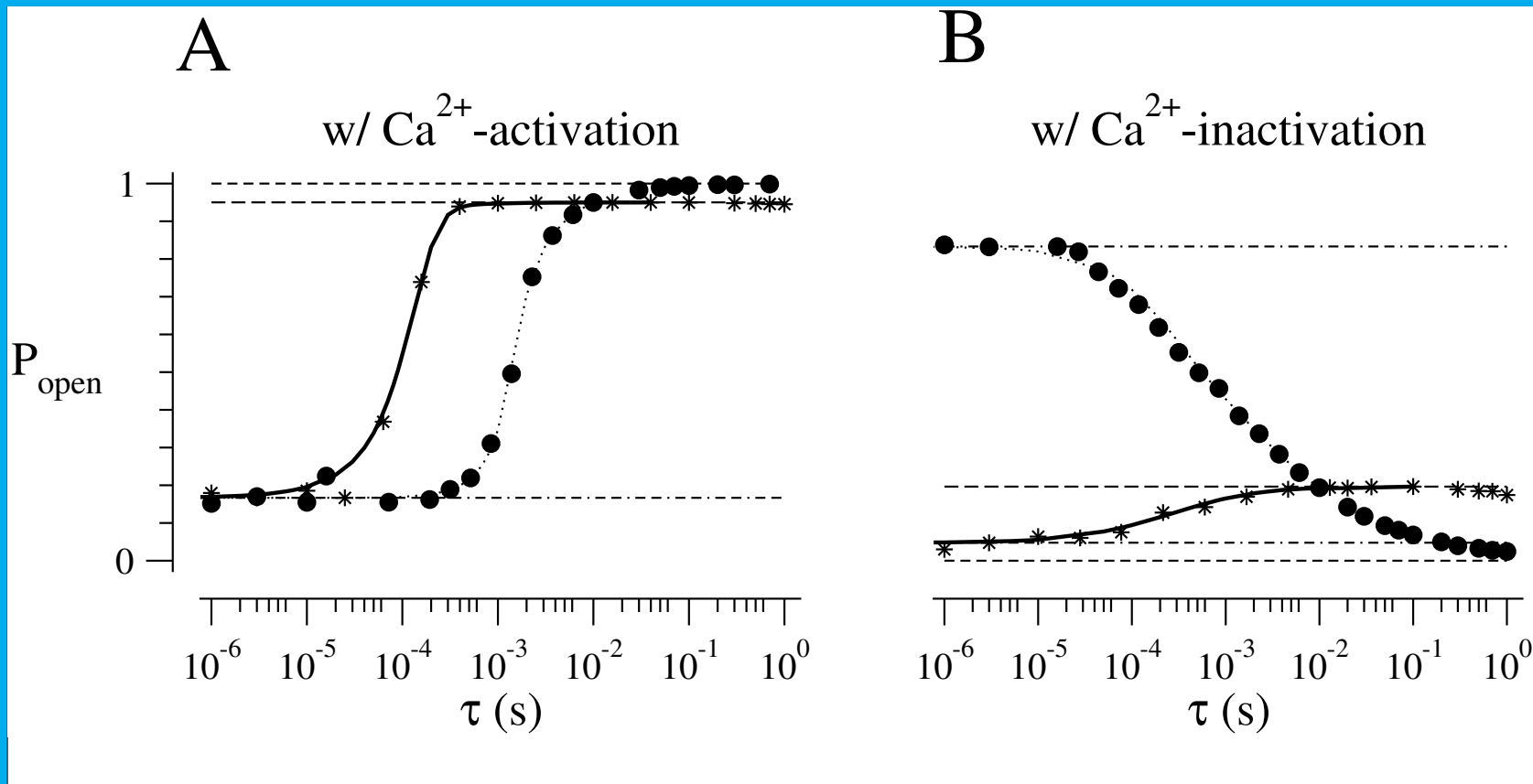
$$p_{open} = \frac{k_+ c_\infty}{k_+ c_\infty + k_-} = \frac{c_\infty}{c_\infty + K}$$

Slow domain (large τ):

$$p_{open} = \frac{c_*}{c_* + K}$$

$$c_* = c_\infty(1 - p_{open}) + c_{ss}p_{open}.$$

Dependence of p_{open} on τ



Single channel in a dynamics Ca^{2+} -domain

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- Results: understanding the dependence of p_{open} on the temporal scales of the problem

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 - ★ p_{open} increases as domain becomes slow compared to channel gating
 - ★ More complicated behavior when channel model contains two different time scales
 - ★ Qualitatively similar behavior for PDE domain

Thank you!

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